A Five-Equation Dissipative Model for Two-Phase Flows

Mathieu Labois*, Hervé Guillard, Marc Grandotto***

** DEN/DER/SSTH/LMDL, CEA Cadarache, F-13108 Saint-Paul-lez-Durance*

*** Projet SMASH, INRIA Sophia-Antipolis, F-06902 Sophia-Antipolis*

ABSTRACT

This work deals with the design of a five-equation dissipative model for the simulation of two-phase flows and its numerical approximation. It is derived from the standard six-equation two-fluid model [1] using a first order Chapman-Enskog expansion technique [2]. Dissipative terms arise in this expansion and enable the model to deal with velocity disequilibriums, even if it features only one velocity. A finite-volume numerical approximation of this model using a fractional step approach is proposed. First, a convective step is performed, taking into account the hyperbolicity of the convective part, then the resolution of the dissipative terms is performed. Different numerical tests are proposed, where the capability of the model to deal with flow featuring phenomena due to velocity disequilibriums is shown.

Keywords: Two-phase flows, Hyperbolic models, Chapman-Enskog expansion, Velocity disequilibriums

One-pressure, two-velocity model [1]

$$\begin{aligned}
\partial_t \alpha_1\rho_1 &+ \mathrm{div}(\alpha_1\rho_1\underline{u_1}) &&= 0\\
\partial_t \alpha_2\rho_2 &+ \mathrm{div}(\alpha_2\rho_2\underline{u_2}) &&= 0\\
\partial_t \alpha_1\rho_1\underline{u_1} &+ \mathrm{div}(\alpha_1\rho_1\underline{u_1}\otimes\underline{u_1}) + \alpha_1\nabla p &&= \underline{M}^d\\
\partial_t \alpha_2\rho_2\underline{u_2} &+ \mathrm{div}(\alpha_2\rho_2\underline{u_2}\otimes\underline{u_2}) + \alpha_2\nabla p &&= -\underline{M}^d\\
\partial_t \alpha_1\rho_1 e_1 &+ \mathrm{div}(\alpha_1(\rho_1 e_1 + p)\underline{u_1}) &&= p\partial_t\alpha_1 + \underline{u}_I.\underline{M}^d\\
\partial_t \alpha_2\rho_2 e_2 &+ \mathrm{div}(\alpha_2(\rho_2 e_2 + p)\underline{u_2}) &&= p\partial_t\alpha_1 - \underline{u}.\underline{M}^d
\end{aligned}$$

Where the drag force is modelled by $\underline{M}^d = \frac{\rho}{\varepsilon}(\underline{u}_2 - \underline{u}_1)$

This model is not hyperbolic.

⇒ Research of alternative, hyperbolic models for the simulation of two-phase flows

Chapman-Enskog expansion [2]

Noting the state vector $\underline{W}\in\mathbb{R}^N$, the system is written $\partial_t \underline{W} + A(\underline{W})\partial_x \underline{W} = \frac{R(\underline{W})}{\varepsilon} + S(\underline{W})$

Using a parameterization $\underline{W}\in\mathbb{R}^N$, with $\underline{w}\in\mathbb{R}^n$, of the space $\{\underline{W}\in\mathbb{R}^N,\ R(\underline{W})=0\}$, the state vectors writes $\underline{W} = M(\underline{w}) + \varepsilon\underline{V}$ with $\underline{V}\in Rng\left(R'\left(M(\underline{w})\right)\right)$.

Assuming $Rng\left(R'\left(M(\underline{w})\right)\right)\oplus ker\left(R'\left(M(\underline{w})\right)\right)=\mathbb{R}^N$, the projection matrices P on $Rng\left(R'\left(M(\underline{w})\right)\right)$ and Q on $Rng\left(R'\left(M(\underline{w})\right)\right)$ are determined.

Thus, the two following systems are obtained

- $Q A(M(\underline{w}))\partial_x M(\underline{w}) - Q R'(M(\underline{w}))\underline{V} = Q S(M(\underline{w}))$
- $\partial_t \underline{w} + P A(M(\underline{w}))\partial_x M(\underline{w})$
  $+\varepsilon P\left(\partial_t \underline{V} + A(M(\underline{w}))\partial_x \underline{V} + \left[\partial_{\underline{w}_i}\underline{V}_i\right]\partial_x M(\underline{w}) - 1/2\,R''(M(\underline{w}))(\underline{V},\underline{V})\right) =$
  $P S(M(\underline{w})) + P S'(M(\underline{w}))\underline{V} + O(\varepsilon^2)$

The first equation has a unique solution $\underline{V}$, and introducing this result in the last equation, we obtain a reduced system with second-order terms.

One-pressure, one-velocity model with dissipative terms

From the six-equation model and taking $R(\underline{W}) = \underline{M}^d$, the following system is obtained, with $\underline{u_r} = \varepsilon\frac{\alpha_1\alpha_2}{\rho}(\rho_1-\rho_2)\nabla p$

$$\begin{aligned}
\frac{\partial\alpha_1\rho_1}{\partial t} &+ div\,\alpha_1\rho_1\underline{u} &&= -div(\rho Y_1 Y_2 \underline{u_r}) && +O(\varepsilon_u^2)\\
\frac{\partial\alpha_2\rho_2}{\partial t} &+ div\,\alpha_2\rho_2\underline{u} &&= div(\rho Y_1 Y_2 \underline{u_r}) && +O(\varepsilon_u^2)\\
\frac{\partial\rho\underline{u}}{\partial t} &+ div(\rho\underline{u}\otimes\underline{u}) + \nabla p &&= \rho\underline{g} && +O(\varepsilon_u^2)\\
\frac{\partial\rho e}{\partial t} &+ div((\rho e + p)\underline{u}) &&= div((h_2-h_1)\rho Y_1 Y_2\underline{u_r}) + \rho\underline{u}.\underline{g} && +O(\varepsilon_u^2)\\
(\alpha_1 C_2 + \alpha_2 C_1)\frac{D\alpha_1}{Dt} &+ \alpha_1\alpha_2(C_1 - C_2)div\,\underline{u} &&= -\alpha_1\alpha_2(C_1 div(Y_2\underline{u_r}) + C_2 div(Y_1\underline{u_r})) &&\\
& && \quad +(Y_1\alpha_1 C_2 - Y_2\alpha_2 C_1)\underline{u_r}.\nabla\alpha_1 &&\\
& && \quad -\left(\frac{\alpha_1\alpha_2}{\rho}\right)^2(\rho_1-\rho_2)\left(1+\left(\frac{\alpha_1 Y_1}{\rho_2\kappa_2}-\frac{\alpha_2 Y_2}{\rho_1\kappa_1}\right)\frac{(\rho_1-\rho_2)}{\rho}\right)\|\nabla p\|^2 && +O(\varepsilon_u^2)
\end{aligned}$$

Properties

- The system is *unconditionnaly hyperbolic* $\lambda_1 = u - \hat{c}$ $\quad\lambda_2=\lambda_3=\lambda_4=u$ $\quad\lambda_5 = u+\hat{c}$ $\quad$ where $\frac{1}{\rho\hat{c}^2} = \frac{\alpha_1}{\rho_1 c_1^2} + \frac{\alpha_2}{\rho_2 c_2^2}$

- The system is *dissipative*

$$\frac{\partial\rho Y_1 s_1}{\partial t} + div\left(\rho Y_1 s_1\left(\underline{u} + Y_2\underline{u}_r\right)\right) = \varepsilon_u\frac{\rho_1 Y_2}{T_1}\left(\frac{Y_1-\alpha_1}{\rho}\right)\|\nabla p\|^2$$

$$\frac{\partial\rho Y_2 s_2}{\partial t} + div\left(\rho Y_2 s_2\left(\underline{u} - Y_1\underline{u}_r\right)\right) = \varepsilon_u\frac{\rho_2 Y_1}{T_2}\left(\frac{Y_1-\alpha_1}{\rho}\right)\|\nabla p\|^2$$

Numerical approximation

- Use of a fractional step approach [3]
- *Explicit resolution of the hyperbolic part*

  ⇒ Use of an acoustic solver [4]

  To avoid numerical instabilities, the equation on the volume fraction is rewritten $\frac{\partial\alpha_1}{\partial t} + div\,\alpha_1\underline{u} - \frac{\alpha_1 C_2}{\alpha_1 C_2 + \alpha_2 C_1}div\,\underline{u} = 0$

  The conservative part is solved in the same time as the other equations of the system ; the non-conservative part is then solved using a Newton algorithm, wich ensures the positivity of α_1.

Semi-implicit resolution of the dissipative parts

➔ Resolution of the "dissipative advection terms"

$$\begin{aligned}
\partial_t\rho Y_1 &= 0\\
\partial_t\rho e &= 0\\
(\alpha_1 C_2 + \alpha_2 C_1)\partial_t\alpha_1 &= \varepsilon_u(Y_1\alpha_1 C_2 - Y_2\alpha_2 C_1)\underline{u_r}.\nabla\alpha_1
\end{aligned}$$

This system is solved using an explicit upwind scheme. A CFL condition must be applied to ensure stability; if necessary, subcycles are performed.

➔ Resolution of the "dissipative source terms"

$$\begin{aligned}
\partial_t\rho Y_1 &= 0\\
\partial_t\rho e &= 0\\
(\alpha_1 C_2 + \alpha_2 C_1)\partial_t\alpha_1 &= -\left(\frac{\alpha_1\alpha_2}{\rho}\right)^2(\rho_1-\rho_2)\left(1+\left(\frac{\alpha_1 Y_1}{\rho_2\kappa_2}-\frac{\alpha_2 Y_2}{\rho_1\kappa_1}\right)\frac{(\rho_1-\rho_2)}{\rho}\right)\|\nabla p\|^2
\end{aligned}$$

This system is solved using a Newton method. A careful discretization ensures the stability of this step.

➔ Resolution of the "dissipative convection terms"

$$\begin{aligned}
\partial_t\rho Y_1 &= -\varepsilon_u div(\rho Y_1 Y_2\underline{u_r})\\
\partial_t\rho e &= \varepsilon_u div((h_2-h_1)\rho Y_1 Y_2\underline{u_r})\\
(\alpha_1 C_2 + \alpha_2 C_1)\partial_t\alpha_1 &= -\varepsilon_u\alpha_1\alpha_2(C_1 div(Y_2\underline{u_r}) + C_2 div(Y_1\underline{u_r}))
\end{aligned}$$

This system is solved using an implicit scheme.

Numerical results

Air- water shock tube

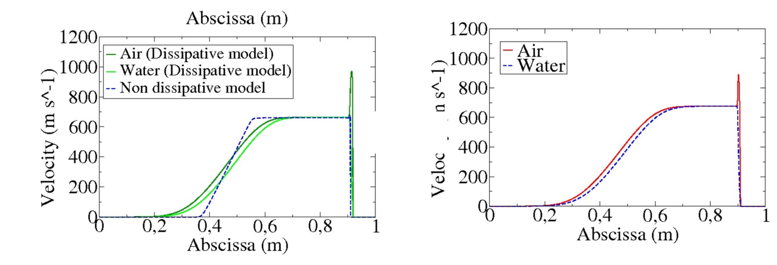

Figure 1: Comparison between the five-equation models (left) and seven-velocity model (right) : Velocities.

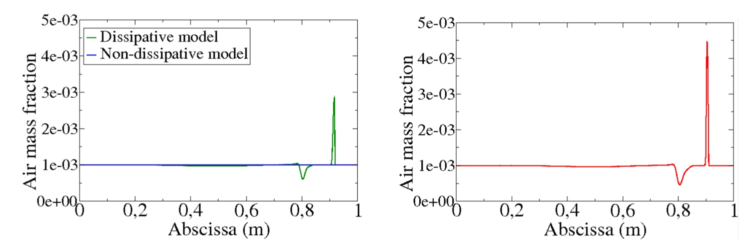

Figure 2: Comparison between the five-equation models (left) and seven-velocity model (right) : Air mass fraction

Sedimentation

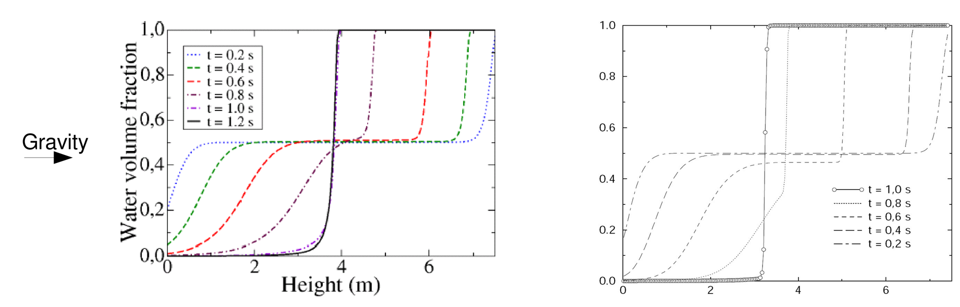

Figure 3: Sedimentation test-case using the five-equation dissipative model (left) and the two-pressure, two-velocity model (right ; results from [5]).

Air-water flow through a pipe with sudden contraction and expansion

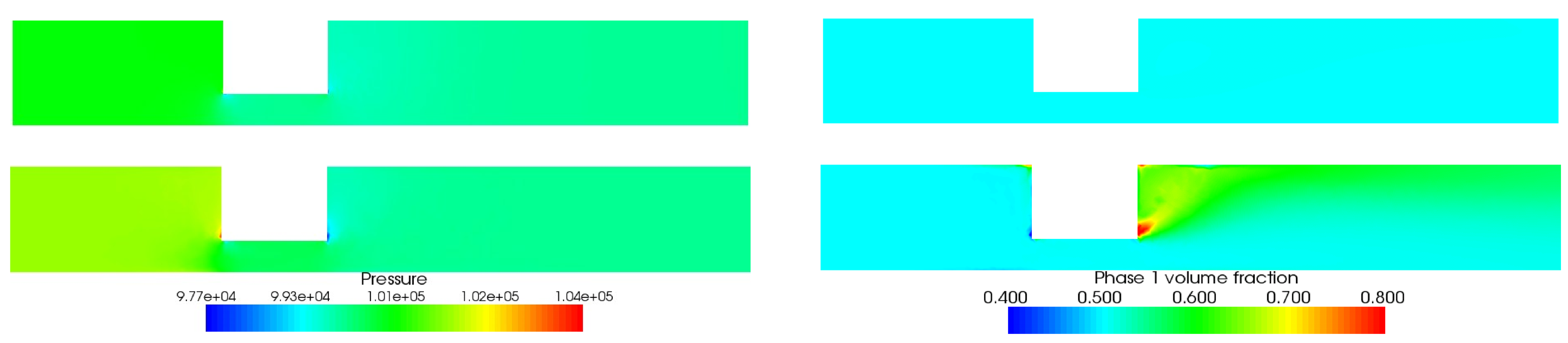

Figure 4: Comparison between the five-equation non-dissipative model (top) and the five-equation dissipative model (bottom): Pressure and air volume fraction

CONCLUSION

A large range of two-phase flows can be characterized by an equality of the phase pressures and velocity disequilibriums. Six-equation two-fluid models, generally used to solve these problems, are usually not hyperbolic, which can lead to numerical failures. We have presented a one-pressure one-velocity model whose dissipative terms enable us to take into account the mechanical disequilibriums. The convective part of this model is hyperbolic, and it has been shown that this model can handle complex flows with counter-currents, even if the model contains only one macroscopic velocity. Numerical results for this model are promising, both for flows with shocks and for flows including counter-currents.

The future work on this model is to perform multidimensional computations, and to implement heat and mass exchanges so that it can deal with all the interfacial phenomena.

[1] A. Guelfi, D. Bestion, M. Boucker, P. Boudier, P. Fillion, M. Grandotto, J.-M. Hérard, E. Hervieu, P. Péturaud. NEPTUNE - A New Software Platform for Advanced Nuclear Thermal-Hydraulics, *Nuclear Science and Engineering*, 156 (3), p. 281-324, 2007

[2] R. Natalini. Recent mathematical results on hyperbolic relaxation problems, *Analysis of systems of conservation laws*, Aachen, 1997, Chapman and Hall/CRC, Boca Raton, p. 128-198.

[3] H. Guillard and F. Duval. A Darcy law for the drift velocity in a two-phase flow model, *Journal of Computational Physics*, 224 (1), p. 288-313, 2007.

[4] A. Murrone and H. Guillard. A five-equation reduced model for compressible two-phase flow computations, *Journal of Computational Physics*, 202 (2), p. 664-698, 2005.

[5] T. Gallouët, J.-M. Hérard and N. Seguin. Numerical modeling of two-phase flows using the two-fluid two-pressure approach, *Mathematical Models and Methods in Applied Sciences*, 14, p. 663-700, 2004.

The work of Mathieu Labois and Marc Grandotto has been achieved in the framework of the NEPTUNE project, financially supported by CEA (Commissariat à l'Énergie Atomique), EDF (Électricité de France), IRSN (Institut de Radioprotection et de Sûreté Nucléaire) and AREVA-NP.