Bench of anisotropic problems

Numerical results with a non-linear cell-centered finite volume scheme (VFPMMD)

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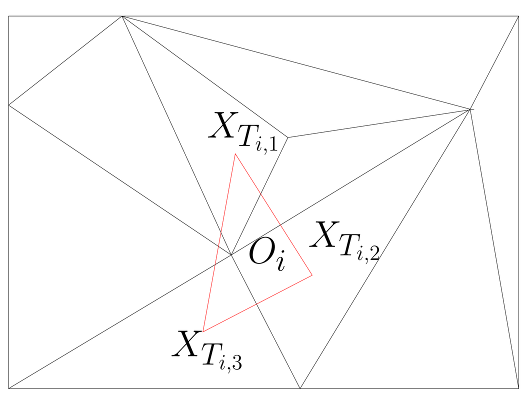
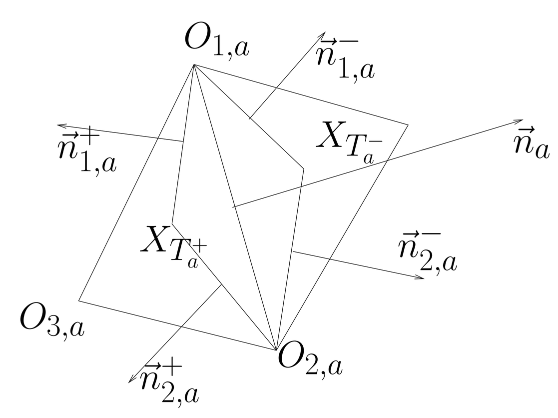
Description of the scheme

We consider a grid $\mathcal{T}$ made up of N_{ma} triangular cells and N_f boundary edges. We define $\mathcal{B} = \{X_{j_{\{1\le j\le N_{ma}+N_f\}}}\}$ as the set of the points which are the intersection of the angle bisectors of each triangular cell and the points located on the middles of the edges of the boundary. For any node O_i of the grid (not located on the boundary), there exists a triangular cell $(X_{T_{i,1}}, X_{T_{i,2}}, X_{T_{i,3}})$ (with $X_{T_{i,j_{j=1,2,3}}} \in \mathcal{B}$) so that the node O_i is inside this triangular cell. In practice, we choose the points closest to O_i. So, there exists three positive or zero coefficients $\lambda_{i,j;j=1,2,3}$ with $\sum_{\{1\le j\le 3\}} \lambda_{i,j} = 1$, such that $\sum_{\{1\le j\le 3\}} \lambda_{i,j} O_i \vec{X}_{T_{i,j}} = \vec{0}$. We approximate the value of the concentration C at the point O_i with the expression : $C_{O_i} = \sum_{\{1\le j\le 3\}} \lambda_{i,j} C_{T_{i,j}}$.



Grid composed of triangular cells

Description of the scheme



Homogeneous case

Calculation of the flux

For an edge a belonging to $\mathcal{A}_{int}$, the integration of the first equation of system (1) on $PT_{i,a}$ using Green's formula leads to, for i=1,2:

$$\int_{PT_{i,a}} \vec{q}\, d\Omega = \int_{PT_{i,a}} \vec{\nabla} C\, d\Omega = \int_{\partial PT_{i,a}} C \vec{n}\, d\Gamma \qquad (1)$$

Using a second order in space formula, we get, for $i = 1, 2$:

$$\vec{q}_{i,a} = \frac{1}{2SF_{i,a}} C_{O_{i,a}} (\vec{n}^+_{i,a} + \vec{n}^-_{i,a}) + \frac{1}{2SF_{i,a}} (-C_{T_a^-} \vec{n}^+_{i,a} - C_{T_a^+} \vec{n}^-_{i,a}) \qquad (2)$$

Description of the scheme

The terms $\vec{q}_{1,a}.\vec{n}_a$ and $\vec{q}_{2,a}.\vec{n}_a$ can be written:

$$\vec{q}_{1,a}.\vec{n}_a = \gamma_1 (C_{O_{1,a}} - C_{T_a^+}) + \beta_1 (C_{T_a^-} - C_{T_a^+}) \qquad (3)$$

and

$$\vec{q}_{2,a}.\vec{n}_a = \gamma_2 (C_{O_{2,a}} - C_{T_a^-}) + \beta_2 (C_{T_a^-} - C_{T_a^+}) \qquad (4)$$

with β_1, β_2 positive coefficients, $\gamma_1 \gamma_2 \le 0$.
Let us assume $\gamma_2 \le 0$, $\gamma_1 \ge 0$.
If $C_{O_{1,a}} - C_{T_a^+} = 0$ (resp. $C_{O_{2,a}} - C_{T_a^-} = 0$), we choose $\vec{q}.\vec{n}_a = \vec{q}_{1,a}.\vec{n}_a$ (resp. $\vec{q}.\vec{n}_a = \vec{q}_{2,a}.\vec{n}_a$). Otherwise, we define:

$$\vec{q}.\vec{n}_a = \frac{-\gamma_2 |C_{O_{2,a}} - C_{T_a^-}| \vec{q}_{1,a}.\vec{n}_a + \gamma_1 |C_{O_{1,a}} - C_{T_a^+}| \vec{q}_{2,a}.\vec{n}_a}{-\gamma_2 |C_{O_{2,a}} - C_{T_a^-}| + \gamma_1 |C_{O_{1,a}} - C_{T_a^+}|} \qquad (5)$$

Note: if $\gamma_1 < 0$, the roles of $\vec{q}_{1,a}.\vec{n}_a$ and $\vec{q}_{2,a}.\vec{n}_a$ are reversed.

Calculation of the main unknown C_T

Let us integrate the mass conservation equation (the second equation of the system (1) over T. We get:

$$\int_T div \vec{q}\, d\Omega = \int_{\partial T} \vec{q}.\vec{n}\, d\Gamma = \sum_{j=1}^{j=3} \vec{q}.\vec{n}_{T,j} = \int_T f\, d\Omega = S(T) f_T \qquad (6)$$

Results for `Test 1.1`

`umin`= 0.0, `umax`=1.0.

- Triangular mesh `mesh1` $\rightsquigarrow$ **ocvl2**= 1.97, **ocvgradl2**= 1.00.

i	nunkw	nnmat	sumflux	erl2	ergrad	ratiol2	ratiograd
1	56	362	ϵ	5.96E-02	2.12E-01		
2	224	1586	ϵ	1.60E-02	9.72E-02	1.90E+00	1.12E+00
3	896	6654	ϵ	4.27E-03	4.71E-02	1.90E+00	1.04E+00
4	3584	27257	ϵ	1.13E-03	2.33E-02	1.91E+00	1.01E+00
5	14336	110297	ϵ	2.84E-04	1.16E-02	1.99E+00	1.00E+00
6	57344	445604	ϵ	7.07E-05	5.76E-03	2.00E+00	1.00E+00
7	229376	1824761	ϵ	1.80E-05	2.88E-03	1.97E+00	1.00E+00

i	erflx0	erflx1	erfly0	erfly1	erflm	umin	umax
1	1.78E-02	1.76E-02	1.63E-02	1.57E-02	4.67E-01	9.89E-02	9.61E-01
2	4.46E-03	3.57E-03	4.54E-03	4.89E-03	2.34E-01	2.80E-02	9.99E-01
3	1.26E-03	1.02E-03	8.78E-04	1.20E-03	1.23E-01	7.38E-03	9.99E-01
4	3.34E-04	2.29E-04	1.98E-04	3.32E-04	6.74E-02	1.88E-03	9.99E-01
5	8.60E-05	4.39E-05	3.64E-05	1.07E-04	3.76E-02	4.77E-04	9.99E-01
6	2.63E-05	7.72E-06	3.40E-06	3.10E-05	1.82E-02	1.20E-04	9.99E-01
7	9.15E-06	3.21E-07	2.02E-06	9.64E-06	9.49E-03	3.00E-05	9.99E-01

- **Comments**
  We perform 3 iterations in the fixed point algorithm.

Results for `Test 1.2`

`umin`= 0.0, `umax`=$1 + \sin(1)$.

- Triangular mesh `mesh1` $\rightsquigarrow$ **ocvl2**= 1.98, **ocvgradl2**= 1.00.

i	nunkw	nnmat	sumflux	erl2	ergrad	ratiol2	ratiograd
1	56	362	ϵ	2.45E-02	1.40E-01		
2	224	1583	ϵ	6.31E-03	6.78E-02	1.95E+00	1.04E+00
3	896	6652	ϵ	1.60E-03	3.34E-02	1.98E+00	1.02E+00
4	3584	27272	ϵ	4.01E-04	1.65E-02	1.99E+00	1.01E+00
5	14336	110210	ϵ	1.00E-04	8.23E-03	2.00E+00	1.01E+00
6	57344	445313	ϵ	2.50E-05	4.10E-03	2.00E+00	1.00E+00
7	229376	1818801	ϵ	6.32E-06	2.05E-03	1.98E+00	1.00E+00

i	erflx0	erflx1	erfly0	erfly1	erflm	umin	umax
1	1.56E-03	4.15E-02	4.97E-02	7.34E-02	2.46E-01	8.39E-03	1.37E+00
2	1.85E-04	1.11E-02	1.52E-02	2.38E-02	1.47E-01	2.00E-03	1.60E+00
3	1.38E-04	3.04E-03	4.46E-03	7.35E-03	7.96E-02	4.92E-04	1.72E+00
4	5.06E-05	8.05E-04	1.26E-03	2.16E-03	4.13E-02	1.22E-04	1.78E+00
5	1.03E-05	2.07E-04	3.42E-04	6.24E-04	2.10E-02	3.06E-05	1.81E+00
6	8.26E-07	6.17E-05	9.00E-05	1.73E-04	1.06E-02	7.62E-06	1.82E+00
7	2.28E-06	1.84E-05	2.19E-05	4.68E-05	5.32E-03	1.89E-06	1.83E+00

- **Comments**
  We perform 3 iterations in the fixed point algorithm.

Results for `Test 5` : Heterogeneous rotating anisotropy

- Non conforming rectangular mesh `mesh5`. `umin`= 0.0, `umax`= 1.0. $\rightsquigarrow$ **ocvl2**= 1.90.

i	nunkw	nnmat	sumflux	erl2	ratiol2
1	32	193	ϵ	3.78E-01	
2	128	921	ϵ	8.99E-02	2.07E+00
3	512	4009	ϵ	2.48E-02	1.85E+00
4	2048	16713	ϵ	6.82E-03	1.86E+00
5	8192	68233	ϵ	1.96E-03	1.80E+00
6	32768	275721	ϵ	5.66E-04	1.80E+00
7	131072	1108489	ϵ	1.52E-04	1.90E+00

i	erflx0	erflx1	erfly0	erfly1	erflm	umin	umax
1	9.10E-02	3.65E-01	1.06E-01	3.66E-01	1.11E+00	9.63E-03	1.36E+00
2	1.92E-02	1.24E-01	2.55E-02	1.31E-01	5.61E-01	3.02E-03	1.13E+00
3	3.13E-03	4.59E-02	8.90E-03	4.42E-02	2.77E-01	9.03E-04	1.04E+00
4	5.52E-04	1.53E-02	2.87E-03	1.45E-02	1.36E-01	2.71E-04	1.01E+00
5	6.51E-05	4.62E-03	8.38E-04	4.42E-03	6.64E-02	8.17E-05	1.00E+00
6	8.02E-06	1.30E-03	2.26E-04	1.26E-03	3.41E-02	2.35E-05	1.00E+00
7	1.10E-05	3.45E-04	5.74E-05	3.38E-04	1.92E-02	6.39E-06	1.00E+00

- **Comments**
  For test 5, we divide each quadrangular cells into two triangular cells. We check that we obtain a positive solution. We perform 10 iterations in the fixed point algorithm.

Results for `Test 8` and `Test 9`

- `Test 8` Perturbed parallelograms mesh `mesh8`, `umin`= 0.0, `umax`= 1.0.

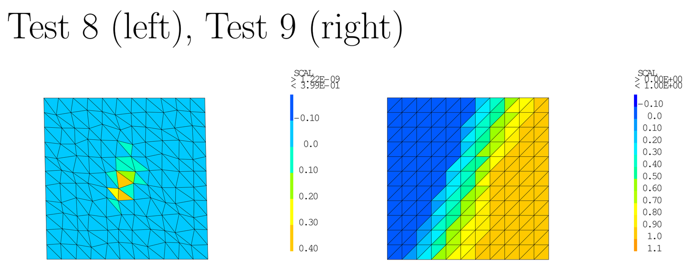
nunkw	nnmat	sumflux	umin	umax
242	1824	ϵ	1.22E-09	3.99E-01

flux0	flux1	fluy0	fluy1
1.76E-06	3.50E-06	4.55E-01	5.44E-01

- `Test 9` Anisotropy with wells. Square uniform grid `mesh9`, `umin`= 0.0, `umax`= 1.0,

nunkw	nnmat	sumflux	umin	umax
242	904	ϵ	0.00E+00	1.00E+00

- Solutions of Test 8 (left), Test 9 (right)



- **Comments**
  For test 8 and 9, we divide each quadrangular cells into two triangular cells. We observe that all the oscillations disappear with the VFPMMD scheme.