Modeling and simulation of turbidity currents

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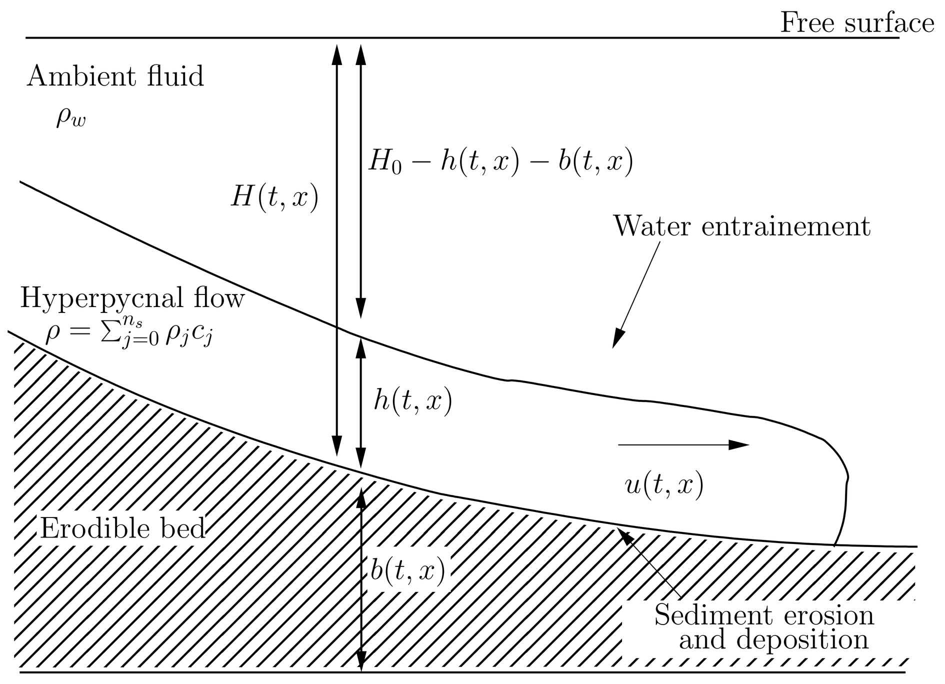
Introduction

When a river contains an elevated concentration of suspended sediment, to the extent that the river density is greater than that of the receiving water body, the river can plunge and create a hyperpycnal plume or turbidity current. This hyperpycnal plume can travel significant distances until it loses its identity by entraining surrounding ambient water and dropping its sediment load.
There is great interest in turbidity currents because of their profound impact on the morphology of the continental shelves and ocean basins of the world. It is commonly accepted that they are one of the potential processes through which sediments can be transferred to the deep sea environments. An additional concern is the destructive effect that turbidity currents have on underwater structures, such as cables, pipelines and foundations.
A numerical model of hyperpycnal flow generated by the plunging of a river is presented. It incorporates the interaction between the turbidity current and bottom, considering eroding and deposit effects as well as solid transport due to the velocity of the current.

1. Model description



$$\begin{cases} \partial_t h + \partial_x(hu) = \phi_\eta + \phi_b, \\ \partial_t(hu) + \partial_x\left(hu^2 + g\left(R_0+R_c\right)\dfrac{h^2}{2}\right) = g\left(R_0+R_c\right)h\partial_x H + u\phi_\eta + \dfrac{u}{2}\phi_b + \tau, \\ \partial_t(hc_j) + \partial_x(hu\,c_j) = \phi_b^j, \text{ for } j=1,\dots,n_s \\ \partial_t H - \xi\partial_x q_b = \xi\phi_b. \end{cases}$$

- Water entrainment: $\phi_\eta = E_w u$

$$E_w = \frac{0.00153}{0.0204 + \mathcal{R}i}, \text{ and } \mathcal{R}i = \frac{R_c g h}{u^2}$$

- $\phi_b^j = F_e^j - F_d^j$ and $\phi_b = \sum_{j=1}^{n_s} \phi_b^j$
- Rate of deposition: $F_d^j = v_{s_j} c_{b_j}$
  v_{s_j} is the settling velocity and c_{b_j} is the near bed concentration

- Rate of erosion: $F_e^j = v_{s_j} p_j E_{s_j}$

$$E_{s_j} = \frac{1.3\cdot 10^{-7} Z_j^5}{1+4.3\cdot 10^{-7} Z_j^5}, \quad Z_j = \alpha_1 \frac{\sqrt{c_D}|u|}{v_{s_j}} R_{p_j}^{\alpha_2}, \quad \mathcal{R}_{p_j} = \frac{\sqrt{R_j g D_j} D_j}{\nu}$$

- Sediment transport of bed-load particles: q_b (Grass, Meyer-Peter& Müller, Nielsen, etc.)

$$q_b = A_g u|u|^{m_g - 1}, 1 \le m_g \le 4, \text{ (Grass model)}$$

2. Reformulation and properties of the model

$$\partial_t W + \mathcal{A}(W)\partial_x W = S(W),$$

where

$$\mathcal{A}(W) = \begin{pmatrix} 0 & 1 & 0 & \dots & 0 & 0 \\ gh(R_0 + \frac{1}{2}R_c) - u^2 & 2u & \frac{g}{2}R_1 h & \dots & \frac{g}{2}R_{n_s} h & -g(R_0+R_c)h \\ -c_1 u & c_1 & u & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ -c_{n_s} u & c_{n_s} & 0 & \dots & u & 0 \\ -\xi\frac{\partial q_b}{\partial h} & -\xi\frac{\partial q_b}{\partial(hu)} & 0 & \dots & 0 & 0 \end{pmatrix}$$

and

$$S(W) = (\phi_\eta + \phi_b, u\phi_\eta + \tfrac{u}{2}\phi_b + \tau, \phi_b^1, \dots, \phi_b^{n_s}, \xi\phi_b)^t$$

Theorem. *Suppose $R_0 h + R_c h > 0$ and ∇q_b sufficiently small. Then, the system is hyperbolic. Moreover, even though it is not strictly hyperbolic for $n_s > 1$, one can always find a complete set of eigenvector for $\mathcal{A}$ in $\mathbb{R}^{n_s+3}$.*

3. Numerical scheme

1. Path conservative scheme

$$\begin{cases} \partial_t W + \mathcal{A}(W)\partial_x W = 0, \\ W(x, t=t^n) = W_i^n \text{ for } x \in I_i. \end{cases} \Longrightarrow W_i^{n+1/2}$$

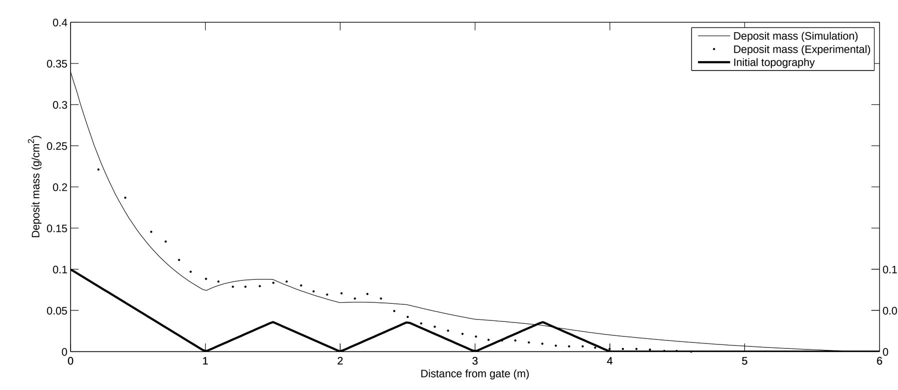
In particular, a Roe linearization based on a family of *paths* is selected and the scheme writes

$$W_i^{n+1/2} = W_i^n - \frac{\Delta t}{\Delta x}\left(\mathcal{A}_{i-1/2}^+ \cdot (W_i^n - W_{i-1}^n) + \mathcal{A}_{i+1/2}^- \cdot (W_{i+1}^n - W_i^n)\right),$$

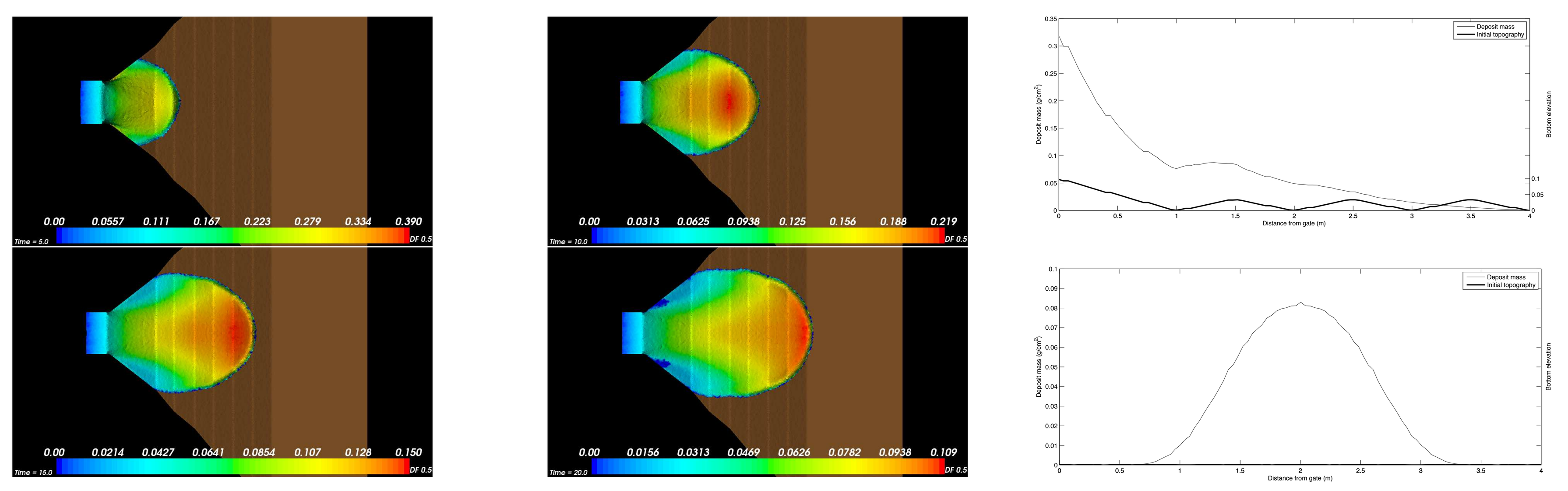
2. Erosion and deposition source terms

$$W_i^{n+1} = W_i^{n+1/2} + \triangle t S(W_i^n)$$

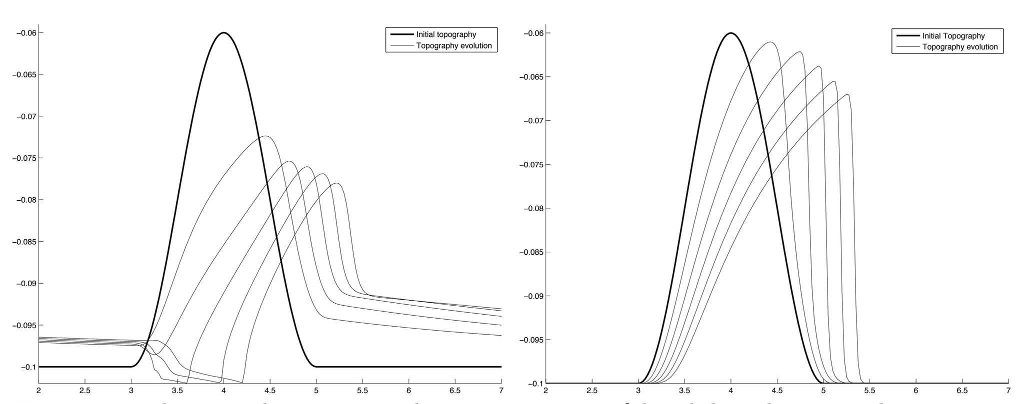
4. Numerical simulations



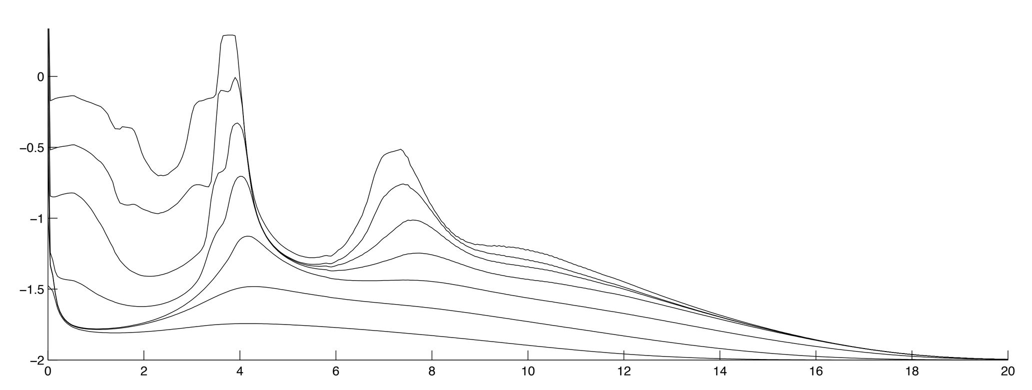
Comparison with laboratory experiments: experiment C5



2D example



Topography evolution with transport of bed-load particles using Grass model: Cases where erosion/deposition is or is not present



Successive turbidity currents over an initial flat bottom: Topography evolution.

References

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