Recursive Mesh Refinement for Vertex Centered FVM applied to a 1-D Phase-Change Problem

Mohamad MUHIEDDINE (IRISA/Université de Rennes 1)

Édouard CANOT (IRISA/CNRS)

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Domain of research

- Prehistoric fire
- Human behavior

Introduction

- Melting processes are moving boundary problems
- A large number of current applications of engineering interest
- Nonlinearity of the interface conditions
- Unkown locations of the moving boundaries
- Vertex-centered finite volume method
- Results are compared with an existing analytical solution

Background

- Various techniques have been employed
- Two schemes are predominant
  1. Front-Tracking method (Askar 1987)
  2. Apparent Capacity method (Bonacina 1973)

$\longrightarrow$ more complete physical model

$\longrightarrow$ suitable mathematical models and numerical schemes

Physical modelling I

$$\begin{cases} \rho C_l \dfrac{\partial T}{\partial t} = \dfrac{\partial}{\partial x}\left[k_l \dfrac{\partial T}{\partial x}\right] & 0 < x < \xi\left(t\right), \quad t > 0 \\ T_l(0,t) = T_w & t > 0 \end{cases} \tag{1}$$

$$\begin{cases} \rho C_s \dfrac{\partial T}{\partial t} = \dfrac{\partial}{\partial x}\left[k_s \dfrac{\partial T}{\partial x}\right] & \xi(t) < x < +\infty, \quad t > 0 \\ T_s(\infty,t) = T_0 & t > 0 \end{cases} \tag{2}$$

and for $t = 0$

$$T(x,0) = T_0 \qquad \forall x \in [0,\infty[$$

Physical modelling II

At the interface

$$k_l \frac{\partial T_l}{\partial x} - k_s \frac{\partial T_s}{\partial x} = -\rho L \frac{d\xi}{dt} \qquad \text{at } x = \xi(t) \tag{3}$$

Moreover

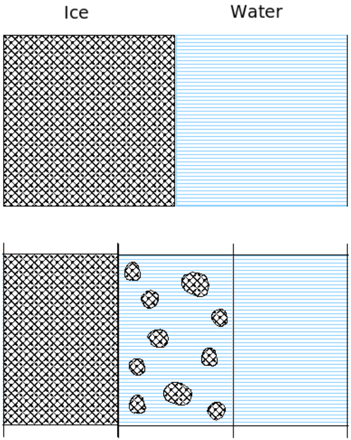
$$T_l = T_s = T_f \qquad \text{at } x = \xi(t)$$

Problem reformulation

$$H = \begin{cases} \rho C_s T, & T < T_f \\ \rho C_s T_f + \rho C_l (T - T_f) + \rho L, & T \geq T_f \end{cases} \tag{4}$$

where H is the enthalpy.

Latent Heat Accumulation (LHA) method I



Latent Heat Accumulation (LHA) method II

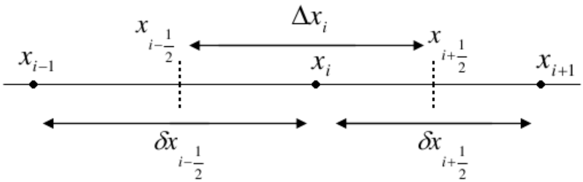
(Prapainop and Maneeratana, 2004)

- Initialization of the total latent heat
- Checking the phase status at each time step
- The saturated volume control is tagged
- Face conductivity
- $\Delta Q_i = \rho C \left(T_i - T_f \right) V_i$
- Accumulation of the latent heat, and $Q_i^{tot} = \int_{V_i} \rho L dv$
- Phase change and the tag is removed

Finite Volume Method

Usual Finite Volume form, using a vertex centered formulation:

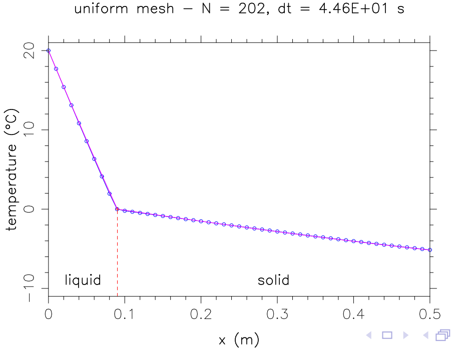
$$\rho C \Delta x_i \left(T_i - T_i^0 \right) = \left[k_{i+\frac{1}{2}} \frac{\left(T_{i+1}^0 - T_i^0 \right)}{\delta x_{i+\frac{1}{2}}} - k_{i-\frac{1}{2}} \frac{\left(T_i^0 - T_{i-1}^0 \right)}{\delta x_{i-\frac{1}{2}}} \right] \Delta t. \quad (5)$$



Basic Model with Uniform Mesh I

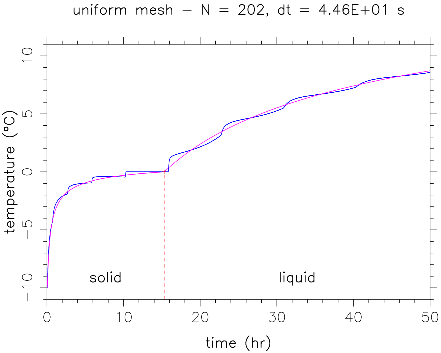
CFL stability criterion (explicit scheme):

$$\Delta t < \Delta t_c = \frac{1}{2} \frac{\rho_s C_s \Delta x^2}{k_s}$$



Basic Model with Uniform Mesh II

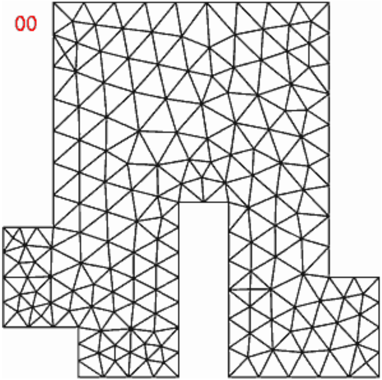
Time evolution presents some fluctuations



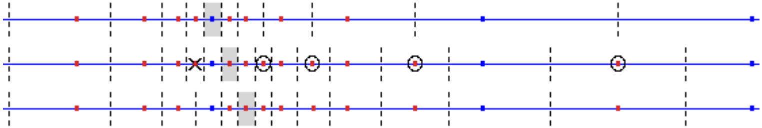
Improved Model with Recursive Mesh Refinement

- Global refinement requires high memory allocation
- Two types of adaptive techniques are mostly used
  1. Local refinement method (Askar 1987)
  2. Moving mesh technique (Mériaux, Piperno 2000)
- Recursive mesh refinement which is a classical "insert/delete nodes" technique for the primal mesh
- It is used to follow the phase change front

Homard (EDF software) adaptive technique (FEM)



Recursive Mesh Refinement (details)



- The mesh rolls because some nodes are added whereas others are deleted
- Only the primal mesh is locally refined; then the dual mesh is updated
- It is not a moving mesh

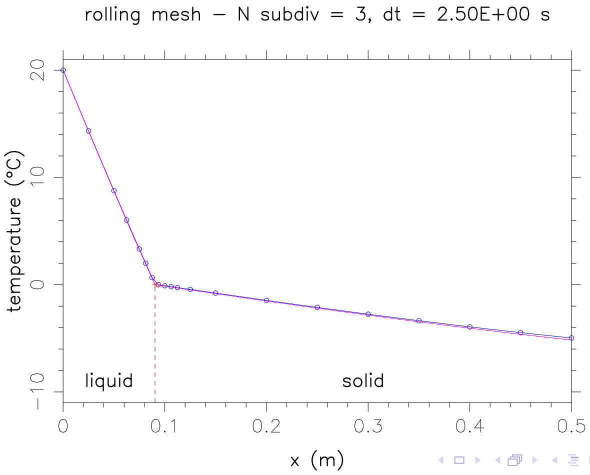
Rolling mesh

Algorithm: repeat for N subdivision levels (fixed):

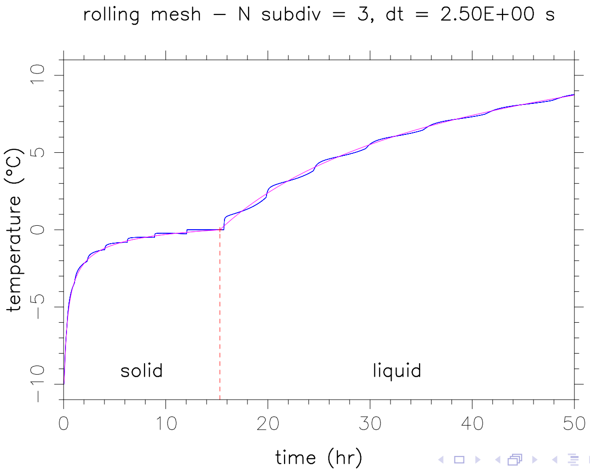
- insert/delete nodes (minimum number to respect a progressiveness constraint)

Build boundary cells between nodes

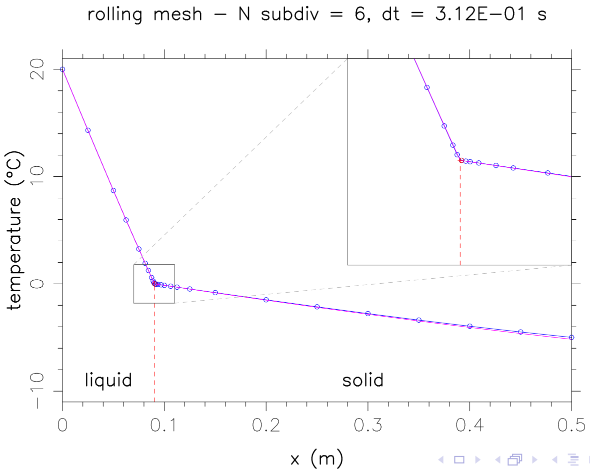
Results and comments (1)



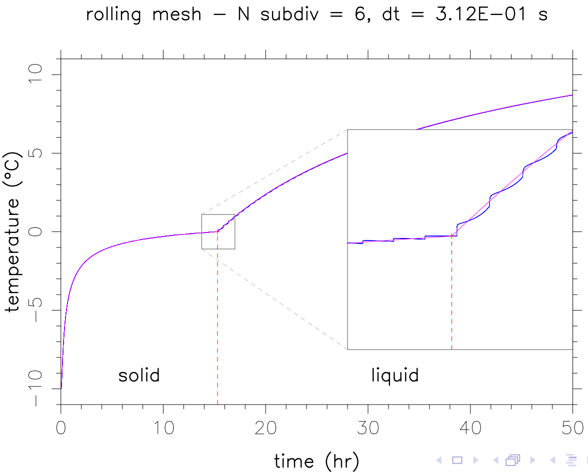
Results and comments (2)



Results and comments (3)

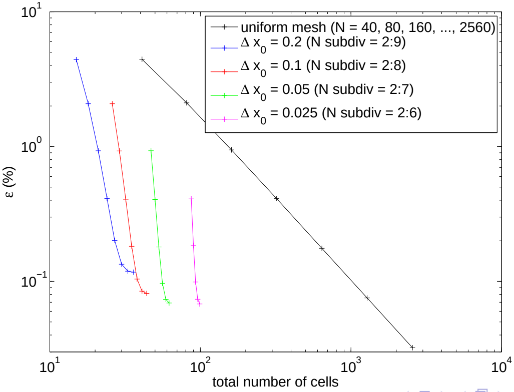


Results and comments (4)

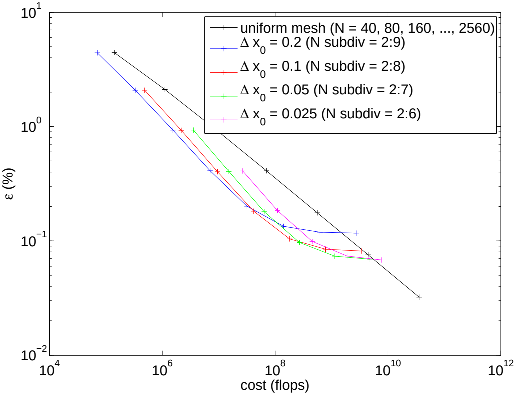


Results and comments (5)

$$Error_{RMS} = \sqrt{\frac{1}{N}\sum_{j=1}^{N}\left(T(t_j) - T(t_j)_{exact}\right)^2}.$$



Results and comments (6)



Conclusion

- Finite volume method for a phase-change problem
- New technique of recursive mesh refinement
- Algorithm used is easy to implement (1D case)
- Verification of the precision and flexibility