Bench of anisotropic problems
Tests with the Discrete Duality Finite Volume Method

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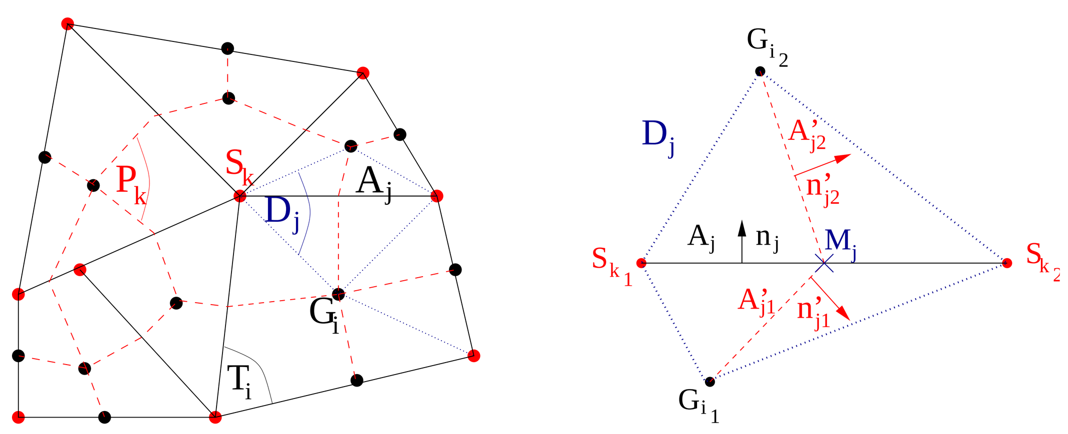
Description of the scheme



Unknowns u_i at G_i and unknowns u_k at S_k.
Integration of $-\nabla\cdot\mathbf{K}\nabla u = f$ over T_i and P_k.
Evaluation of the fluxes $\int \mathbf{K}\nabla u\cdot\mathbf{n}$ over ∂T_i or ∂P_k in D_j using the discrete gradient $(\nabla_h u)_j$ defined over D_j:

$$(\nabla_h u)_j := \frac{1}{2\,|D_j|}\Big\{\ \left[u_{k_2}-u_{k_1}\right]\left(|A'_{j1}|\mathbf{n}'_{j1}+|A'_{j2}|\mathbf{n}'_{j2}\right) + \left[u_{i_2}-u_{i_1}\right]|A_j|\mathbf{n}_j\Big\}.$$

The scheme reads: Find (u_i, u_k) such that for all T_i and P_k

$$-\sum_{j\in\partial T_i}\mathbf{K}_j(\nabla_h u)_j\cdot|A_j|\mathbf{n}_j = |T_i|\bar{f}_i\,,$$

$$-\sum_{j\in\partial P_k}\mathbf{K}_j(\nabla_h u)_j\cdot\left(|A'_{j1}|\mathbf{n}'_{j1}+|A'_{j2}|\mathbf{n}'_{j2}\right) = |P_k|\bar{f}^P_k\,.$$

Results for Test 1.1

umin= 0.0, umax=1.0.

- Triangular mesh mesh1 ⇝ **ocvl2**= 2.0, **ocvgradl2**= 1.0.

i	nunkw	nnmat	sumfluxel	sumfluxpo	erl2el	erl2po	ergrad	ratiol2	ratiograd
1	77	589	-3.55E-15	-7.25E-02	2.90E-02	2.90E-02	4.05E-02		
2	321	2665	-4.44E-15	-2.32E-02	7.76E-03	6.92E-03	1.53E-02	1.85	1.36
3	1313	11353	-3.11E-14	-6.42E-03	1.97E-03	1.72E-03	6.39E-03	1.95	1.24
4	5313	46873	-5.06E-14	-1.68E-03	4.93E-04	4.29E-04	2.89E-03	1.98	1.14
5	21377	190489	-1.89E-13	-4.30E-04	1.23E-04	1.07E-04	1.37E-03	1.99	1.07
6	85761	768025	-1.06E-12	-1.09E-04	3.08E-05	2.68E-05	6.70E-04	1.99	1.03
7	343553	3084313	-4.01E-12	-2.73E-05	7.71E-06	6.69E-06	3.30E-04	2.00	1.02

i	erflx0	erflx1	erfly0	erfly1	erflm	erflmd	uminel	uminpo	umaxel	umaxpo
1	1.06E-02	1.06E-02	1.06E-02	1.06E-02	7.94E-02	4.80E-01	8.57E-02	0.	9.42E-01	1.03E+00
2	2.65E-03	2.65E-03	2.65E-03	2.65E-03	7.52E-03	1.47E-01	6.04E-03	0.	9.96E-01	1.00E+00
3	6.63E-04	6.63E-04	6.63E-04	6.63E-04	1.97E-03	7.55E-02	1.54E-03	0.	9.99E-01	1.00E+00
4	1.66E-04	1.66E-04	1.66E-04	1.66E-04	2.67E-02	2.78E-01	2.33E-02	0.	9.85E-01	1.01E+00
5	4.14E-05	4.14E-05	4.14E-05	4.14E-05	5.00E-04	3.82E-02	3.87E-04	0.	1.00E+00	1.00E+00
6	1.04E-05	1.04E-05	1.04E-05	1.04E-05	1.25E-04	1.92E-02	9.72E-05	0.	1.00E+00	1.00E+00
7	2.59E-06	2.59E-06	2.59E-06	2.59E-06	3.13E-05	9.63E-03	2.44E-05	0.	1.00E+00	1.00E+00

- Distorted quadrangular mesh mesh4_1_i ⇝ **ocvl2**= 2.0, **ocvgradl2**= 1.5.

i	nunkw	nnmat	sumfluxel	sumfluxpo	erl2el	erl2po	ergrad	ratiol2	ratiograd
1	545	4705	-1.42E-14	3.92E-02	3.18E-02	1.17E-02	4.87E-02		
2	2245	19801	-7.11E-15	4.99E-03 2	8.19E-03	3.05E-03	1.64E-02	1.92	1.54
3	5101	45301	-2.80E-14	1.48E-03	3.65E-03	1.39E-03	8.77E-03	1.97	1.53
4	9113	81205	-1.33E-14	6.25E-04	2.06E-03	7.91E-04	5.64E-03	1.97	1.52
5	14281	127513	4.22E-14	3.20E-04	1.32E-03	5.11E-04	4.01E-03	1.98	1.52
6	20605	184225	-3.15E-14	1.85E-04 6	9.14E-04	3.57E-04	3.03E-03	2.01	1.53

i	erflx0	erflx1	erfly0	erfly1	erflm	erflmd	uminel	uminpo	umaxel	umaxpo
1	5.07E-04	1.44E-02	4.93E-03	4.27E-04	1.86E-01	3.59E-01	1.34E-02	0.	1.03E+00	1.01E+00
2	2.86E-04	3.86E-03	1.17E-03	2.03E-04	5.87E-02	1.81E-01	3.41E-03	0.	1.01E+00	1.00E+00
3	1.33E-04	1.73E-03	5.14E-04	1.08E-04	2.81E-02	1.21E-01	1.52E-03	0.	1.00E+00	1.00E+00
4	7.53E-05	9.80E-04	2.87E-04	6.29E-05	1.71E-02	9.13E-02	8.59E-04	0.	1.00E+00	1.00E+00
5	4.83E-05	6.28E-04	1.83E-04	4.07E-05	1.30E-02	7.35E-02	5.51E-04	0.	1.00E+00	1.00E+00
6	3.35E-05	4.37E-04	1.27E-04	2.83E-05	1.02E-02	6.16E-02	3.83E-04	0.	1.00E+00	1.00E+00

- **Comments** Numerical order of convergence greater or equal than expected (superconvergence on distorted quadrangular meshes). Positivity principle respected. Slight overshoots.

Results for Test 1.2

umin= 0.0, umax=$1+\sin(1)$.

- Triangular mesh mesh1 ⇝ **ocvl2**= 2.0, **ocvgradl2**= 1.0.

i	nunkw	nnmat	sumfluxel	sumfluxpo	erl2el	erl2po	ergrad	ratiol2	ratiograd
1	77	589	-6.99E-15	-6.77E-03	3.70E-03	2.85E-03	1.77E-02		
2	321	2665	-9.88E-15	-2.17E-03	8.87E-04	6.97E-04	6.71E-03	2.00	1.36
3	1313	11353	1.35E-14	-5.99E-04	2.21E-04	1.74E-04	2.85E-03	1.97	1.22
4	5313	46873	1.00E-13	-1.57E-04	5.54E-05	4.35E-05	1.31E-03	1.98	1.11
5	21377	190489	-9.58E-14	-4.01E-05	1.39E-05	1.09E-05	6.24E-04	1.99	1.07
6	85761	768025	1.45E-13	-1.01E-05	3.47E-06	2.72E-06	3.05E-04	2.00	1.03
7	343553	3084313	8.99E-15	-2.55E-06	8.68E-07	6.79E-07	1.51E-04	2.00	1.01

i	erflx0	erflx1	erfly0	erfly1	erflm	erflmd	uminel	uminpo	umaxel	umaxpo
1	4.09E-03	4.83E-03	5.44E-03	1.62E-02	6.13E-02	2.22E-01	6.34E-03	0.	1.37E+00	1.84E+00
2	1.21E-03	1.32E-03	2.03E-03	5.20E-03	3.00E-02	1.27E-01	1.57E-03	0.	1.60E+00	1.84E+00
3	3.51E-04	3.38E-04	6.75E-04	1.60E-03	1.47E-02	7.18E-02	3.91E-04	0.	1.72E+00	1.84E+00
4	1.01E-04	8.50E-05	2.11E-04	4.74E-04	7.23E-03	3.82E-02	9.77E-05	0.	1.78E+00	1.84E+00
5	2.84E-05	2.13E-05	6.33E-05	1.37E-04	3.59E-03	1.97E-02	2.44E-05	0.	1.81E+00	1.84E+00
6	7.90E-06	5.33E-06	1.85E-05	3.90E-05	1.79E-03	1.01E-02	6.10E-06	0.	1.83E+00	1.84E+00
7	2.18E-06	1.33E-06	5.28E-06	1.09E-05	8.92E-04	5.12E-03	1.53E-06	0.	1.83E+00	1.84E+00

- Locally refined mesh mesh3 ⇝ **ocvl2**= 2.0, **ocvgradl2**= 1.5.

i	nunkw	nnmat	sumfluxel	sumfluxpo	erl2el	erl2po	ergrad	ratiol2	ratiograd
1	73	589	-1.67E-15	9.10E-03	7.60E-03	7.92E-03	2.92E-02		
2	305	2605	-1.39E-14	1.15E-03	1.94E-03	2.07E-03	1.06E-02	1.91	1.42
3	1249	10957	1.25E-14	1.45E-04	4.92E-04	5.28E-04	3.83E-03	1.95	1.44
4	5057	44941	1.33E-13	1.82E-05	1.24E-04	1.34E-04	1.37E-03	1.97	1.47
5	20353	182029	-2.09E-14	2.27E-06	3.10E-05	3.36E-05	4.87E-04	1.99	1.49

i	erflx0	erflx1	erfly0	erfly1	erflm	erflmd	uminel	uminpo	umaxel	umaxpo
1	7.83E-03	2.19E-03	1.12E-03	1.40E-02	2.04E-01	2.95E-01	1.56E-02	0.	1.66E+00	1.84E+00
2	1.31E-03	3.14E-04	3.34E-05	6.58E-03	1.06E-01	1.69E-01	3.91E-03	0.	1.75E+00	1.84E+00
3	1.52E-04	7.14E-05	3.43E-05	2.47E-03	5.30E-02	9.06E-02	9.78E-04	0.	1.79E+00	1.84E+00
4	7.29E-06	1.88E-05	1.79E-05	8.27E-04	2.63E-02	4.70E-02	2.44E-04	0.	1.82E+00	1.84E+00
5	1.33E-05	4.94E-06	6.75E-06	2.60E-04	1.31E-02	2.39E-02	6.10E-05	0.	1.83E+00	1.84E+00

- **Comments** Numerical order of convergence greater or equal than expected (superconvergence on non-conforming meshes). Positivity principle respected. No overshoots.

Results for Test 2 Numerical locking

Triangular mesh mesh1. umin= -1, umax= 1.

- $\delta = 10^5$ ⇝ **ocvl2**= 2.0, **ocvgradl2**= 1.0.

i	nunkw	nnmat	sumfluxel	sumfluxpo	erl2el	erl2po	ergrad	ratiol2	ratiograd
1	93	797	-5.13E-11	-5.13E-11	1.35E-01	1.99E-01	2.23E-01		
2	353	3105	-1.16E-10	-1.16E-10	3.61E-02	4.86E-02	6.00E-02	1.98	1.97
3	1377	12257	-6.10E-11	-6.10E-11	9.03E-03	1.22E-02	1.66E-02	2.04	1.89
4	5441	48705	6.52E-11	6.52E-11	2.26E-03	3.06E-03	5.46E-03	2.02	1.62
5	21633	194177	8.34E-11	8.34E-11	5.66E-04	7.67E-04	2.33E-03	2.01	1.23
6	86273	775425	-1.31E-10	-1.31E-10	1.42E-04	1.92E-04	1.14E-03	2.00	1.03
7	344577	3099137	-3.12E-10	-3.12E-10	3.59E-05	4.77E-05	5.39E-04	2.01	1.08

i	erflx0	erflx1	fluy0	fluy1	erflm	erflmd	uminel	uminpo	umaxel	umaxpo
1	-1.86E-16	1.43E-16	-3.01E-11	-7.16E-12	6.10E+02	9.33E+02	-8.24E-01	-7.90E-01	8.05E-01	7.76E-01
2	1.43E-16	1.43E-16	-6.64E-11	-1.20E-11	1.21E+02	9.10E+02	-9.45E-01	-9.49E-01	9.46E-01	9.49E-01
3	4.28E-16	0.00E+00	-6.96E-11	-6.68E-11	4.03E+01	5.32E+02	-9.86E-01	-9.87E-01	9.86E-01	9.87E-01
4	4.28E-16	1.43E-16	6.61E-12	6.57E-13	1.08E+01	2.76E+02	-9.96E-01	-9.97E-01	9.96E-01	9.97E-01
5	4.28E-16	8.57E-16	6.01E-11	2.32E-11	2.73E+00	1.39E+02	-9.99E-01	-9.99E-01	9.99E-01	9.99E-01
6	1.28E-15	1.43E-15	-1.13E-10	-1.80E-11	2.11E+00	6.98E+01	-1.00E+00	-1.00E+00	1.00E+00	1.00E+00
7	0.00E+00	9.99E-16	-4.86E-11	-2.63E-10	2.11E+00	3.48E+01	-1.00E+00	-1.00E+00	1.00E+00	1.00E+00

- $\delta = 10^6$ ⇝ **ocvl2**= 2.0, **ocvgradl2**= 1.0.

i	nunkw	nnmat	sumfluxel	sumfluxpo	erl2el	erl2po	ergrad	ratiol2	ratiograd
1	93	797	-1.92E-10	-1.92E-10	1.35E-01	1.99E-01	2.22E-01		
2	353	3105	8.62E-10	8.62E-10	3.59E-02	4.85E-02	5.84E-02	1.99	2.00
3	1377	12257	-1.22E-10	-1.22E-10	9.01E-03	1.22E-02	1.60E-02	2.03	1.90
4	5441	48705	8.40E-11	8.40E-11	2.25E-03	3.05E-03	5.05E-03	2.02	1.68
5	21633	194177	1.04E-09	1.04E-09	5.64E-04	7.63E-04	2.01E-03	2.00	1.33
6	86273	775425	4.75E-10	4.75E-10	1.42E-04	1.90E-04	9.45E-04	1.99	1.09
7	344577	3099137	-1.20E-09	-1.20E-09	3.60E-05	4.71E-05	4.76E-04	2.01	0.99

i	erflx0	erflx1	fluy0	fluy1	erflm	erflmd	uminel	uminpo	umaxel	umaxpo
1	1.42E-16	0.00E+00	-9.35E-11	-9.87E-11	1.93E+03	2.98E+03	-8.19E-01	-7.87E-01	8.13E-01	7.82E-01
2	1.42E-16	2.84E-16	5.10E-10	3.52E-10	3.86E+02	2.91E+03	-9.45E-01	-9.50E-01	9.46E-01	9.50E-01
3	0.00E+00	1.42E-16	6.92E-11	-1.92E-10	1.28E+02	1.70E+03	-9.86E-01	-9.88E-01	9.86E-01	9.88E-01
4	1.42E-16	1.28E-15	-5.82E-11	1.42E-10	3.40E+01	8.80E+02	-9.97E-01	-9.97E-01	9.97E-01	9.97E-01
5	1.42E-16	2.84E-16	5.03E-10	5.37E-10	8.65E+00	4.44E+02	-9.99E-01	-9.99E-01	9.99E-01	9.99E-01
6	0.00E+00	1.56E-15	-2.15E-09	2.63E-09	2.17E+00	2.23E+02	-1.00E+00	-1.00E+00	1.00E+00	1.00E+00
7	1.99E-15	2.98E-15	-6.82E-10	-5.14E-10	1.62E+00	1.11E+02	-1.00E+00	-1.00E+00	1.00E+00	1.00E+00

- **Comments** No numerical locking. Order of convergence greater than expected on the coarser meshes. Better convergence for $\delta = 10^6$. No under or overshoots.

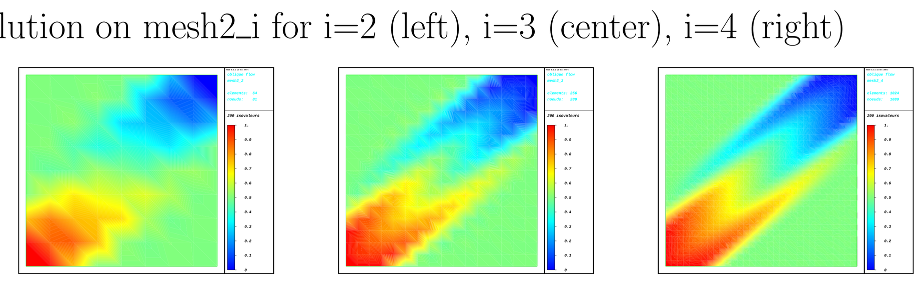
Results for Test 3 : Oblique flow

- Uniform rectangular mesh mesh2. umin= 0.0, umax= 1.0.

i	nunkw	nnmat	sumfluxel	sumfluxpo	uminel	uminpo	umaxel	umaxpo
1	25	181	-1.11E-16	-1.11E-16	1.04E-01	0.	8.96E-01	1.00E+00
2	113	925	2.10E-15	2.10E-15	3.86E-02	0.	9.61E-01	1.00E+00
3	481	4141	-3.33E-15	-3.33E-15	5.43E-03	0.	9.95E-01	1.00E+00
4	1985	17485	4.58E-15	4.58E-15	5.99E-03	0.	9.94E-01	1.00E+00
5	8065	71821	-9.96E-15	-9.96E-15	5.05E-03	0.	9.95E-01	1.00E+00
6	32513	291085	-2.42E-14	-2.42E-14	2.42E-03	0.	9.98E-01	1.00E+00
7	130561	1171981	-1.22E-13	-1.22E-13	1.01E-03	0.	9.99E-01	1.00E+00
ref	204161	1833613	-5.21E-14	-5.21E-14	7.14E-04	-4.44E-16	9.99E-01	1.00E+00

i	flux0	flux1	fluy0	fluy1	ener1	ener2	eren
1	-1.80E-01	1.80E-01	-1.35E-01	1.35E-01	1.81E-01	1.80E-01	3.68E-03
2	-1.91E-01	1.91E-01	-1.07E-01	1.07E-01	2.20E-01	2.19E-01	2.24E-03
3	-1.94E-01	1.94E-01	-9.86E-02	9.86E-02	2.38E-01	2.38E-01	1.09E-03
4	-1.94E-01	1.94E-01	-9.81E-02	9.81E-02	2.40E-01	2.40E-01	1.02E-04
5	-1.94E-01	1.94E-01	-9.83E-02	9.83E-02	2.42E-01	2.42E-01	5.52E-05
6	-1.93E-01	1.93E-01	-9.85E-02	9.85E-02	2.42E-01	2.42E-01	1.02E-06
7	-1.93E-01	1.93E-01	-9.86E-02	9.86E-02	2.42E-01	2.42E-01	1.77E-06
ref	-1.93E-01	1.93E-01	-9.86E-02	9.86E-02	2.42E-01	2.42E-01	4.40E-06

- Solution on mesh2_i for i=2 (left), i=3 (center), i=4 (right)



- **Comments** No under or overshoots. Boundary fluxes and energy already well converged on rough grids.

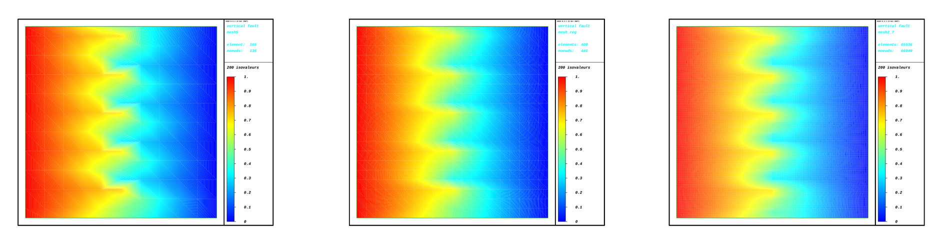
Results for Test 4 : Vertical fault

- Non conforming rectangular mesh mesh5. umin= 0.0, umax= 1.0.

i	nunkw	nnmat	sumfluxel	sumfluxpo	uminel	uminpo	umaxel	umaxpo
1	200	1681	3.32E-11	3.32E-11	4.04E-02	0.	9.62E-01	1.00E+00
reg	761	6613	-4.25E-10	-4.25E-10	1.99E-02	0.	9.82E-01	1.00E+00
ref	204161	1833613	-2.99E-08	-2.99E-08	1.32E-03	0.	9.99E-01	1.00E+00

i	flux0	flux1	fluy0	fluy1	ener1	ener2	eren
1	-4.00E+01	4.18E+01	-1.81E+00	9.08E-04	4.22E+01	4.06E+01	3.65E-02
reg	-3.99E+01	4.26E+01	-2.68E+00	8.01E-04	4.32E+01	4.10E+01	5.08E-02
ref	-4.21E+01	4.44E+01	-2.33E+00	7.97E-04	4.32E+01	4.32E+01	1.28E-03

- Solution for the vertical fault on the meshes: (Left)mesh5 (center) mesh2_7. (Right) mesh5_ref.



- **Comments** The non-conforming rough mesh already provides satisfying results.

Results for Test 5 : Heterogeneous rotating anisotropy

- Non conforming rectangular mesh mesh5. umin= 0.0, umax= 1.0. ⇝ **ocvl2**= 2.0, **ocvgradl2**= 1.5.

i	nunkw	nnmat	sumfluxel	sumfluxpo	erl2el	erl2po	ergrad	ratiol2	ratiograd
1	25	181	-2.78E-16	2.73E-01	7.39E-02	2.63E-02	1.02E-01		
2	113	925	-2.44E-15	7.24E-02	1.43E-02	9.07E-03	3.26E-02	2.18	1.51
3	481	4141	1.72E-15	1.83E-02	3.04E-03	2.48E-03	1.04E-02	2.14	1.58
4	1985	17485	-8.10E-15	4.58E-03	7.10E-04	6.40E-04	3.42E-03	2.05	1.57
5	8065	71821	1.91E-14	1.15E-03	1.73E-04	1.62E-04	1.15E-03	2.01	1.55
6	32513	291085	-2.10E-14	2.86E-04	4.27E-05	4.07E-05	3.97E-04	2.01	1.53
7	130561	1171981	2.17E-14	7.16E-05	1.06E-05	1.02E-05	1.38E-04	2.00	1.52

i	erflx0	erflx1	erfly0	erfly1	erflm	erflmd	uminel	uminpo	umaxel	umaxpo
1	1.21E-01	6.58E-02	1.21E-01	6.58E-02	5.03E-01	5.61E-01	1.41E-01	0.	9.32E-01	1.00E+00
2	3.97E-02	1.56E-02	3.97E-02	1.56E-02	2.42E-01	3.02E-01	3.66E-02	0.	9.76E-01	1.00E+00
3	1.22E-02	3.82E-03	1.22E-02	3.82E-03	1.19E-01	1.53E-01	9.38E-03	0.	9.93E-01	1.00E+00
4	3.60E-03	9.49E-04	3.60E-03	9.49E-04	5.95E-02	7.70E-02	2.38E-03	0.	9.98E-01	1.00E+00
5	1.04E-03	2.36E-04	1.04E-03	2.36E-04	2.97E-02	3.85E-02	5.98E-04	0.	1.00E+00	1.00E+00
6	2.95E-04	5.90E-05	2.95E-04	5.90E-05	1.49E-02	1.93E-02	1.50E-04	0.	1.00E+00	1.00E+00
7	8.24E-05	1.47E-05	8.24E-05	1.47E-05	7.43E-03	9.64E-03	3.76E-05	0.	1.00E+00	1.00E+00

- **Comments** Since the tensor $\mathbf{K}$ is continuous, its values at the centers of gravity of the diamond cells were used for $\mathbf{K_j}$ in the above scheme. This provides a faster converging approximation than assuming a constant $\mathbf{K}$ in each primal cell and splitting the diamond cells into two half diamonds and invoking flux continuity (order 1 only).

Results for Test 6 and Test 7

- Test 6 Oblique drain, min $= -1.2$, max $= 0$, coarse (C) and fine (F) oblique meshes, mesh6 and mesh7

grid	nunkw	nnmat	sumfluxel	sumfluxpo	erl2el	erl2po	ergrad
C	390	3328	1.38E-10	1.38E-10	4.75E-13	3.66E-14	2.57E-12
F	450	3868	-4.04E-10	-4.04E-10	7.42E-12	8.85E-12	3.60E-09

grid	erflx0	erflx1	erfly0	erfly1	erflm	erflmd	uminel	uminpo	umaxel	umaxpo
C	1.48E-12	2.17E-11	7.92E-14	9.23E-14	2.54E-09	6.41E-11	-1.15E+00	-1.20E+00	-5.43E-02	0.00E+00
F	3.36E-12	6.46E-11	1.77E-12	1.45E-12	2.31E-07	3.16E-08	-1.15E+00	-1.20E+00	-5.43E-02	0.00E+00

- Test 7 Oblique barrier, min $= -5.575$, max $= 0.575$, coarse oblique mesh mesh6

nunkw	nnmat	sumfluxel	sumfluxpo	erl2el	erl2po	ergrad
390	3328	-2.83E-12	-2.83E-12	3.79E-08	3.12E-08	4.51E-08

erflx0	erflx1	erfly0	erfly1	erflm	erflmd	uminel	uminpo	umaxel	umaxpo
2.26E-07	5.91E-08	4.67E-08	1.04E-08	5.76E-06	1.65E-05	-5.54E+00	-5.58E+00	5.37E-01	5.75E-01

- **Comments** Exact values are recovered since the solution is piecewise linear.

Results for Test 8 and Test 9

- Test 8 Perturbed parallelograms mesh mesh8, umin= 0.0, umax= 1.0.

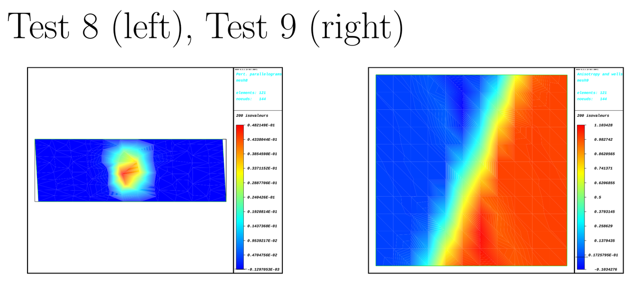
nunkw	nnmat	sumfluxel	sumfluxpo	uminel	uminpo	umaxel	umaxpo
221	1861	-2.92E-16	-2.92E-16	-1.77E-03	-1.30E-04	8.36E-02	4.82E-02

flux0	flux1	fluy0	fluy1
-6.86E-10	-4.86E-10	4.98E-01	5.02E-01

- Test 9 Anisotropy with wells. Square uniform grid mesh9. umin= 0.0, umax= 1.0,

nunkw	nnmat	sumfluxel	sumfluxpo	uminel	uminpo	umaxel	umaxpo
265	2289	7.74E-16	7.74E-16	-7.07E-02	-1.03E-01	1.07E+00	1.10E+00

- Solutions of Test 8 (left), Test 9 (right)



- **Comments** Violation of the maximum principle.