A brief Report on the article [BOY 10] “Nonoverlapping Schwarz algorithm for solving two-dimensional m-DDFV schemes.”

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Last update: Tuesday 28th December, 2010; not finished yet and my hope I come back again to this article

Abstract: This article deals with a m-DDFV (modified discrete duality finite volume) scheme for anisotropic elliptic problems with mixed Dirichlet/Fourier boundary conditions. As a result, the authors provide a nonoverlapping Schwarz algorithm associated with a subdomain decomposition of the problem domain for solving m-DDFV scheme on the whole domain. The convergence of the Schwarz algorithm is proved to converge to the solution of the m-DDFV scheme on the whole domain. The properties of the algorithm were illustrated by numerical results on anisotropic elliptic equations.

Key words and phrases: finite volume methods; Nonoverlapping Schwarz algorithm; discrete duality finite volume (DDFV) schemes; mixed Dirichlet/Fourier boundary conditions

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Some remarks

1. idea on m-DDFV: m-DDFV (“m” is for modified) is a discrete duality finite volume introduced to take into account the possible discontinuities in the coefficients of the elliptic equation under study. It is first introduced in [BOY 08].

The DDFV method has been developed to approximate anisotropic diffusion problems on general meshes. It was first introduced and studied in [HER 03, DOM 05] to approximate the Laplace equation with Dirichlet boundary conditions or homogeneous Neumann boundary conditions on a general class of meshes.

2. equation solved: the following problem is considered in [BOY 10]:

\[- \nabla \cdot (\Lambda(x) \nabla u(x)) = f(x), \quad x \in \Omega,\]

with the following boundary conditions
(a) Dirichlet boundary conditions
\[ u(x) = h(x), \quad x \in \partial x \in \Omega \setminus \Gamma, \quad [2] \]

(b) Fourier boundary conditions
\[ - (\Lambda(x) \cdot \nabla u(x)) = \lambda u(x) - g(x), \quad x \in \Gamma, \quad [3] \]

where \( \Omega \) is an open bounded polygonal domain of \( \mathbb{R}^2 \). The measurable matrix-valued map \( \Lambda : \Omega \rightarrow \mathcal{M}_{2,2} \) is assumed to fulfil the following assumptions: there exists \( C_A > 0 \) such that
\[ (\Lambda(x) \xi, \xi) \geq \frac{1}{C_A} |\xi|^2 \quad \text{and} \quad |\Lambda(x) \xi| \leq C_A |\xi|, \quad \forall \xi \in \mathbb{R}^2, \forall \text{ a.e. } x \in \Omega, \quad [4] \]
\[ f \in H^{-1}(\Omega), \text{ and } g, h \in H^2(\Omega). \]

References


