University of Annaba–Department of Technology

First year undergraduation **2009–2010**

Analysis

Supplementary problems

Complex numbers

Exercise 1. Consider the function $f(z) = z^2$ and consider $z_1 = -2 + i$, $z_2 = 1 - 3i$. Put under the plar form, the following complex numbers:

- Drow $f(z_1)$ and $f(z_2)$
- Drow $f(D)$, where

$$D = \{z \in \mathbb{C} : |z| = 2, 0 \leq \theta \leq \frac{\pi}{2}\}.$$

Exercise 2. Compute the limits: :

1. $$\lim_{z \to 0} \frac{z - \sin z}{z^2}, \tag{1}$$
2. $$\lim_{z \to 1} z^{\frac{1}{z^2-1}}. \tag{2}$$

Exercise 3. Compute the derivatives of the following functions:

1. by definition
$$f(z) = z^3,$$
2. $$f(z) = \cos(\log(z + i)),$$
3. $$f(z) = \frac{(\mathrm{ch}\, z)^3}{\mathrm{ch}\, z^3}.$$

   Recall that
$$\mathrm{ch}\, z = \frac{\exp(z) + \exp(-z)}{2},$$
$$\mathrm{sh}\, z = \frac{\exp(z) - \exp(-z)}{2}.$$

Exercise 4. Let f be an hlomorphic function on an open subset D. Let $z_0 \in D$. Prove that there exists a neighborhood of z_0 such that :

$$f(z) = f(z_0) + f'(z_0)(z - z_0) + \eta(z), \tag{3}$$

and

$$\lim_{z \to z_0} \eta(z) = 0. \tag{4}$$

Exercise 5. Check the Cauchy–Riemann conditions for the following functions:

1.

$$f(z) = i \exp(-z),$$

2.

$$f(z) = \sin\, z.$$

Exercise 6.

1. Show that the following function is harmonic:

$$u(x, y) = \exp(-x)(\,x\, \sin y - y\, \cos y).$$

2. find the function $v(x, y)$ such that $f(z) = u(x, y) + iv(x, y)$ is an holomorphic function on $\mathbb{C}$.

3. Express f as a function in z.

4. Is the function $u(x, y) = x^2 - y^2$ is harmonic?