An introduction to the subject of the project of my student "L'approximation numérique des valeurs et vecteurs propres et applications"

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1 Some remarks on the introduction presented by the students

- there is no title
- the students have not taken habit yet with internet: they could find some available introduction and motivation in internet
- introduction (which is written in French) is useful but some sentences are not complete.
- no references!

2 A suggestion of an introduction (would say motivation)

I suggest the following introduction (it is wriiten in English):

In Mathematics, it is difficult to resolve (find the solution) exactly of some problems, e.g. computation of the exact solution of differential equations. Numerical analysis is a useful tool allows us to compute approximatly the solutions of some problems.

Among the interesting subjects in Mathematics (Applied Mathematics) is the study of eigenvalues and eigenfunctions. Indeed, the eigenvalues and eigenfunctions appear for instance in the Sturm–Liouville theory. As an example of Sturm–Liouville equation is:

$$-\left(p(x)y'(x)\right)' + q(x)y'(x) = \lambda\, w(x)y(x), \qquad [1]$$

where λ and y are unknown. Therefore λ and y appear in [1] as the eigenvalue and eigenfunction of some self–adjoint operator in an Hilbertian space.

Eigenvalues and eigenfunctions appear, for instance, when we study the Wave–equation (it is a model of second order hyperbolic equations) $u_t t(x,t) - \Delta\, u(x,t) = 0$. Indeed, if we assume that the solution u of the Wave equation can be written as $\sin(wt)\psi(x)$, where $w \in \mathbb{R}$, we obtain the equation $\Delta\,\psi(x) + w^2\psi(x) = 0$, i.e. w^2 is an eigenvalue of the operator $-\Delta$ associated to the eigenfunction ψ.

From this simple introduction, we understand that the study of eigenvalues and eigenfunctions has a variety of applications.

We then shall present this modest review on the numerical approximation of Eigenvalues and eigenvectors. It is obvious to say that the discetization, using finite difference, finite element, finite element methodes, of the *spectral problem* $\mathcal{L}\,u = \lambda\,u$, where $\mathcal{L}$ is a differential operator, leads to system to be solved $\mathcal{A}\,u = \lambda\,u$, where $\mathcal{A}$ is a matrix with finite dimension. So we will focus on the numerical approximation Eigenvalues and eigenvectors of problems like $\mathcal{A}\,u = \lambda\,u$, where $\mathcal{A}$ is a matrix with finite dimension.