Rupelton layer

Abstract

In this work, we use the finite volume code pdelib/thermoha2 of Enchery and Fuhrmann [2] to understand the influence of layers of small thichness on the behaviour of the fluid. To this end consider the following case:

- K_1 Material for bottom layer which represents the Upper Cretaceous Layer; will be refereed to in the sequel as to the first layer
- Ru Material for Rupel layer, will be refereed to in the sequel as the second layer
- Cz Material for top layer which represents Cenozoic Layer, will be refereed to in the sequel as the third layer

Let us denote k_2 the permeability of Rupelton, phi_2 the porosity of Rupelton and D_2 the dispersion coefficient in Rupelton.

Key words : simulations, termal Rayleigh number, buoyancy ratio, Lewis number

1 An introduction to the problem and some basic definitions and an introduction to the problem

For the sake of completness, let us recall the following basic definition:

- Salinity: is thwe saltness or dissolved salt content of a body of water.
- The salt content of most natural lakes, rivers and streams is small that these waters are termed **fresh** even **sweet** water.
  The actual amount of salt in **fresh** water is, by definition, less than 0.05 percent. Otherwise, the water is regarded as **brackish** or defined as **saline** if contains 3 until 5 percent salt by volume. If the percent is over 5 percent, the water is considered **brine**.

Groundwater is a source necessary for human life, therefore the studying of the mechanisms of solute transport in aquifers is essential. Numerical simulations can lead to a better understanding of driving forces behind transport phneomenas, cf. [2]. For instance, in various parts of the North East German Basin, sline springs occur at the surface; in addition to this, some of the saline springs even occur in areas where no salt deposits or salt domes are known.

A hydrochemical analysis (cf. [2]) proves that the main source of salinity in the basin is the salt dissolution from the "Zechstein layer". The salt content in water increases with depth until a staturation point of $345 kg \cdot m^{-3}$. The chemical composition of water samples suggests the existence of communication between aquifers characterized by upward brine flows.

Further experimental results tend to suppose that these flows (which caracterize the communication between aquifers cited before) are driven by thermal effects.

To justify this Hypothesis, a two–dimensional coupled three parabolic equation, under the data cited above, is considered in [6]. Magri used the commercial software FEFLOW© (which is based on finite element method) to approximate this model.

Various numerical simulation based on this approximation, show thermal instabilities. These lead to convection cells which make saline water rise up to the surface.

The Paper [2] gives numerical insight, based on the use of finite volume method ([5]), on the mechanisms which drive the **saline** waters up to surface.

[2] perfomed simulations, using finite volume methods, where the transport of salt by water and no temperature was taken into account, and, in that case, they only observed the **downward flow of saline waters** along the dome slopes and the progessive diffusion of the salt through the basin. this confirms that the plumes that observed are **thermally driven**.

The project is concerned with numerical simulations of thermohaline convection in the North–East Germany Basin, and the identification of the qualitative state regime.

The tasks are:

- the developement and validation of a simulation code for thermohaline convection in porous media on the base of a more general software framework for finite volume methods for systems of diffusion-convection-reaction equations
- the application of this code to the aformentioned problem.

In this work,we use the finite volume code pdelib/thermoha2 of Enchery and Fuhrmann [2] to understand the behaviour of the fluid when a thick layer is introduced (Rupelton layer) between two layers.

The results of the simulations show that the convection force drives salt plumes reaching the surface whose tips are moving apperantly randomly.

We remarke oscillations in theses simulations which confirm some known results in thermohaline.

2 Description of the mathematical model and the parameters

continuity equation

$$\partial_t(\rho_f \phi) + \nabla \cdot (\rho_f \vec{v}) = 0 \tag{1}$$

Darcy law

$$\vec{v} = -\frac{k}{\mu}(\nabla p - \rho g \vec{e}_z) \tag{2}$$

Energy transport

$$\partial_t \left((\phi \rho_f c_f + (1-\phi)\rho_s c_s) T \right) - \nabla \cdot \Big((\phi \lambda_f + (1-\phi)\lambda_s) \nabla T - T \rho_f c_f \vec{v} \Big) = 0 \tag{3}$$

Solute transport

$$\partial_t(\phi C) + \nabla \cdot \big(C\vec{v} - D\phi \nabla C \big) = 0 \tag{4}$$

The system is closed by appropriate boundary conditions and solved for p, T, C.

3 Boussinesq approximation and a dimensionless formulation

In order to gather principal insight into the phenomena occuring during for this type of problems, we regard a simplified situation, the so called Boussinesq approximation [].

In our context, this approximation consists in the assumptions, that all solid properties are constant and that

$$\rho = \rho(p, T, C) = \rho_0(1 - \alpha(T - T_0) + \beta(C - C_0)) \tag{5}$$

$$\rho_f = \rho_0 \tag{6}$$

$$\mu = \mu_0 \tag{7}$$

As ρ_f is now constant, the time derivative vanishes, and we can divide the energy transport equation by $\rho_f c_f$, leading to

$$\nabla \cdot \vec{v} = 0 \tag{8}$$

$$\vec{v} = -\frac{k}{\mu}(\nabla p - \rho g \nabla z) \tag{9}$$

$$\partial_t(\sigma T) - \nabla \cdot \big(\kappa \nabla T - T\vec{v}\big) = 0 \tag{10}$$

$$\partial_t(\phi C) + \nabla \cdot \big(C\vec{v} - D\phi\nabla C\big) = 0, \tag{11}$$

where

$$\sigma = \frac{\phi \rho_f c_f + (1-\phi)\rho_s c_s}{\rho_f c_f} \tag{12}$$

$$\kappa = \frac{\phi \lambda_f + (1-\phi)\lambda_s}{\rho_f c_f} \tag{13}$$

We will use the accent   in order to mark dimensionless quantities. In order to introduce dimensionless quantities, we make the following substitutions [8]:

$$x = h\hat{x}, z = h\hat{z} \tag{14}$$

$$t = \frac{h^2\sigma}{\kappa}\hat{t} \tag{15}$$

$$p = \frac{\mu\kappa}{k}\hat{p} + \rho_0 g z \tag{16}$$

$$T = T_0 + (T_1 - T_0)\hat{T} \tag{17}$$

$$C = C_0 + (C_1 - C_0)\hat{C} \tag{18}$$

$$\vec{v} = \frac{\kappa}{h}\hat{\vec{v}}, \tag{19}$$

which imply $\nabla = \frac{1}{h}\hat{\nabla}, \nabla \cdot \; = \frac{1}{h}\hat{\nabla} \cdot$ and $\partial_t = \dfrac{\kappa}{h^2\sigma}\partial_{\hat{t}}$.
Inserting these substitutions leads to the dimensionless equations

$$\hat{\nabla} \cdot \hat{\vec{v}} = 0 \tag{20}$$

$$\hat{\vec{v}} = -\hat{\nabla}\hat{p} - (\mathrm{Ra}_T\,\hat{T} - \mathrm{Ra}_S\,\hat{C})\hat{\nabla}\hat{z} \tag{21}$$

$$\partial_{\hat{t}}\hat{T} - \hat{\nabla} \cdot \big(\hat{\nabla}\hat{T} - \hat{T}\hat{\vec{v}}\big) = 0 \tag{22}$$

$$\partial_{\hat{t}}(\phi^\star \hat{C}) + \hat{\nabla} \cdot \big(\hat{C}\hat{\vec{v}} - \frac{1}{\mathrm{Le}}\hat{\nabla}\hat{C}\big) = 0. \tag{23}$$

They are controlled by four dimensionless numbers:

$$\mathrm{Ra}_T = \frac{khg\rho_0}{\mu\kappa}\alpha(T_1 - T_0) \quad \text{Thermal Rayleigh number}$$

$$\mathrm{Ra}_S = \frac{khg\rho_0}{\mu\kappa}\beta(C_1 - C_0) \quad \text{Solutal Rayleigh number}$$

$$\mathrm{Le} = \frac{\kappa}{D\,\phi} \quad \text{Lewis number}$$

$$\phi^* = \frac{\phi}{\kappa}$$

Some authors substitute Ra_S for the bouyancy ratio $R\rho = \frac{\beta(C_1-C_0)}{\alpha(T_1-T_0)}$

In the sequal, when the context is clear, we will omit the $\hat{}$ in the dimensionless equations.

A Symbols and their units

p	Pa	Fluid pressure
T	K	Temperature
C	kg/m^3	Salt concentration
T_0	K	Reference temperature
C_0	kg/m^3	Reference concentration
T_1	K	Maximal temperature
C_1	kg/m^3	Maximal concentration
$\vec{v}$	m/s	Volumetric flux
ρ	kg/m^3	Density of fluid
ρ_f	kg/m^3	Approximate density of fluid
ρ_0	kg/m^3	Reference density of fluid
c_f	J/kg·K	Heat capacity of fluid
λ_f	J/K·m·s	Heat conductivity of fluid
λ_s	J/K·m·s	Heat conductivity of solid
g	m/s^2	Gravity acceleration
ϕ	1	Porosity
k	m^2	Permeability
D	m^2/s	Diffusivity
μ_0	Pa·s	Reference viscosity
μ	Pa·s	Viscosity
ρ_s	kg/m^3	Density of solid matrix
c_s	J/kg·K	Heat capacity of solid matrix
α	K^{-1}	Heat expansion coefficient of fluid
β	m^3/kg	Solutal expansion coefficient of fluid
z	m	Coordinate function
h	m	Characteristic length
κ		Thermal diffusivity of saturated porous medium

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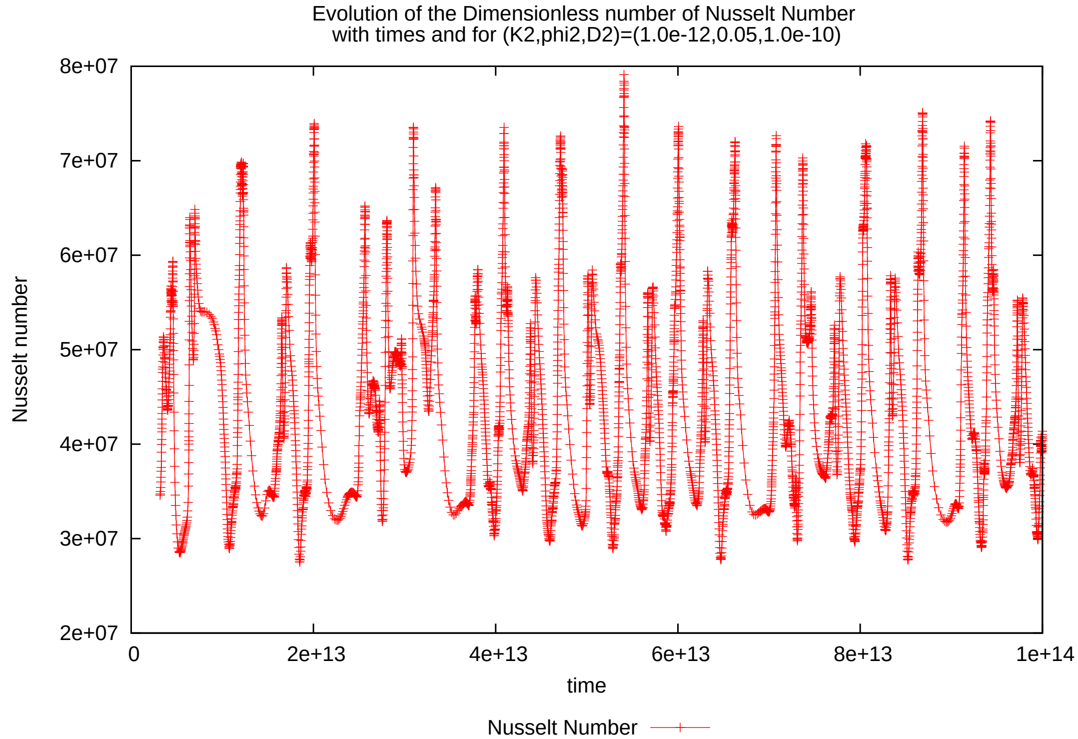
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with times and for (K2,phi2,D2)=(1.0e-12,0.05,1.0e-10)
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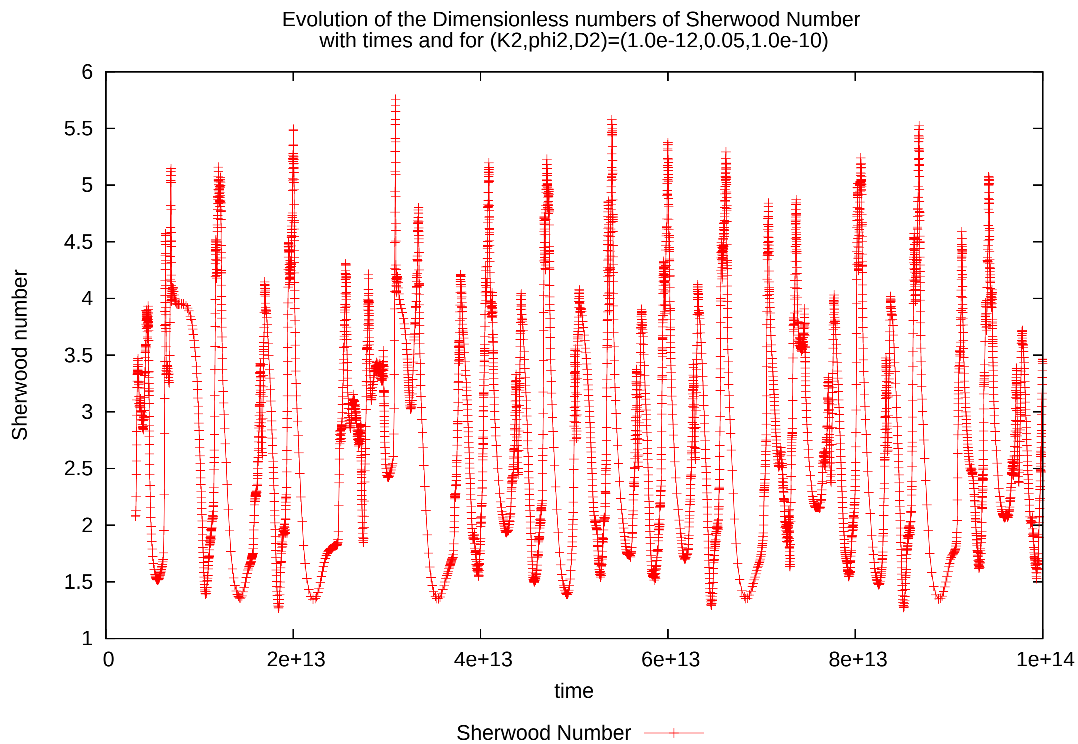
7e+07
6e+07
5e+07
4e+07
3e+07
2e+07
0
2e+13
4e+13
6e+13
8e+13
1e+14
time
Nusselt Number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5
4.5
4
3.5
3
2.5
2
1.5
1
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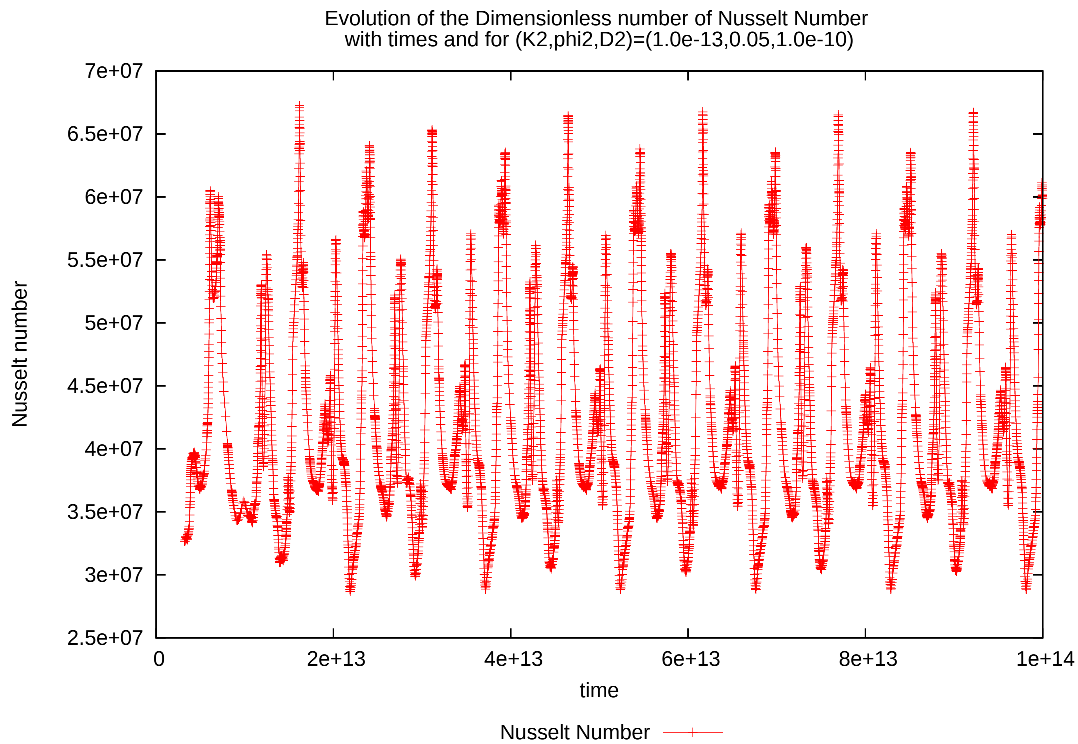
4e+13
6e+13
8e+13
1e+14
time
Sherwood Number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5e+07
4.5e+07
4e+07
3.5e+07
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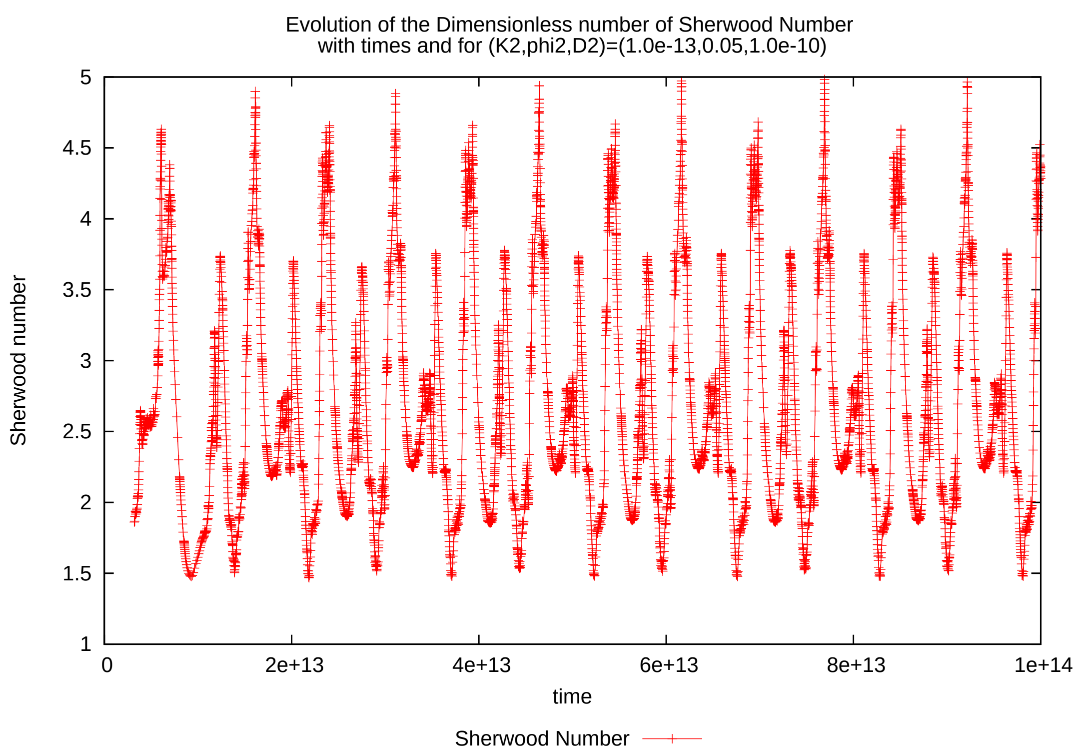
8e+13
1e+14
time
Nusselt Number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4
3.5
3
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1.5
1
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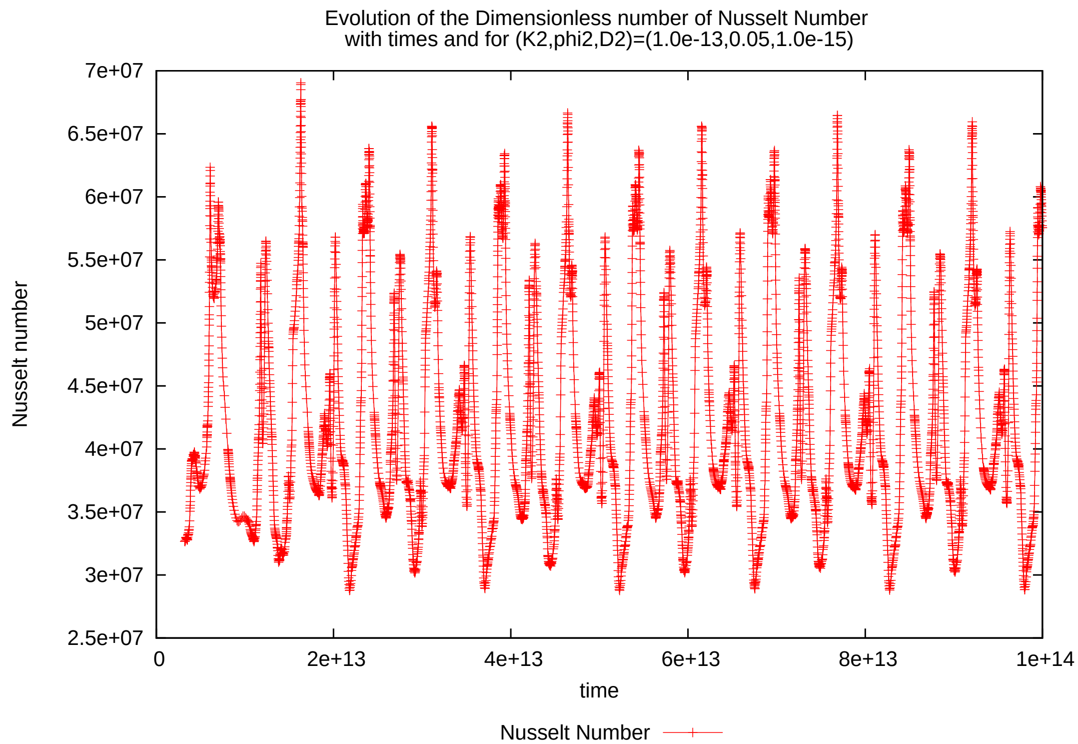
4e+13
6e+13
8e+13
1e+14
time
Sherwood Number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6.5e+07
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4e+07
3.5e+07
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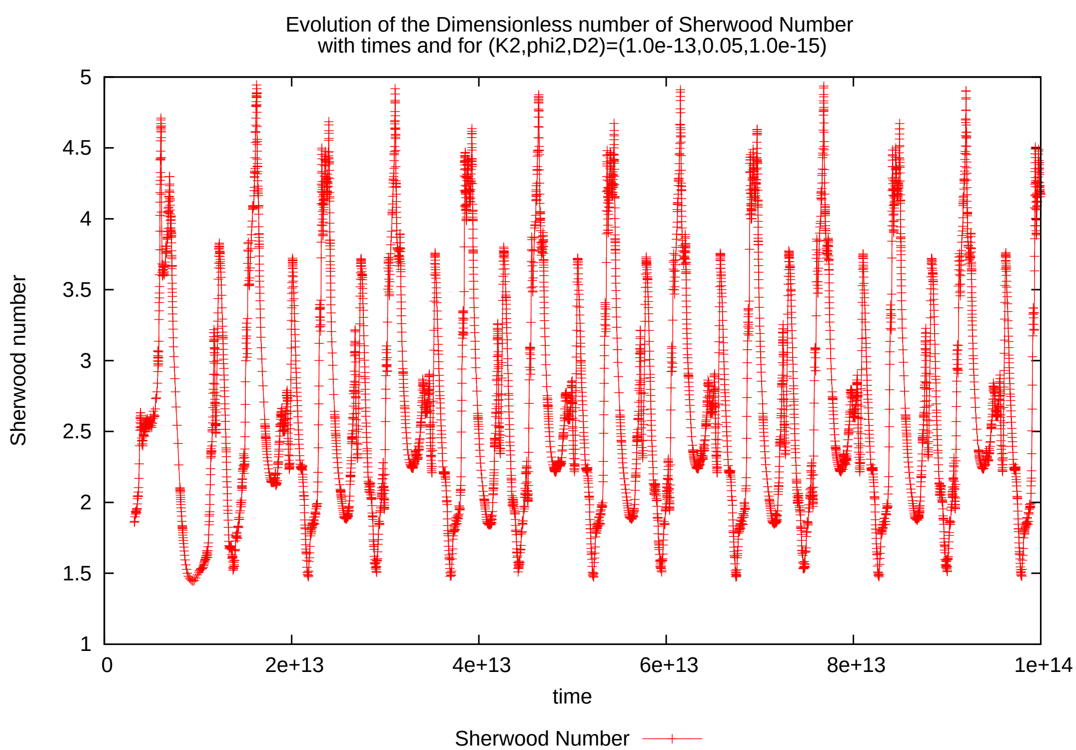
4e+13
6e+13
8e+13
1e+14
time
Nusselt Number


Evolution of the Dimensionless number of Sherwood Number
with times and for (K2,phi2,D2)=(1.0e-13,0.05,1.0e-15)
Sherwood number
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4.5
4
3.5
3
2.5
2
1.5
1
0
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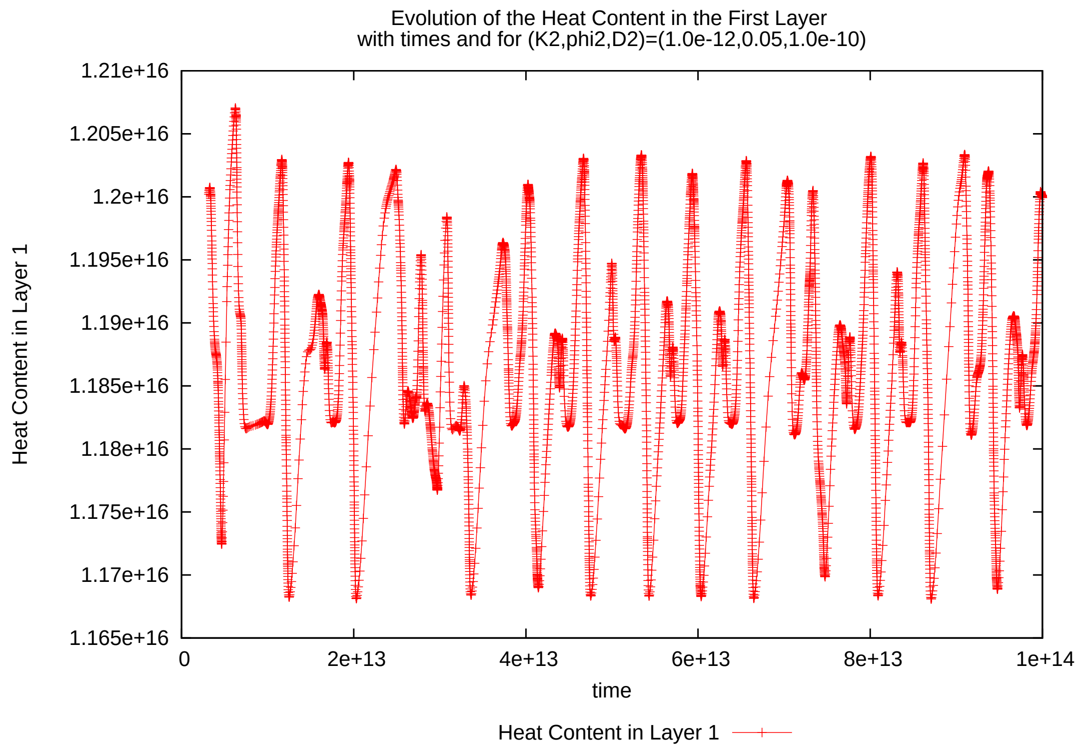
4e+13
6e+13
8e+13
1e+14
time
Sherwood Number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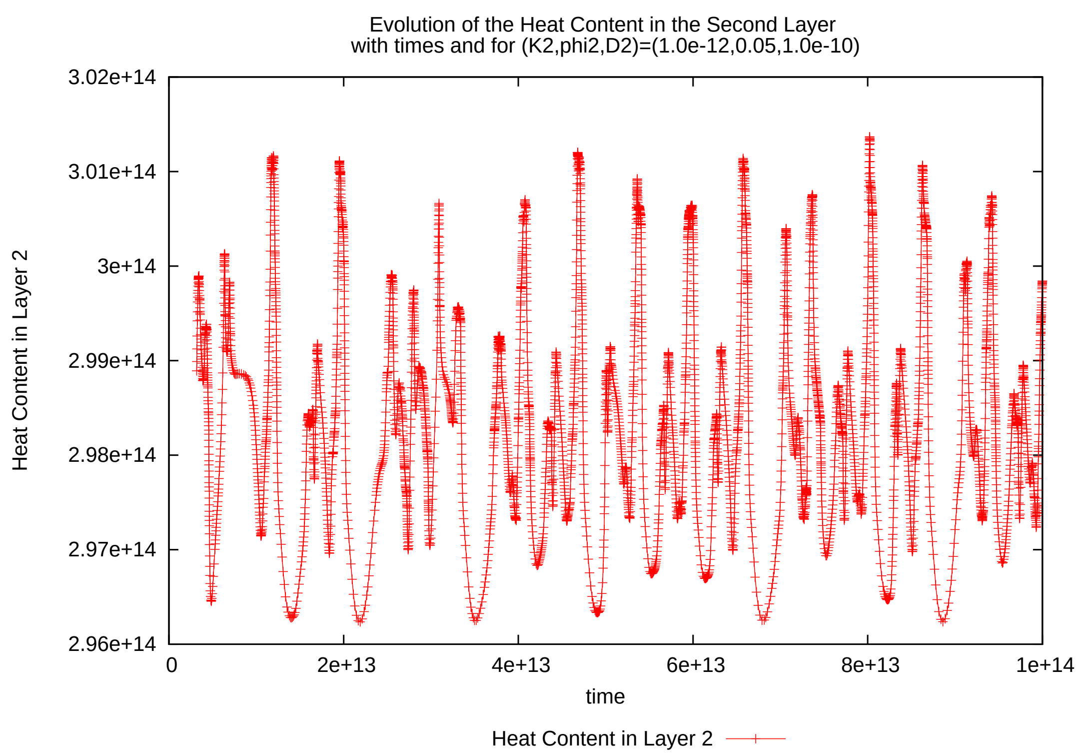
with times and for (K2,phi2,D2)=(1.0e-12,0.05,1.0e-10)
Heat Content in Layer 1
1.21e+16
1.205e+16
1.2e+16
1.195e+16
1.19e+16
1.185e+16
1.18e+16
1.175e+16
1.17e+16
1.165e+16
0
2e+13
4e+13
6e+13
8e+13
1e+14
time
Heat Content in Layer 1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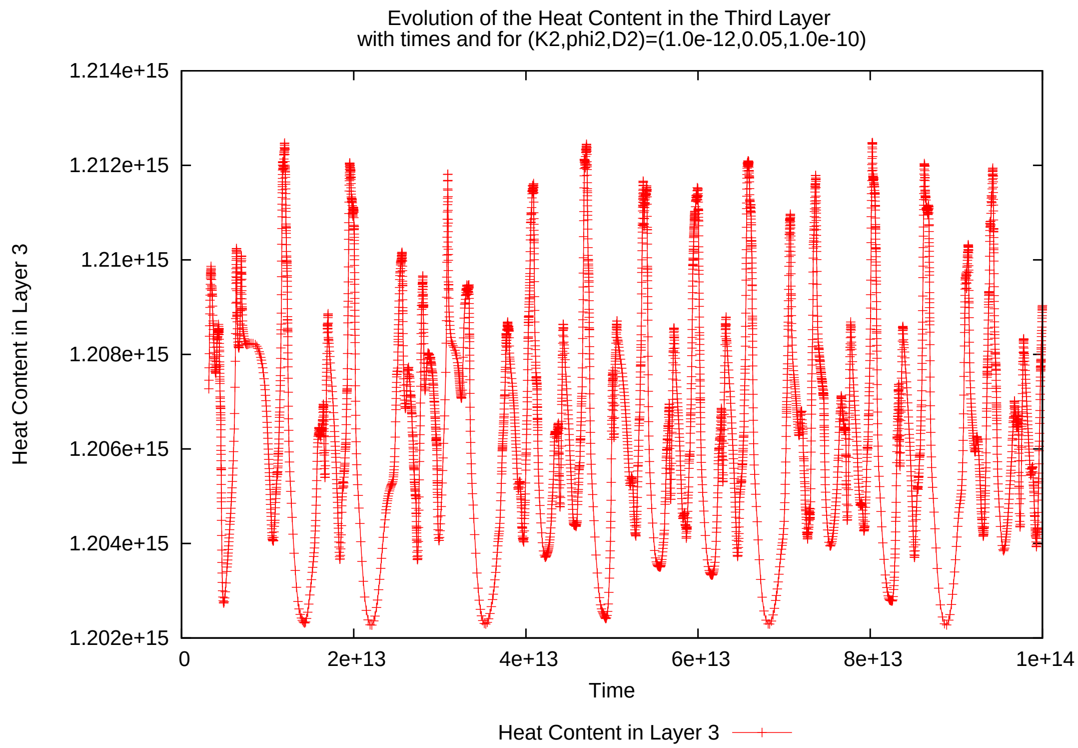
with times and for (K2,phi2,D2)=(1.0e-12,0.05,1.0e-10)
Heat Content in Layer 2
3.02e+14
3.01e+14
3e+14
2.99e+14
2.98e+14
2.97e+14
2.96e+14
0
2e+13
4e+13
6e+13
8e+13
1e+14
time
Heat Content in Layer 2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with times and for (K2,phi2,D2)=(1.0e-12,0.05,1.0e-10)
Heat Content in Layer 3
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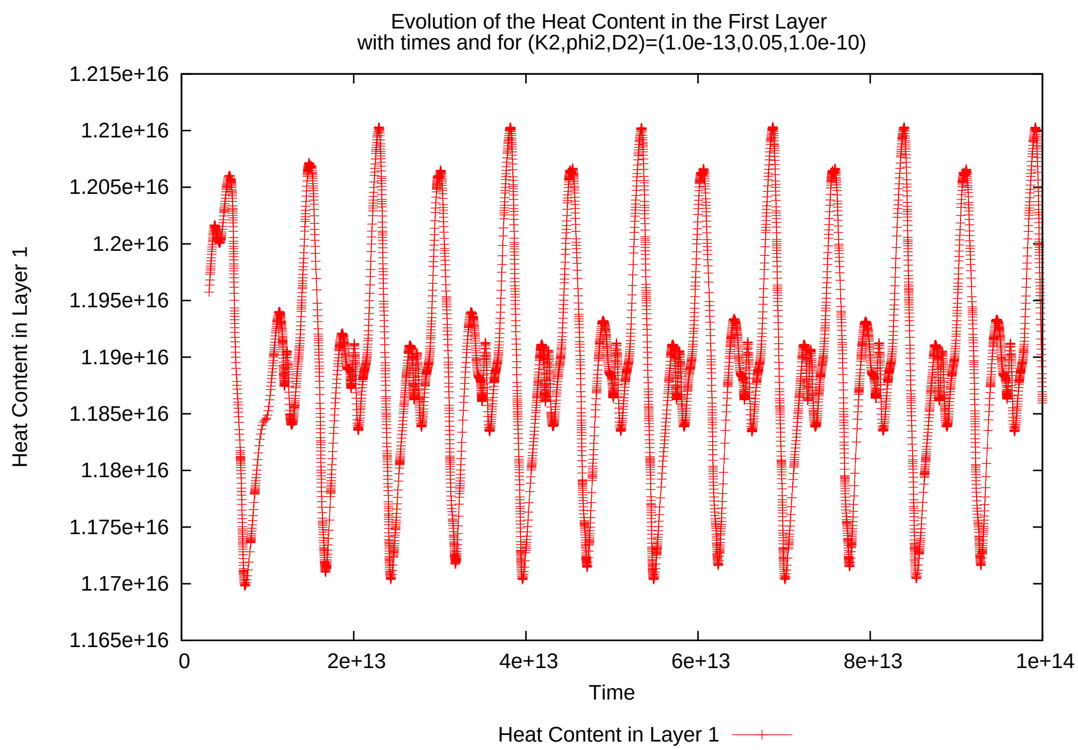
1.212e+15
1.21e+15
1.208e+15
1.206e+15
1.204e+15
1.202e+15
0
2e+13
4e+13
6e+13
8e+13
1e+14
Time
Heat Content in Layer 3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with times and for (K2,phi2,D2)=(1.0e-13,0.05,1.0e-10)
Heat Content in Layer 1
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1.21e+16
1.205e+16
1.2e+16
1.195e+16
1.19e+16
1.185e+16
1.18e+16
1.175e+16
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4e+13
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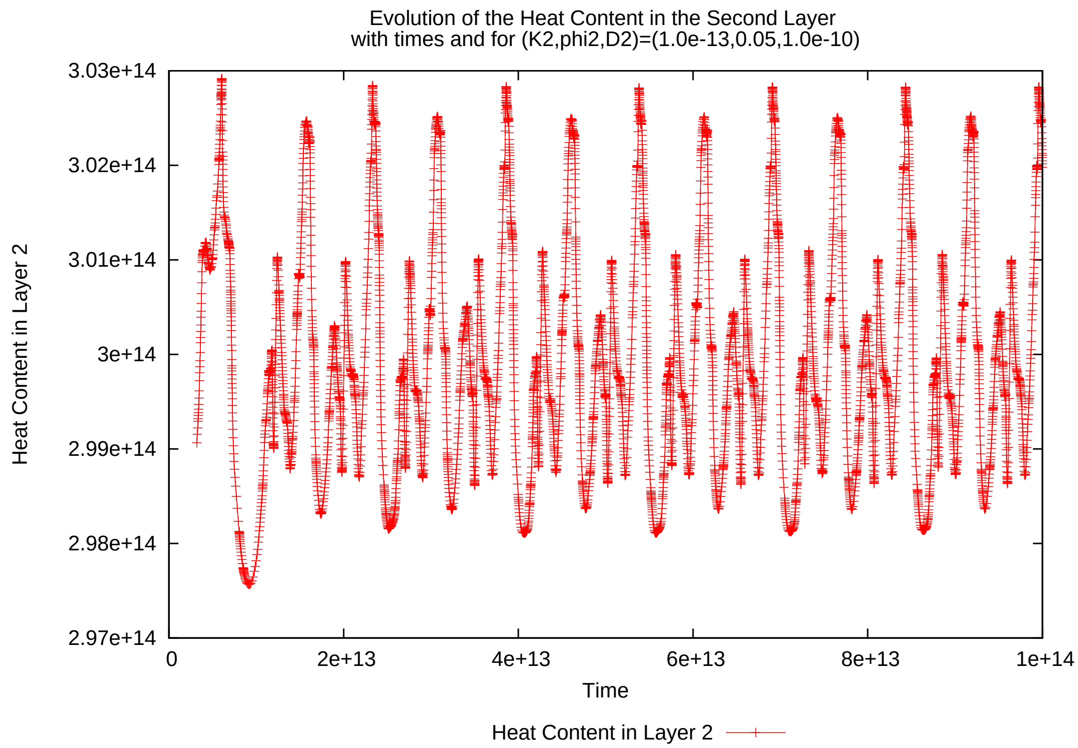
8e+13
1e+14
Time
Heat Content in Layer 1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with times and for (K2,phi2,D2)=(1.0e-13,0.05,1.0e-10)
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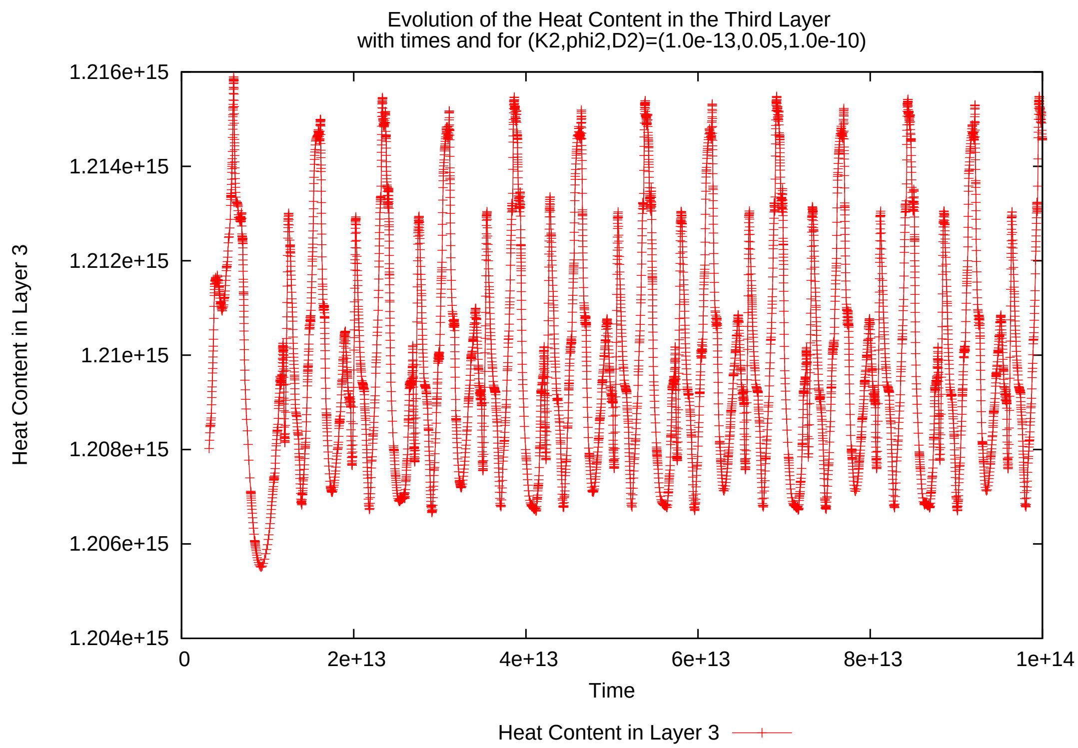
3.02e+14
3.01e+14
3e+14
2.99e+14
2.98e+14
2.97e+14
Heat Content in Layer 2
0
2e+13
4e+13
6e+13
8e+13
1e+14
Time
Heat Content in Layer 2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with times and for (K2,phi2,D2)=(1.0e-13,0.05,1.0e-10)
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1.214e+15
1.212e+15
1.21e+15
1.208e+15
1.206e+15
1.204e+15
Heat Content in Layer 3
0
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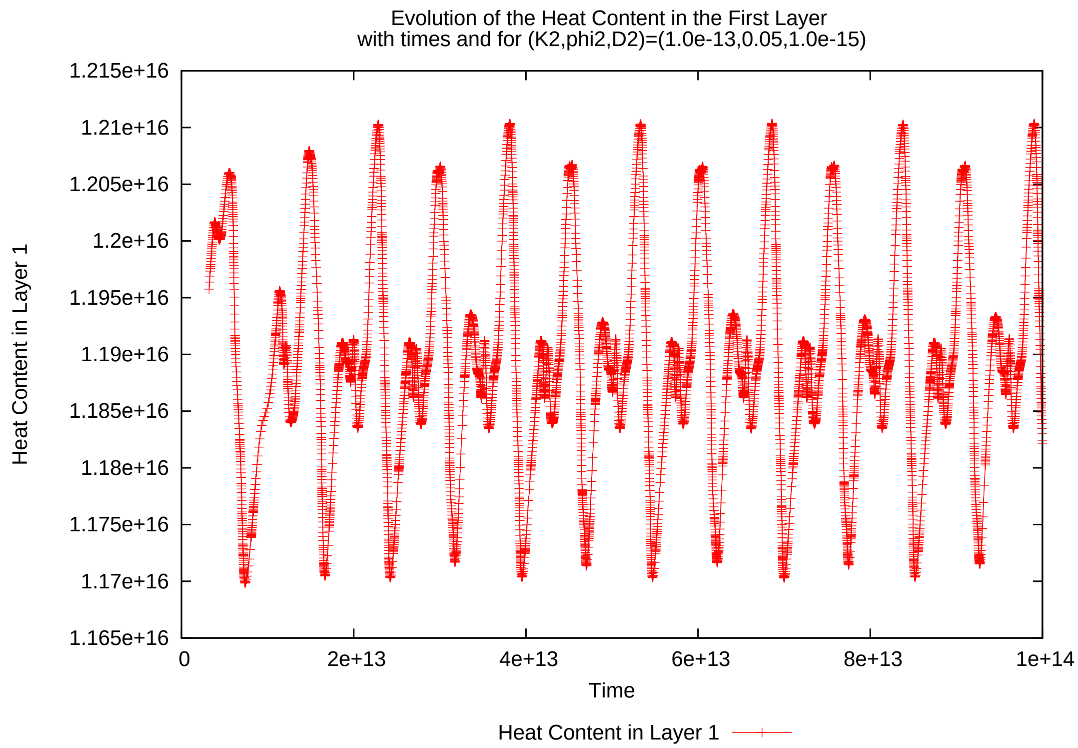
4e+13
6e+13
8e+13
1e+14
Time
Heat Content in Layer 3


Evolution of the Heat Content in the First Layer
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1.205e+16
1.2e+16
1.195e+16
1.19e+16
1.185e+16
1.18e+16
1.175e+16
1.17e+16
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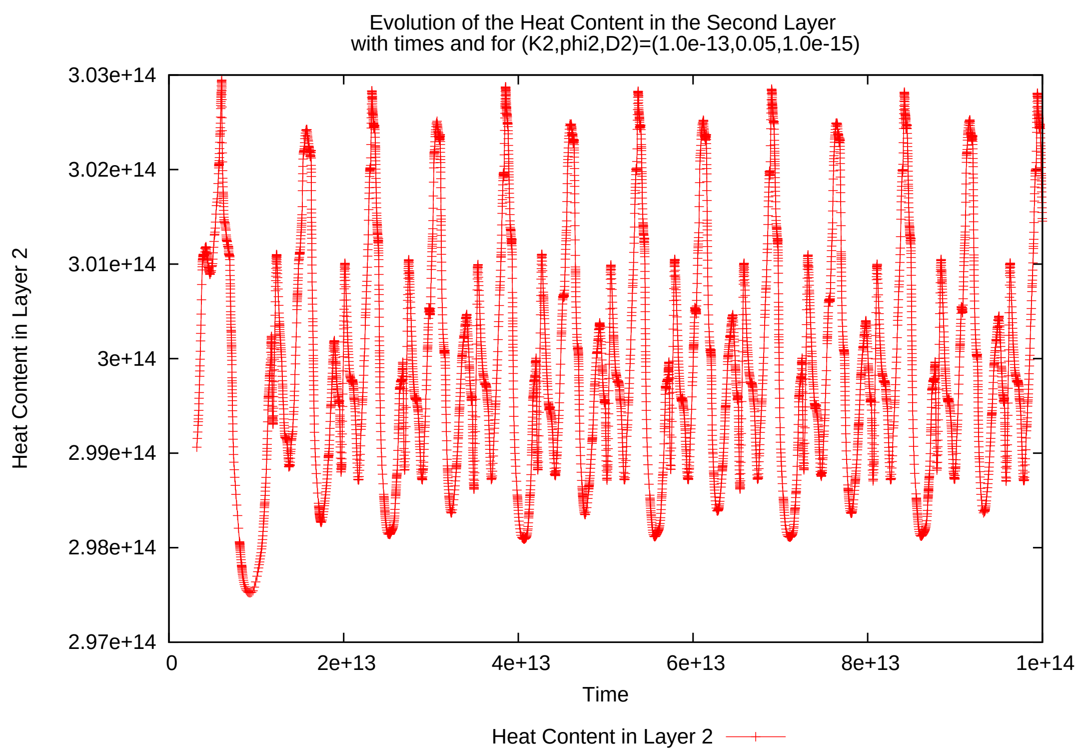
1e+14
Time
Heat Content in Layer 1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3.02e+14
3.01e+14
3e+14
2.99e+14
2.98e+14
2.97e+14
Heat Content in Layer 2
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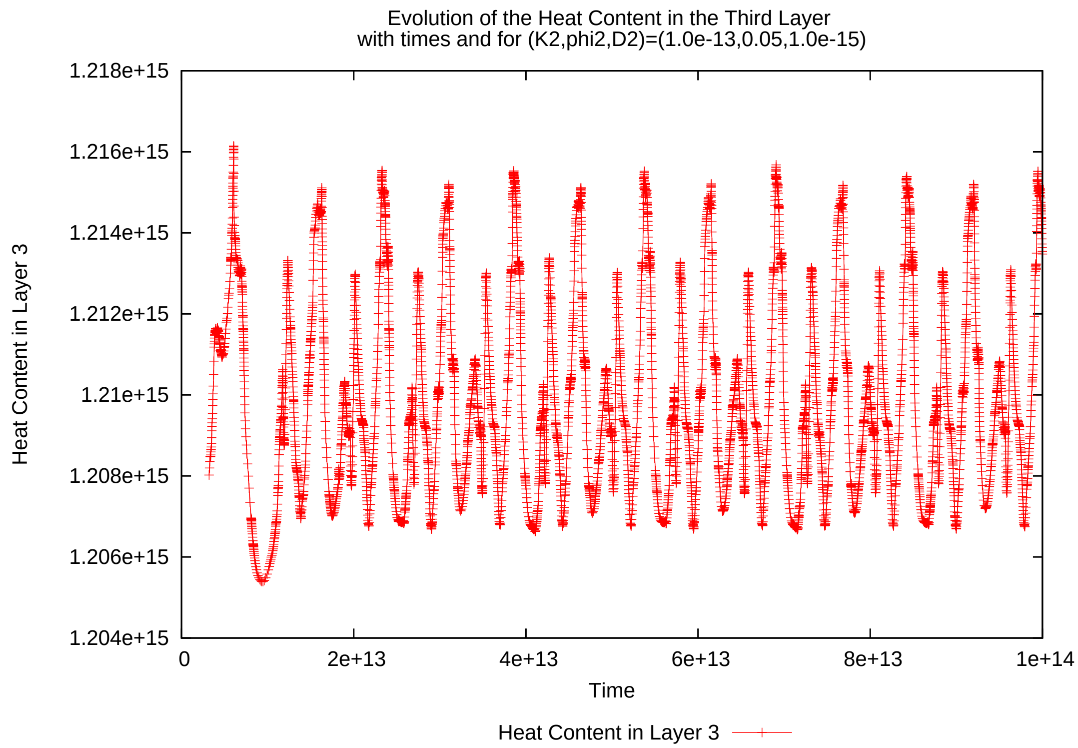
6e+13
8e+13
1e+14
Time
Heat Content in Layer 2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with times and for (K2,phi2,D2)=(1.0e-13,0.05,1.0e-15)
Heat Content in Layer 3
Time
Heat Content in Layer 3
1.218e+15
1.216e+15
1.214e+15
1.212e+15
1.21e+15
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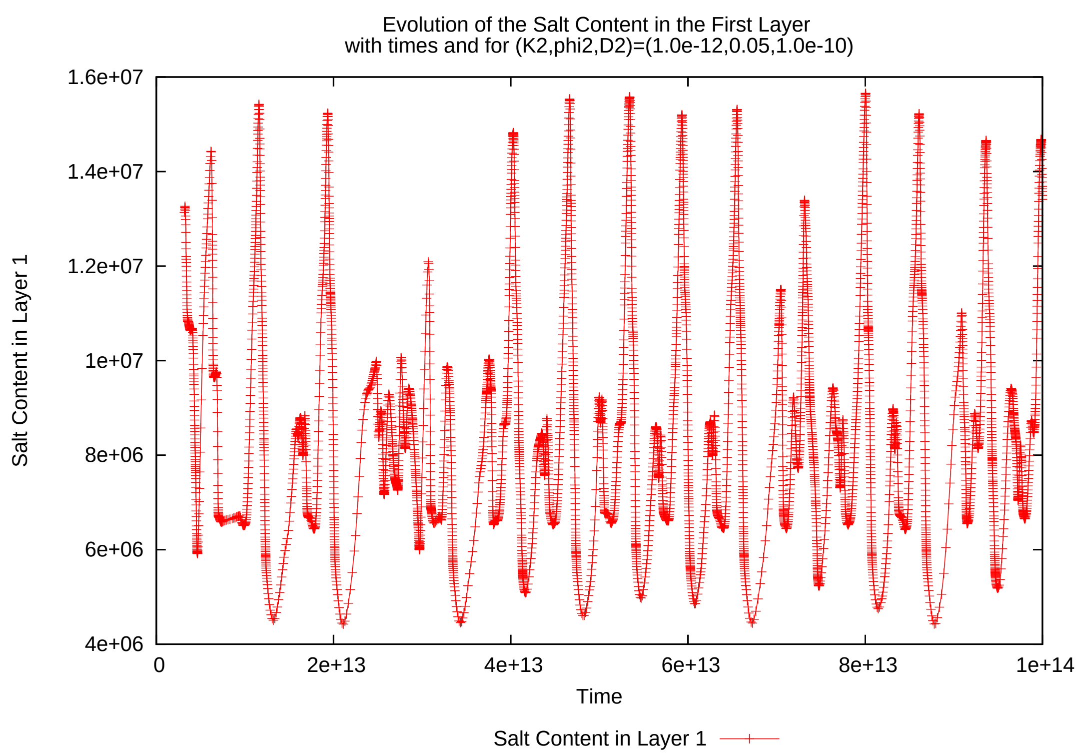
4e+13
6e+13
8e+13
1e+14


Evolution of the Salt Content in the First Layer
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Salt Content in Layer 1
Time
Salt Content in Layer 1
1.6e+07
1.4e+07
1.2e+07
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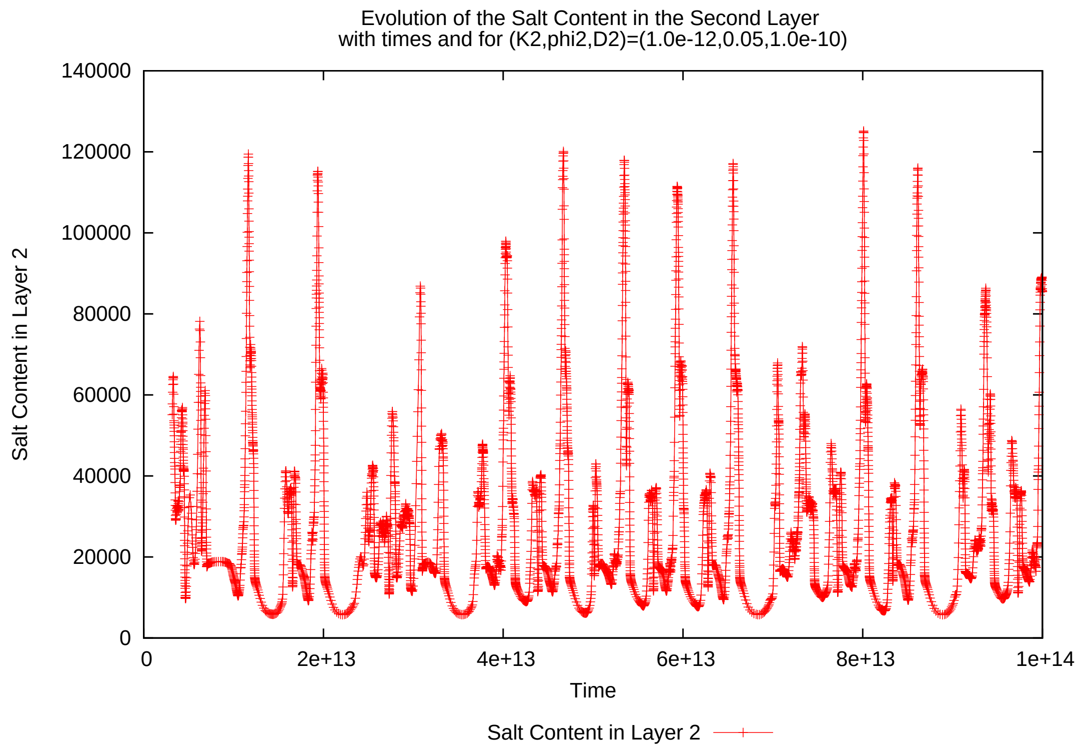
8e+13
1e+14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120000
100000
80000
60000
40000
20000
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Salt Content in Layer 2
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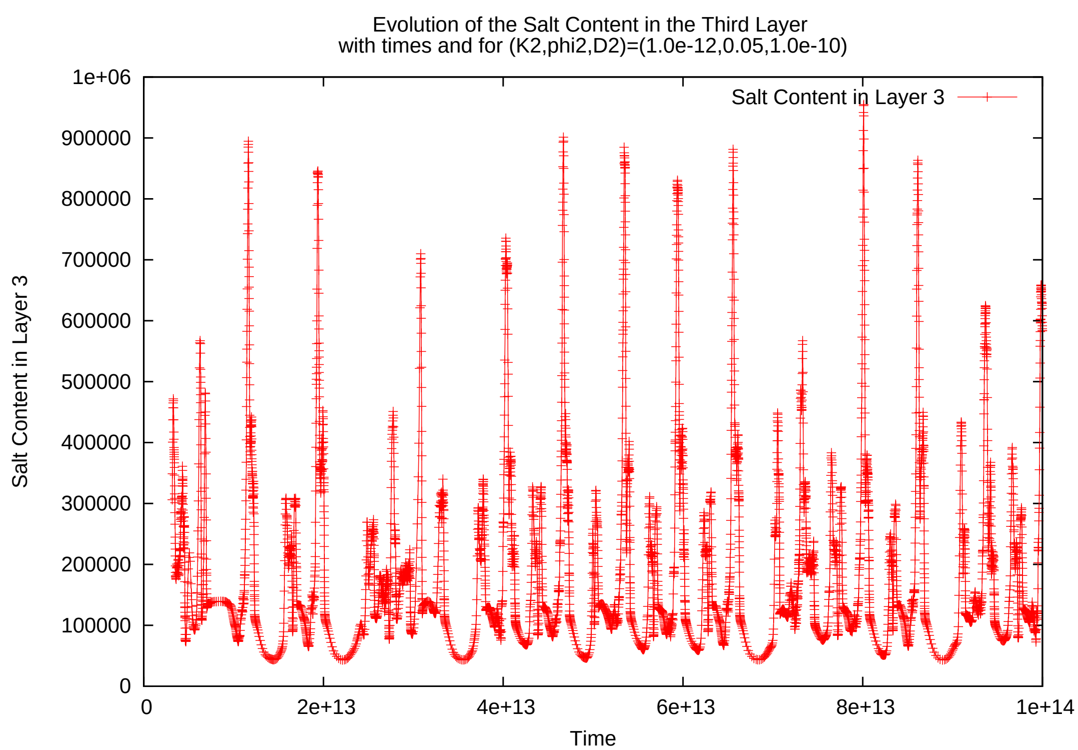
8e+13
1e+14
Time
Salt Content in Layer 2


Evolution of the Salt Content in the Third Layer
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Salt Content in Layer 3
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900000
800000
700000
600000
500000
400000
300000
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100000
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Salt Content in Layer 3
0
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8e+13
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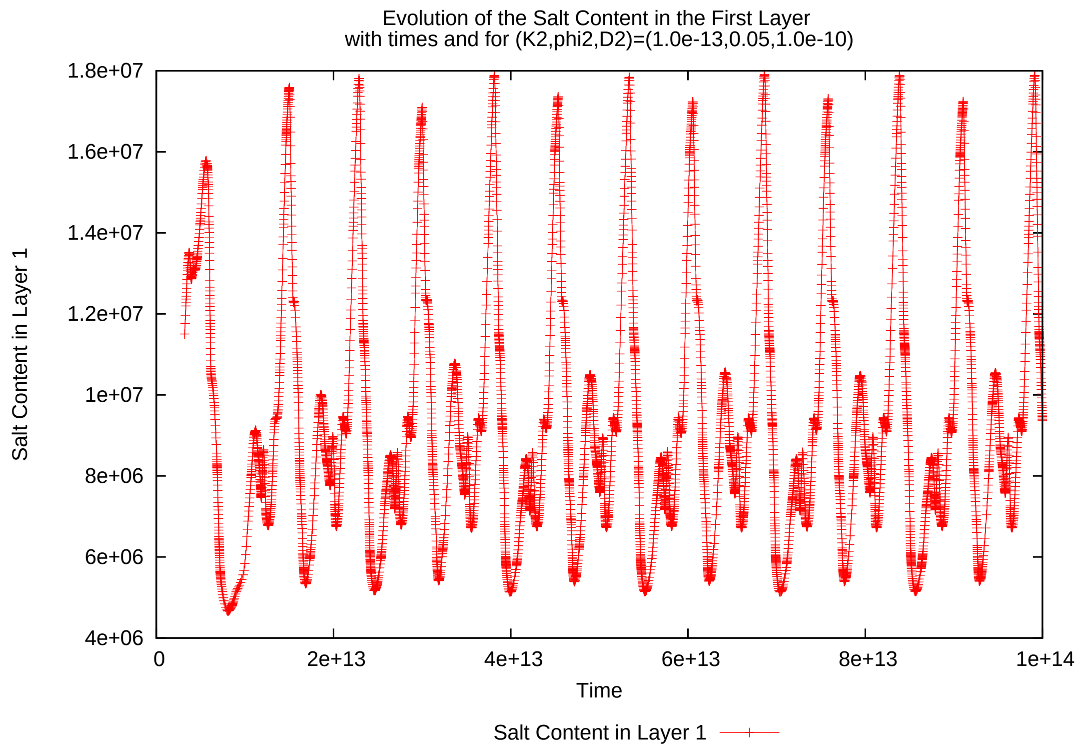
Time



Evolution of the Salt Content in the First Layer
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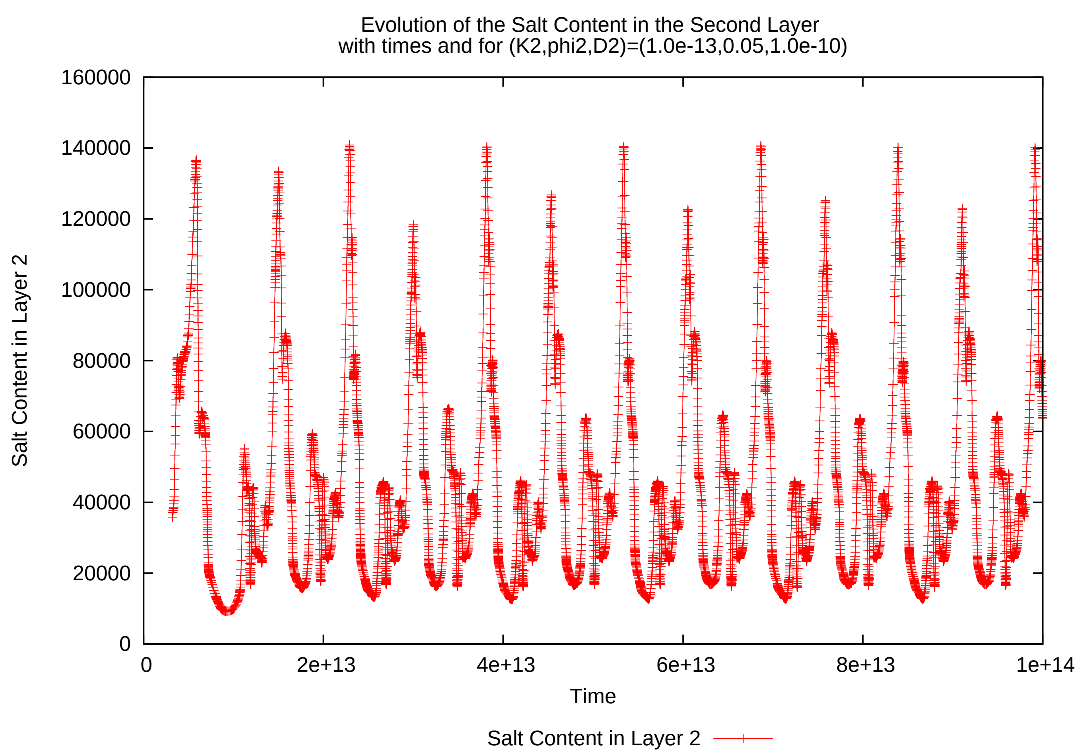
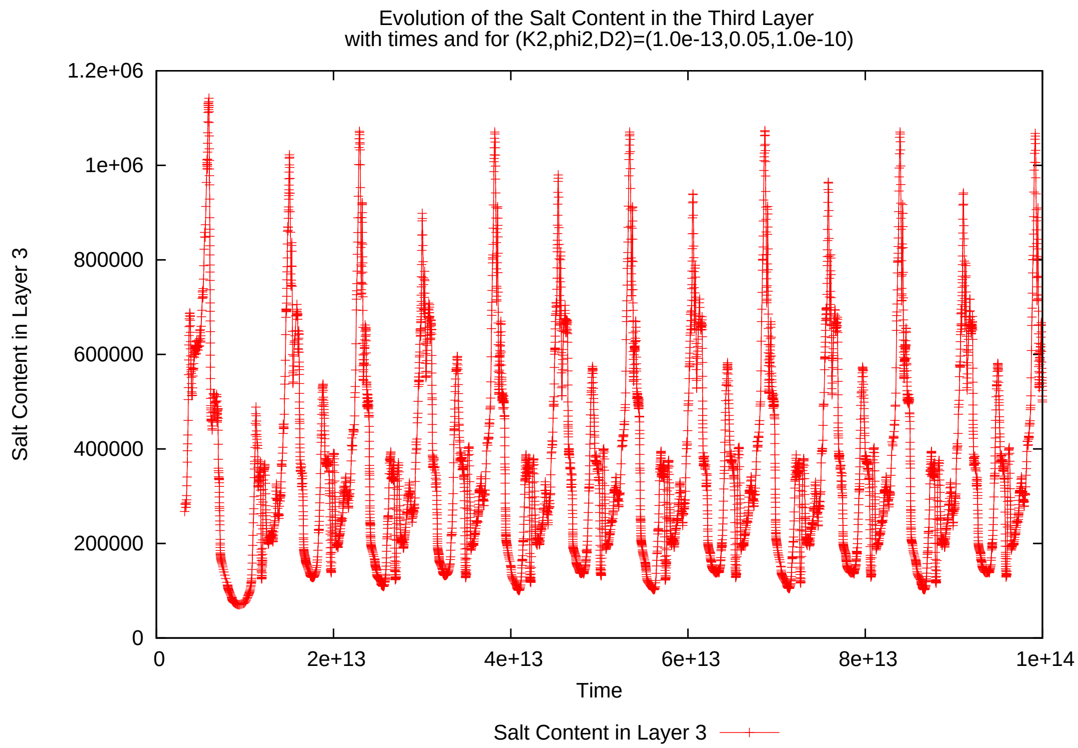
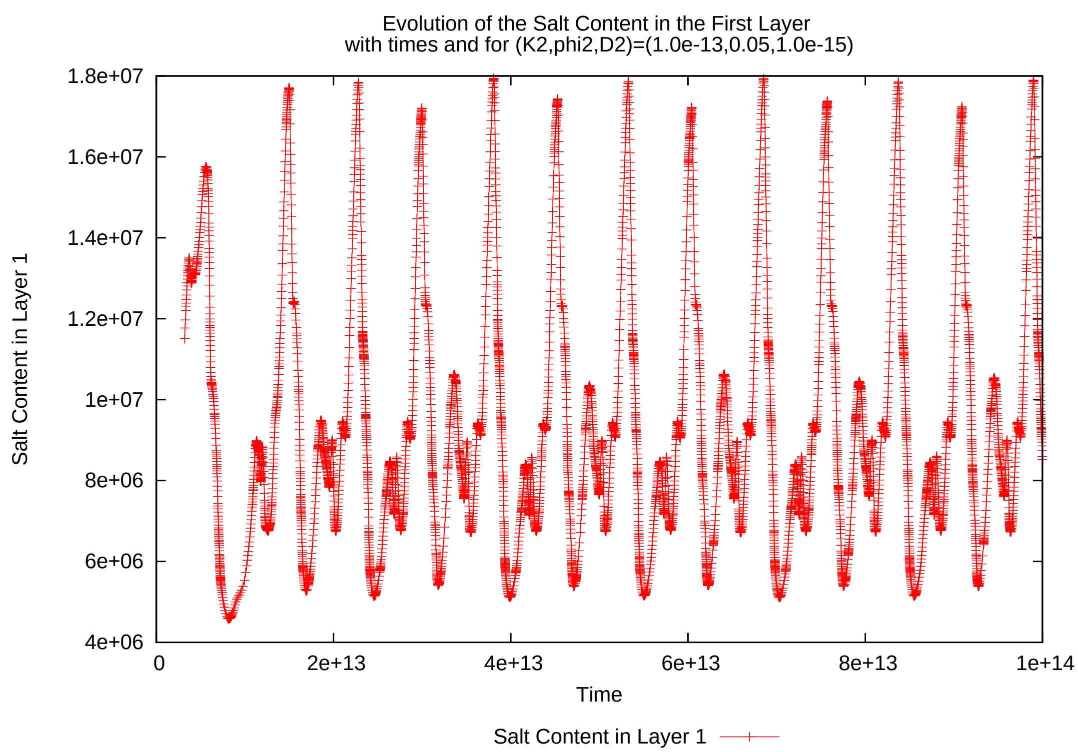
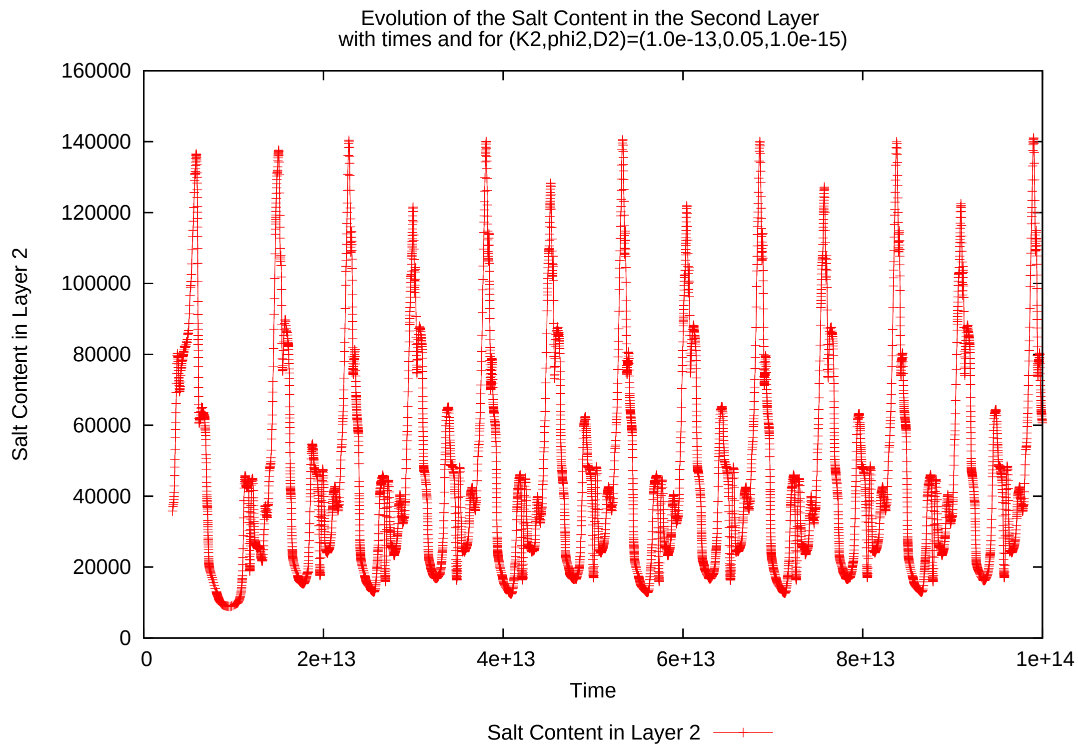
Salt Content in Layer 1
Time
Salt Content in Layer 1

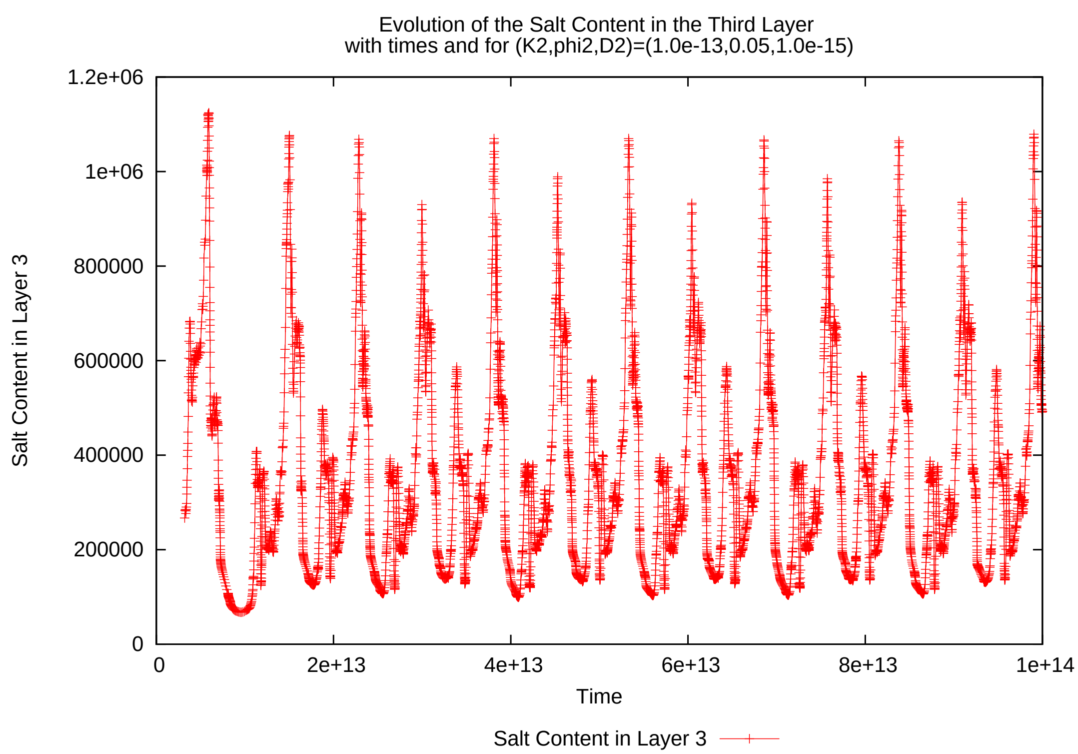


Evolution of the Salt Content in the Second Layer
with times and for (K2,phi2,D2)=(1.0e-13,0.05,1.0e-10)
Salt Content in Layer 2
Time
Salt Content in Layer 2


Evolution of the Salt Content in the Third Layer
with times and for (K2,phi2,D2)=(1.0e-13,0.05,1.0e-10)
Salt Content in Layer 3
1.2e+06
1e+06
800000
600000
400000
200000
0
0
2e+13
4e+13
6e+13
8e+13
1e+14
Time
Salt Content in Layer 3


Evolution of the Salt Content in the First Layer
with times and for (K2,phi2,D2)=(1.0e-13,0.05,1.0e-15)
Salt Content in Layer 1
1.8e+07
1.6e+07
1.4e+07
1.2e+07
1e+07
8e+06
6e+06
4e+06
0
2e+13
4e+13
6e+13
8e+13
1e+14
Time
Salt Content in Layer 1


Evolution of the Salt Content in the Second Layer
with times and for (K2,phi2,D2)=(1.0e-13,0.05,1.0e-15)
Salt Content in Layer 2
160000
140000
120000
100000
80000
60000
40000
20000
0
0
2e+13
4e+13
6e+13
8e+13
1e+14
Time
Salt Content in Layer 2


Evolution of the Salt Content in the Third Layer
with times and for (K2,phi2,D2)=(1.0e-13,0.05,1.0e-15)
Salt Content in Layer 3
1.2e+06
1e+06
800000
600000
400000
200000
0
0
2e+13
4e+13
6e+13
8e+13
1e+14
Time
Salt Content in Layer 3

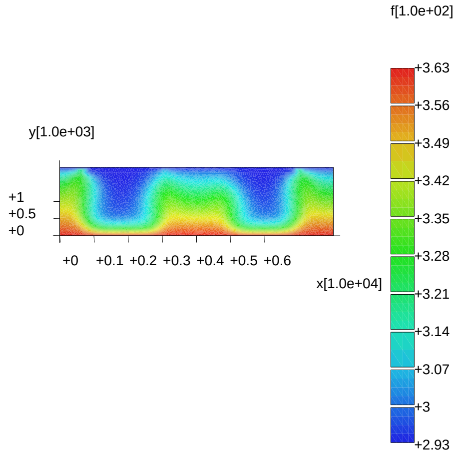


Figure 1: Solution in temperature for $(K_2, phi_2, D_2) = (1.0e-12, 0.05, 1.0e-10)$

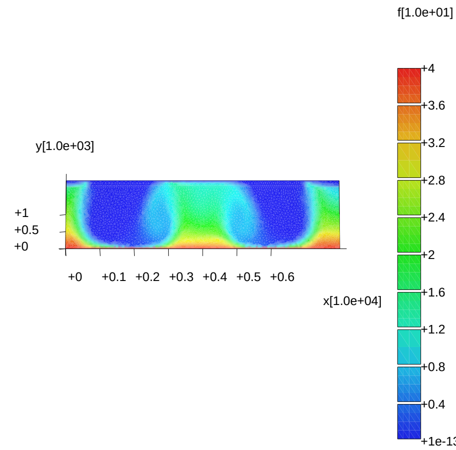
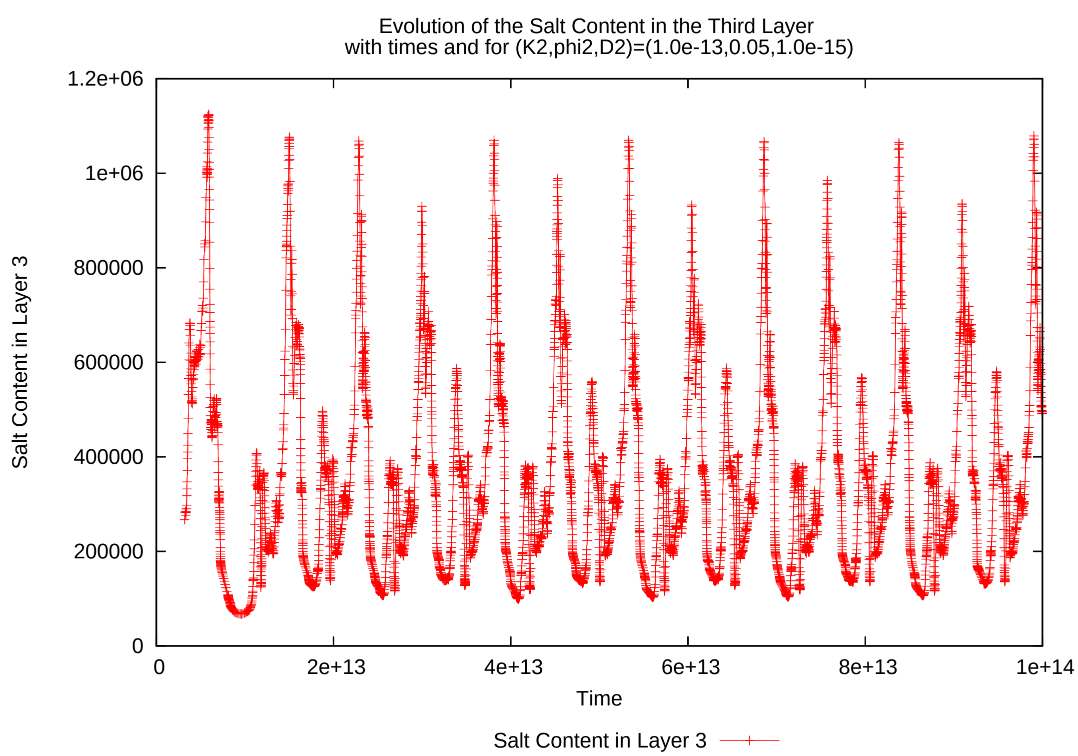


Figure 2: Solution in concentration for $(K_2, phi_2, D_2) = (1.0e-12, 0.05, 1.0e-10)$



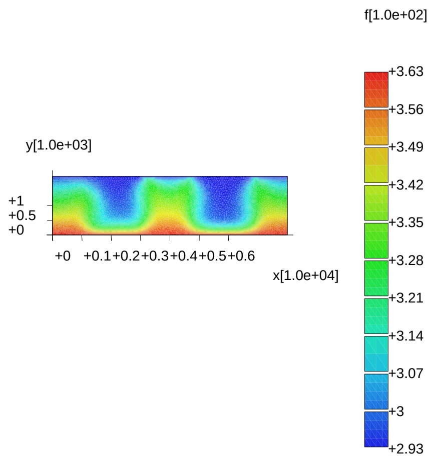


Figure 3: Solution in temperature for $(K_2, phi_2, D_2) = (1.0e-13, 0.05, 1.0e-10)$

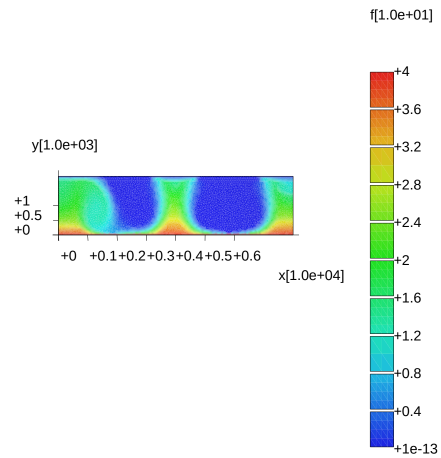


Figure 4: Solution in concentration for $(K_2, phi_2, D_2) = (1.0e-13, 0.05, 1.0e-10)$

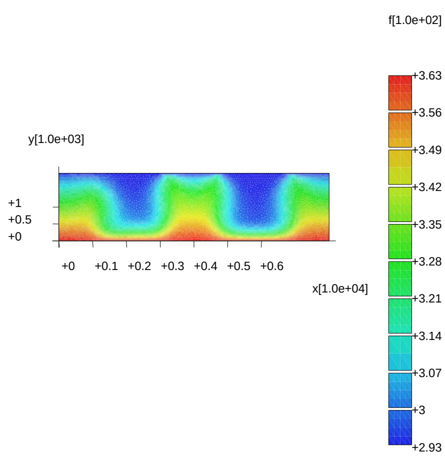


Figure 5: Solution in temperature for $(K_2, phi_2, D_2) = (1.0e-13, 0.05, 1.0e-15)$

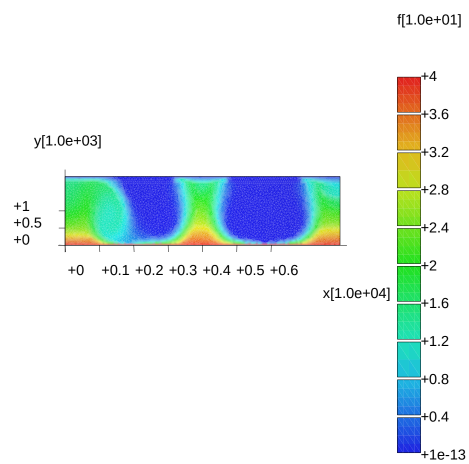


Figure 6: Solution in concentration for $(K_2, phi_2, D_2) = (1.0e-13, 0.05, 1.0e-15)$