Completeness of Systems of Translates and Uniqueness Theorems for Asymptotically Holomorphic Functions

A.A. Borichev

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We consider some completeness problems for systems of right and arbitrary translates in certain weighted spaces of functions on the real line. A generalization of the Titchmarsh convolution theorem and a tauberian theorem for quasianalytic Beurling-type algebras are obtained.

The solution of these problems involves the usage of the so-called generalized Fourier transform. After that, completeness problems turn into uniqueness problems of the theory of functions, which are interesting in themselves.

This report is a short version of the work, one part of which is published in [3], and the other will appear in [1].

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We deal with problems concerning completeness of systems of translates $\{\tau_t f\}$,

$$(\tau_t f)(x) = f(x - t),$$

in spaces A of functions on the real line. Via the Hahn-Banach theorem, such problems are transformed into problems on solving convolution equations: on the whole line,

$$a * b = 0,$$

if we take all translates, and on the half-line,

$$(a * b)_- = 0,$$

if we take only right translates, where $a \in A$, $b \in A^*$,

$$(a * b)(t) = \langle a, \tau_t b \rangle, \quad f_-|\mathbf{R}_- = f|\mathbf{R}_-, \quad f_-|\mathbf{R}_+ = 0, \quad f_+ = f - f_-.$$

Suppose that the Fourier transforms $\mathcal{F}a$ and $\mathcal{F}b$ of the convolutors a and b (as usual,

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$$\mathcal{F}f(z) = \int f(x)e^{ixz}dx)$$

are well-defined on certain sets. If the intersection of these sets is non-empty, then the equations $a * b = 0$, $(a * b)_+ = 0$ can be rewritten accordingly as $\mathcal{F}a \cdot \mathcal{F}b = 0$, $\mathcal{F}a \cdot \mathcal{F}b \in \mathcal{F}(A * A^*)_+$; that is, harmonic analysis problems turn into multiplicative problems from the theory of functions.

In studying the possibility of extending this method to the case when these sets are disjoint (in particular, when the Fourier transform doesn't exist for one of the spaces A, A^*) it turns out that one can construct an analogue of the Fourier transform, that maps convolutions into products. It is applicable to very rapidly growing functions for which the usual Fourier transform cannot be used. Though the Fourier images of these functions are not necessarily analytic, they are asymptotically holomorphic, i.e. they satisfy

$$f \in C^1(\Omega),\ |\overline{\partial} f(z)| < w(\mathrm{dist}(z, \partial\Omega)), \quad w(0) = 0,$$

where $\overline{\partial} = \frac{1}{2}\left(\frac{\partial}{\partial x} + i\frac{\partial}{\partial y}\right)$. We further remark that this "generalized" Fourier transform is consistent; that is, it coincides with the usual Fourier transform on the domain of definition of the latter.

Unfortunately, this technique doesn't allow us to work with Banach spaces. Thus we are lead to study problems of completeness in projective (inductive) limits (of weighted spaces). In addition, for the weighted spaces under consideration, we sometimes need very strong conditions on the regularity of the weight (which shall not be written down precisely in this paper).

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The usage of generalized Fourier transform is particularly efficient in problems of spectral analysis; that is, for cases of empty spectrum. See [13] about the spectral analysis-synthesis problem.

This paper is devoted to two problems, stated in [7], [12, Problem 7.18], and in [12, Problem 7.19].

(a) The classical Titchmarsh convolution theorem claims that the convex hull of the support of the convolution of two functions with compact supports is equal to the sum of their supports. The condition of compactness cannot be omitted in general (it is sufficient to consider $u \equiv 1$, and a function v with $\operatorname{supp} v \subset (0,1)$, $\int_0^1 v(x)dx = 0$).

In the papers of Domar [7] and Ostrovskii [15] the Titchmarsh theorem was extended to the case of functions, decaying rapidly on the negative half-line (approximately as $\exp(-x^2)$, $\exp(-|x|\log|x|)$) and possibly, having some growth on the positive half-line:

$$\inf \operatorname{supp} u + \inf \operatorname{supp} v = \inf \operatorname{supp} u * v. \tag{3.1}$$

It should be mentioned that these papers were motivated by problems from such different domains as radical Banach algebras and probability theory.

When u and v are bounded or their growth is at most exponential, one can apply the Fourier transform and standard analytic techniques such as those in [15] to prove (1).

We introduce a self-adjoint topological algebra $\mathcal{U}$,

$$\mathcal{U} = \left\{ f \in L^2_{\mathrm{loc}}(\mathbf{R}) \ : \ \forall c > 0, \int_{\mathbf{R}_+} |f(x)|^2 (p(x))^{-c} dx < \infty, \right.$$
$$\left. \exists c > 0, \int_{\mathbf{R}_+} |f(-x)|^2 (p(x))^{c} dx < \infty \right\},$$

where $p \in C(\mathbf{R}_+)$, $\log p(x)$ is a convex function, $\lim_{x\to\infty} x^{-1} \log p(x) = \infty$ and for some $c < \infty$ the function $x^{-c} \log p(x)$ decreases (for large x).

A weakened version of (1) for $\mathcal{U}$,

$$u \in \mathcal{U}_+, \ v \in \mathcal{U}_- \Rightarrow \inf \operatorname{supp} u + \inf \operatorname{supp} v = \inf \operatorname{supp} u * v \tag{3.2}$$

is equivalent to the fact that elements f of the algebra $\mathcal{U}_+$ are cyclic (that is $\operatorname{clos} \mathcal{L}\{\tau_t f, \, t \geq 0\} = \mathcal{U}_+$) if and only if $0 \in \operatorname{ess\,supp} f$.

The statement on the non-existence of zero divisors in $\mathcal{U}$,

$$u, v \in \mathcal{U}, \quad u * v = 0 \quad \Rightarrow \quad u = 0 \ \text{ or } \ v = 0 \tag{3.3}$$

is equivalent to the fact that every non-zero element f of $\mathcal{U}$ is cyclic; that is $\operatorname{clos} \mathcal{L}\{\tau_t f, \, t \in \mathbf{R}\} = \mathcal{U}$.

Finally, the equality (1) is equivalent to the fact that

$$\operatorname{clos} \mathcal{L}\{\tau_t f, \, t \geq 0\} = \mathcal{U} \Leftrightarrow \inf \operatorname{supp} f = -\infty.$$

(b) The general tauberian theorem of Wiener claims that every closed ideal $I \subset L^1(\mathbf{R})$ such that the Fourier transforms $\mathcal{F}f(t)$, $f \in I$ do not have common zeros on $\mathbf{R}$ is equal to $L^1(\mathbf{R})$ itself.

A. Beurling, in the late thirties, introduced a class of function algebras, so called Beurling algebras,

$$L^1_p(\mathbf{R}) = \{f : \ fp \in L^1(\mathbf{R})\},$$

where $p(x)p(y) \geq p(x+y)$, $p(x) \geq p(0) = 1$, $p(tx) \geq p(x)$, $t \geq 1$. These conditions on the weight p imply the existence of two limits

$$\alpha_{\pm} = \lim_{x \to \pm\infty} \frac{\log p(x)}{x}.$$

Beurling divided these weights into three groups; the analytic case, $\alpha_+ > \alpha_-$; the quasianalytic case,

$$\alpha_+ = \alpha_-, \quad \int_{-\infty}^{\infty} \frac{\log p(x) - \alpha_+|x|}{1+x^2} dx = \infty;$$

and the non-quasianalytic case,

$$\alpha_+ = \alpha_-, \quad \int_{-\infty}^{\infty} \frac{\log p(x) - \alpha_+|x|}{1+x^2} dx < \infty.$$

This classification is natural enough because the Fourier transform maps elements of $L^1_p(\mathbf{R})$ into functions which are continuous in the strip region $S = \{z : \alpha_- \le \operatorname{Im} z \le \alpha_+\}$, and are analytic in its interior.

Further, the divergence of the integral

$$\int_{-\infty}^{\infty} \frac{\log p(x) - \alpha_+|x|}{1+x^2} dx, \quad \alpha_+ = \alpha_-,$$

is equivalent to the quasianalyticity of the Fourier transforms of elements of $L^1_p(\mathbf{R})$ on the boundary of S (in the sense of the Denjoy-Carleman theorem).

Beurling proved that an analog of Wiener's theorem is valid in the non-quasianalytic case.

A closed ideal I in $L^1_p(\mathbf{R})$ is said to be primary at ∞ if

$$\bigcap_{f\in I} \{z \in S : \mathcal{F}f(z) = 0\} = \emptyset.$$

In 1950 Nyman [14] proved that in some particular (analytic and quasianalytic) cases there exist primary ideals at ∞ in $L^1_p(\mathbf{R})$. Later, Korenblum [11], Vretblad [16], and Domar [5] demonstrated chains of primary ideals in the general case, Hedenmalm [8] described all primary ideals at ∞ in the analytic-non-quasianalytic case (for $p(x) = \exp c|x|$ it was made by Korenblum [10]).

Here the analytic-non-quasianalytic case is the following one:

$$\alpha_+ > \alpha_-, \quad \int^{\infty} \frac{\log p(x) - \alpha_+ x}{1+x^2} dx < \infty, \quad \int_{-\infty} \frac{\log p(x) - \alpha_- x}{1+x^2} dx < \infty.$$

The ideals are parameterized by two numbers as follows:

$$I = I^+_{\delta_+} \cap I^-_{\delta_-}, \ \delta_\pm = \inf_{f\in I} \delta_\pm(f), \tag{3.4}$$

$$I^\pm_\delta = \{f \in L^1_p(\mathbf{R}) : \ \delta_\pm(f) \ge \delta\}, \tag{3.5}$$

$$\delta_\pm(f) = \varliminf_{x\to\infty} \frac{2\alpha}{\pi} \log^+ \log^+ \left| \frac{1}{\mathcal{F}f(\pm x)} \right| - x, \tag{3.6}$$

where, for the sake of simplicity $\alpha = \alpha_+ = -\alpha_-$. The corresponding tauberian theorem is formulated as follows:

Let $f_\alpha \in L^1_p(\mathbf{R})$, $\alpha \in A$. Then $\operatorname{clos} \mathcal{L}\{\tau_t f_\alpha,\ \alpha \in A,\ t \in \mathbf{R}\} = L^1_p(\mathbf{R}) \Leftrightarrow$

$$\Leftrightarrow \inf_{\alpha \in A} \delta_+(f_\alpha) = \inf_{\alpha \in A} \delta_-(f_\alpha) = -\infty \quad \& \quad \bigcap_{\alpha \in A} \{z \in S : \mathcal{F} f_\alpha(z) = 0\} = \emptyset.$$

The fundamental steps in these works concern the possibility of analytic continuation of Carleman's transform of a functional annihilating an ideal I primary at ∞ in $L^1_p(\mathbf{R})$, and the log – log theorem of Levinson. Specifically, if $\varphi \in (L^1_p(\mathbf{R}))^*$, then the elements φ_+ and φ_- have Fourier transforms which are analytic, respectively, in $\mathbf{C}_+$ and $\mathbf{C}_-$.

One should prove that if φ is orthogonal to I, then $\mathcal{F}\varphi_+$ can be continuated analytically across the strip S to $-\mathcal{F}\varphi_-$. This entire function is called the Carleman's transform of φ and is denoted by $\mathcal{F}\varphi$. Then, one should evaluate the growth of this function by using Levinson's log – log theorem. In the quasianalytic case the first step could be made by a method from the theory of commutative Banach algebras, offered by Domar [6].

We show a way of extending both of these steps and, accordingly, give a description of primary ideals at ∞ in the quasianalytic case for the space

$$L^{1,x}_p(\mathbf{R}) = \{f : \forall n,\ f(x)(1 + |x|^n) \in L^1_p(\mathbf{R})\}.$$

One more problem, which can be treated in a similar way, was formulated by Gurarii [12, Problem 7.19] and concerns ideals in $L^1_p(\mathbf{R}_+)$.

Question: Is it true that if $f \in L^1_p(\mathbf{R}_+)$, $0 \in \operatorname{ess\,supp} f$, $\mathcal{F}f(z) \neq 0$, $\operatorname{Im} z \leq 0$, then $\operatorname{clos} \mathcal{L}\{\tau_t f,\ t \geq 0\} = L^1_p(\mathbf{R}_+)$?

We assert that the answer is positive if $L^1_p(\mathbf{R}_+)$ is replaced by the space $L^{1,x}_p(\mathbf{R}_+)$.

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Let us introduce the generalized Fourier transform. It can be defined by different methods. We use a construction proposed by A. Volberg. It should be noted that when solving convolution equations, one can replace the spaces $\mathcal{U}$ from (a) and $L^{1,x}_p(\mathbf{R})$ from (b) by spaces of smooth functions.

In the case (a) put

$$p^*(r) = \max(rx - \log p(x)), \quad p^*(r) = rv(r) - \log p(v(r)),$$

$$\tilde{\mathcal{F}} f(z) = \int_{-\infty}^{v(\operatorname{Im} z)} f(x) e^{ixz} dx, \quad \operatorname{Im} z \geq 0.$$

It can be proved that $\tilde{\mathcal{F}}$ is an isomorphism between the convolution algebra $\mathcal{U}_1$,

$$\mathcal{U}_1 = \{f \in C^\infty(\mathbf{R}) \ : \ \exists c > 0\, \forall k\, \exists c_1, \quad |f^{(k)}(-x)| \leq c_1 (p(x))^{-c}, \ x \geq 0,$$

$$\forall c>0\ \forall k\ \exists c_1,\quad |f^{(k)}(x)|\le c_1(p(x))^{-c},\ \ x\ge 0\},$$

and Q/J, a function algebra relative to pointwise multiplication, where

$$\begin{aligned} Q=\{f\in C^1(\overline{\mathbf{C}}_+)\ \ :\ \ &\forall c<\infty\ \forall k\ \exists c_1,\quad |\bar\partial f(z)|\le c_1(1+|\mathrm{Re}\,z|)^{-k}\\ &\qquad\qquad\qquad\qquad \times\exp(-p^*(c\,\mathrm{Im}\,z)),\\ &\exists c<\infty\ \forall k\ \exists c_1,\quad |f(z)|\le c_1(1+|\mathrm{Re}\,z|)^{-k}\\ &\qquad\qquad\qquad\qquad \times\exp(p^*(c\,\mathrm{Im}\,z))\},\end{aligned}$$

$$J=\{f\in Q:\ \forall c<\infty\ \forall k\ \exists c_1,\ \ |f(z)|\le c_1(1+|\mathrm{Re}\,z|)^{-k}\exp(-p^*(c\,\mathrm{Im}\,z))\}.$$

So the problems (3.1) – (3.3) turn into questions on multiplicative structure of the algebra Q/J.

Further by theoretical-functional methods in [3] the following results can be proved.

Theorem A. *The implications* (3.2) *and* (3.3) *are valid for $\mathcal{U}$. The implication* (3.1) *holds if*

$$\lim_{x\to\infty}\frac{\log p(x)}{x\log x}=\infty.$$

In the case (b), for the sake of simplicity, let p be even, $\alpha=\alpha_+=-\alpha_-$. We define p^*, v, $\tilde{\mathcal{F}}f$:

$$p^*(r)=\max(\log p(x)-rx),\qquad p^*(r)=\log p(v(r))-rv(r),$$

$$\tilde{\mathcal{F}}_0 f(z)=\int_{-v(\mathrm{Im}\,z)}^{\infty} f(x)e^{ixz}dx,\quad \mathrm{Im}\,z>\alpha,$$

$$\tilde{\mathcal{F}}_0 f(z)=\int_{-\infty}^{v(-\mathrm{Im}\,z)} f(x)e^{ixz}dx,\quad \mathrm{Im}\,z<-\alpha.$$

It can be proved that for $f\in L^{1,x}_p(\mathbf{R})$ the generalized Fourier transform $\tilde{\mathcal{F}}f(z)$ (which, by the definition, is equal to $\mathcal{F}f(z)$ for $z\in S$ and to $\tilde{\mathcal{F}}_0 f(z)$ for other z) belongs to $C^1(\mathbf{C})$ and

$$\forall n, |\bar\partial(\tilde{\mathcal{F}}f)(z)|\exp p^*(|\mathrm{Im}\,z|)=o((|\mathrm{Im}\,z|-c)^n(1+|\mathrm{Re}\,z|)^{-2}).$$

Further, for $\varphi\in(L^{1,x}_p(\mathbf{R}))^* * C_0^\infty$

$$|\mathcal{F}\varphi_\pm(z)|\exp(-p^*(|\mathrm{Im}\,z|))=o((1+|\mathrm{Re}\,z|)^{-2}).$$

Finally, it can be proved that a functional φ is orthogonal to a primary ideal I at ∞ if and only if $\mathcal{F}\varphi=\mathcal{F}\varphi_+=-\mathcal{F}\varphi_-$ is an entire function and $\tilde{\mathcal{F}}f\cdot\mathcal{F}\varphi\in L^\infty(\mathbf{C})$.

So by function theoretic methods in [1] the following results are proved (under some regularity conditions).

Theorem B. *In the quasianalytic case all primary ideals at* ∞ *are described as in* (3.4), (3.5), *where instead of* (3.6) *we have*

$$\delta_{\pm}(f) = \varliminf_{x\to\infty} \left[R\left(\log^+ \left|\frac{1}{\mathcal{F}f(\pm x)}\right|\right) - x\right],$$

and $R(x) = (2/\pi)\int^x (\log p(y))/(1+y^2)\,dy$.

Theorem C. *If* $f \in L_p^{1,x}(\mathbf{R}_+)$, $0 \in \operatorname{ess\,supp} f$, $\mathcal{F}f(z) \neq 0, \operatorname{Im} z \leq 0$, *then*

$$\operatorname{clos}\mathcal{L}\{\tau_t f,\, t \geq 0\} = L_p^{1,x}(\mathbf{R}_+).$$

Similar results can be stated in the analytic-quasianalytic case, at least for even weights ($p(x) = p(-x)$),

$$\alpha_+ > 0, \qquad \int^\infty \frac{\log p(x) - \alpha_+ x}{1+x^2}\,dx = \infty.$$

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We now present some uniqueness theorems for analytic and asymptotically holomorphic functions that arise in proofs of Theorems A – C.

(a) The implications (3.1) – (3.3) are equivalent to the following ones for the algebra Q:

$$f,\ g \in Q,\ fg \in J \ \Rightarrow\ f \in J \text{ or } g \in J,$$

$$f \in Q, g, fg \in Q \cap L^\infty(\mathbf{C}_+)\setminus J \Rightarrow \text{for some } c\ |f(z)| + |g(z)| < c\cdot\exp(c\operatorname{Im} z),$$

$$f, g \in Q, fg \in Q \cap L^\infty(\mathbf{C}_+)\setminus J \Rightarrow \text{for some } c\ |f(z)| + |g(z)| < c\cdot\exp(c\operatorname{Im} z).$$

In the proofs of these statements the usual asymptotically holomorphic technique (estimates on the harmonic measure and balayage, see [2],[4]) is used to reduce them to uniqueness theorems for analytic functions (in particular, to some theorem of Ostrovskii [15]).

(b) When proving Theorem B, we state three uniqueness theorems: on asymptotics of quasianalytic functions on $\mathbf{R}$, on asymptotics of entire Carleman's transforms, and on that of their products:

Theorem D. *Let* $f \in L_p^1(\mathbf{R})$, $f \not\equiv 0$ *and* p *quasianalytic. For some set* $E \subset \mathbf{R}_+$, *such that* $m(E \cap (x, x+1)) < 1/3$ *for all* x, *there exist the limits*

$$R_{\pm}(f) = \lim_{\substack{x\to\infty \\ x\notin E}} \left[R\left(\log^+ \left|\frac{1}{\mathcal{F}f(\pm x)}\right|\right) - x\right],$$

which are either finite or equal to $-\infty$. *There is a function* f *such that* $R_{\pm}(f) = 0$.

Theorem E. *Let* $\varphi \in (L^1_p(\mathbf{R}))^*, \varphi \neq 0$, *and let an entire function* $\mathcal{F}\varphi$ *be the Carleman's transform of* φ. *If* p *is quasianalytic, there exist the limits*

$$R^M_{\pm}(\varphi) = \lim_{x\to\pm\infty} \left[R\left(\log \max_{\operatorname{Re} z = x} |\mathcal{F}\varphi(z)| \right) - x \right],$$

which are either finite or equal to $\pm\infty$. $R^M_{\pm}(\varphi)$ *is equal to* $-\infty$ *if and only if* $\mathcal{F}\varphi$ *is bounded in the half-plane* $\{z : \pm\operatorname{Re} z > 0\}$. *There is a functional* φ *such that* $R^M_{\pm}(\varphi) = 0$.

Theorem F. *If* f *and* φ *satisfy the conditions of Theorems* D *and* E, *then*

$$R_+(f) \geq R^M_+(\varphi) \;\;\&\;\; R_-(f) \geq R^M_-(\varphi) \Leftrightarrow \langle f * L^{1,x}_p(\mathbf{R}), \varphi \rangle = 0.$$

Here, besides the asymptotically holomorphic technique, some sharp form of the Warshawskii theorem on the asymptotics of conformal mappings of infinite strips is employed.

It should be noted that weaker estimates on asymptotics of quasianalytically smooth functions were earlier stated in [9], [11], [16]. Theorem E can be considered as an extension of the log-log theorem of Levinson and the Phragmen-Lindelof theorem for the strip.

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A.A. Borichev
Steklov Mathematical Institute
St. Petersburg Branch
Fontanka 27
St. Petersburg, 191011
RUSSIA