On Second Main Theorems and Uniqueness Theorems of Meromorphic Maps $f : \mathbb{C}^m - \rightarrow \mathbb{CP}^n$ for Moving Targets
j.w.w. Tan Van Tran (MCF ENS Hanoi)

Introduction: Let $f : \mathbb{C}^m - \rightarrow \mathbb{CP}^n$ be a meromorphic map and $D \subset \mathbb{CP}^n$ a divisor not containing the image of f. Very roughly speaking in value distribution theory one compares $T_f(r)$, measuring the 'growth' of f, with $N_f(D)$, measuring the intersection of the image of f with D (normally counting multiplicities). It is quite easy to bound $N_f(D)$ by $T_f(r)$ (First Main Theorems) but it seems usually quite difficult to bound $T_f(r)$ by $N_f(D)$ (Second Main Theorems), even if certain restrictions on D, known to be necessary, hold. So far Second Main Theorems have only been obtained in very particular situations.

If $D = D_1 \cup ... \cup D_l$ has l components, and $f^{-1}(D_1), ..., f^{-1}(D_l)$ are their invers images under f (possibly counted with multiplicities), we can ask how many different meromorphic maps g exist sharing these invers images (possibly with multiplicities). Results of this type are called uniqueness theorems.

Moving targets means that the divisor D or its components are not fixed, any more, but, as f does, depend on $z \in \mathbb{C}^m$.

Results: Our first result is a Second Main Theorem for meromorphic mappings $f : \mathbb{C}^m - \rightarrow \mathbb{CP}^n$ for $n+2$ moving hyperplane targets D_j with intersection multiplicities of $N_f(D_j)$ bounded by n. Using this and a generalization of the Borel lemma from nonvanishing holomorphic functions to meromorphic functions, we obtain a uniqueness theorem for $3n+1$ moving hyperplane targets ($n \geq 2$), while before this existed only for $3n+2$ moving hyperplane targets. Compared to previous results of Tu '02, this also improves truncation of multiplicities and reduces the number of restrictions.

The main result of this talk is a generalization to moving targets of Ru's Second Main Theorem for hypersurfaces from '04. We will discuss in some details the key difficulties to pass from the case of fixed hypersurface targets to moving ones, coming among others from the fact that one has to do the algebraic geometry part of this proof over the field of small meromorphic functions (compared to a given meromorphic map $f : \mathbb{C}^m - \rightarrow \mathbb{CP}^n$), which is not algebraically closed, any more.