

Introduction à l'analyse**CUPGE****PLANCHE 3BIS DÉRIVATION**

Donner les domaines de définition et de dérivabilité de ces fonctions, puis les dériver :

1. $f_1(x) = 3x^4 - 2x^3 + 5x - 4$;
2. $f_2(x) = \frac{x^2+5}{x+1}$;
3. $f_3(x) = \frac{x+3}{x^2+2}$;
4. $f_4(x) = \sqrt{x}(1-x)$;
5. $f_5(x) = \sqrt{x}(1 - \frac{1}{x^2})$;
6. $f_6(x) = (2x^3 - x^2 + 5)^5$;
7. $f_7(x) = \frac{\sqrt{x}-1}{\sqrt{x}+1}$;
8. $f_8(x) = 5 \cos(4x) + 6 \sin\left(\frac{x}{2}\right)$;
9. $f_9(x) = \cos\left(\frac{\pi}{3} - 2x\right) \sin(3x)$;
10. $f_{10}(x) = \tan(3x)$;
11. $f_{11}(x) = \frac{3 \cos(2x)}{1+\tan(x)}$;
12. $f_{12}(x) = (\cos(x) + 2 \tan(3x))^2$;
13. $f_{13}(x) = \ln(3x+1)$;
14. $f_{14}(x) = x \ln(x)$;
15. $f_{15}(x) = \ln(\sin(x))$;
16. $f_{16}(x) = \ln\left(\frac{3x+1}{5x+5}\right)$;
17. $f_{17}(x) = e^{x^2+1}$;
18. $f_{18}(x) = (2x+3)e^{5x^2-1}$;
19. $f_{19}(x) = \ln(x)e^{2x} \sin(3x)$;
20. $f_{20}(x) = \operatorname{ch}(x) \ln(x)$;
21. $f_{21}(x) = \operatorname{sh}(1 - \operatorname{ch}^2(x))$;
22. $f_{22}(x) = \arctan(\sqrt{x})$;
23. $f_{23}(x) = \arcsin(x) - \operatorname{argsh}(x)$;
24. $f_{24}(x) = \operatorname{argch}(e^x \cos(x)) \ln(\sqrt{1-x^2})$;
25. $f_{25}(x) = \cos\left(\frac{\sqrt{1+x^3}}{\operatorname{argsh}(x-1)}\right)$;
26. $f_{26}(x) = e^{\arcsin(2-x)e^x}$;
27. $f_{27}(x) = \left(e^{\operatorname{ch}(x)} + \frac{e^{\operatorname{th}(x)}}{\cos(x)}\right)^7$;
28. $f_{28}(x) = (1 + \operatorname{th}^2(x))^x$;
29. $f_{29}(x) = \left(\frac{1}{3} \arcsin\left(\ln(1 + \tan(\sqrt{x})) + e^{\operatorname{argsh}(\sqrt{1-x^x})}\right)\right)^{\sin(x)}$.

Solutions :

Toutes les fonctions sont dérivables par composition de fonctions dérivables. On donne d'abord le domaine de définition, puis le domaine dérivabilité, puis la dérivée. Un tiret signifie que les domaines de définition et de dérivabilité sont les mêmes.

1. $\mathbb{R}; -; f'_1(x) = 12x^3 - 6x^2 + 5;$
2. $\mathbb{R} \setminus \{-1\}; -; f'_2(x) = \frac{x^2+2x-5}{(x+1)^2};$
3. $\mathbb{R}; -; f'_3(x) = \frac{2-6x-x^2}{(x^2+2)^2};$
4. $\mathbb{R}_+; \mathbb{R}_+^*; f'_4(x) = \frac{1-3x}{2\sqrt{x}};$
5. $\mathbb{R}_+^*; -; f'_5(x) = \frac{x^2+3}{2x^{\frac{5}{2}}};$
6. $\mathbb{R}; -; f'_6(x) = 5(6x^2 - 2x)(2x^3 - x^2 + 5)^4;$
7. $\mathbb{R}_+; \mathbb{R}_+^*; f'_7(x) = \frac{1}{\sqrt{x}(\sqrt{x+1})^2};$
8. $\mathbb{R}; -; f'_8(x) = 3 \cos\left(\frac{x}{2}\right) - 20 \sin(4x);$
9. $\mathbb{R}; -; f'_9(x) = 2 \cos\left(5x - \frac{\pi}{3}\right) + \cos(3x) \cos\left(2x - \frac{\pi}{3}\right);$
10. $\mathbb{R} \setminus \left\{\frac{\pi}{6}(1+2k) | k \in \mathbb{Z}\right\}; -; f'_{10}(x) = 3 + 3 \tan^2(3x);$
11. $\mathbb{R} \setminus \left(\left\{\frac{\pi}{2} + k\pi | k \in \mathbb{Z}\right\} \cup \left\{\frac{3\pi}{4} + k\pi | k \in \mathbb{Z}\right\}\right); -; f'_{11}(x) = \frac{-3(\sin(2x)+1)(\sin(2x)+\cos(2x))}{(\sin(x)+\cos(x))^2};$
12. $\mathbb{R} \setminus \left\{\frac{\pi}{6}(1+2k) | k \in \mathbb{Z}\right\}; -; f'_{12}(x) = 2(6 + 6 \tan^2(3x) - \sin(x))(\cos(x) + 2 \tan(3x));$
13. $]-\frac{1}{3}, \infty[; -; f'_{13}(x) = \frac{3}{3x+1};$
14. $\mathbb{R}_+^*; -; f'_{14}(x) = 1 + \ln(x);$
15. $\bigcup_{k \in \mathbb{Z}}]2k\pi, (2k+1)\pi[; -; f'_{15}(x) = \frac{1}{\tan(x)};$
16. $\mathbb{R} \setminus [-1, -\frac{1}{3}]; -; f'_{16}(x) = \frac{2}{(x+1)(3x+1)};$
17. $\mathbb{R}; -; f'_{17}(x) = 2xe^{x^2+1};$
18. $\mathbb{R}; -; f'_{18}(x) = 2(10x^2 + 15x + 1)e^{5x^2-1};$
19. $\mathbb{R}_+^*; -; f'_{19}(x) = \frac{e^{2x} \sin(3x)}{x} + 2 \ln(x)e^{2x} \sin(3x) + 3 \ln(x)e^{2x} \cos(3x);$
20. $\mathbb{R}_+^*; -; f'_{20}(x) = \ln(x) \operatorname{sh}(x) + \frac{\operatorname{ch}(x)}{x};$
21. $\mathbb{R}; -; f'_{21}(x) = -\operatorname{sh}(2x) \operatorname{ch}(\operatorname{ch}^2(x) - 1);$
22. $\mathbb{R}_+; \mathbb{R}_+^*; f'_{22}(x) = \frac{1}{2\sqrt{x}(1+x)};$
23. $[-1, 1];]-1, 1[; f'_{23}(x) = \frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1-x^4}};$
24. $[0, 1[;]0, 1[; f'_{24}(x) = \frac{e^x(\cos(x)-\sin(x)) \ln(1-x^2)}{2\sqrt{e^{2x} \cos^2(x)-1}} - \frac{x \cdot \operatorname{argch}(e^x \cos(x))}{1-x^2};$
25. $[-1, \infty[\setminus \{1\};]-1, \infty[\setminus \{1\}; f'_{25}(x) = \frac{2+2x^3-3x^2\sqrt{x^2-2x+2} \cdot \operatorname{argsh}(x-1)}{2\sqrt{(x^2-2x+2)(1+x^3)} \cdot \operatorname{argsh}^2(x-1)} \sin\left(\frac{\sqrt{1+x^3}}{\operatorname{argsh}(x-1)}\right);$
26. $[2 - \frac{\pi}{2}, 2 + \frac{\pi}{2}];]2 - \frac{\pi}{2}, 2 + \frac{\pi}{2}[; f'_{26}(x) = \left(\arcsin(2-x) - \frac{1}{\sqrt{(x-1)(3-x)}}\right) e^{1+e^x \arcsin(2-x)};$
27. $\mathbb{R} \setminus \left\{\frac{\pi}{2} + k\pi | k \in \mathbb{Z}\right\}; -; f'_{27}(x) = 7 \left(e^{\operatorname{ch}(x)} + \frac{e^{\operatorname{th}(x)}}{\cos(x)}\right)^6 \left(\operatorname{sh}(x)e^{\operatorname{ch}(x)} + \frac{e^{\operatorname{th}(x)}}{\cos(x)}(1 - \operatorname{th}^2(x) + \tan(x))\right);$
28. $\mathbb{R}; -; f'_{28}(x) = (1 + \operatorname{th}^2(x))^x \left(\ln(1 + \operatorname{th}^2(x)) + \frac{2x \cdot \operatorname{th}(x)}{\operatorname{ch}(2x)}\right);$
29. $[0, 1]; [0, 1[; f'_{29}(x) = \left(\frac{1}{3} \arcsin\left(\ln(1+\tan(\sqrt{x})) + e^{\operatorname{argsh}(\sqrt{1-x^x})}\right)\right)^{\sin(x)} \left(\cos(x) \ln\left(\frac{1}{3} \arcsin\left(\ln(1+\tan(\sqrt{x})) + e^{\operatorname{argsh}(\sqrt{1-x^x})}\right)\right) + \frac{\frac{\sin(x)}{2\sqrt{x} \cos^2(\sqrt{x})(1+\tan \sqrt{x})} - \frac{\sin(x)e^{\operatorname{argsh}(\sqrt{1-x^x})}(1+\ln(x))x^x}{2\sqrt{(2-x^x)(1-x^x)}}}{\arcsin\left(\ln(1+\tan \sqrt{x}) + e^{\operatorname{argsh}(\sqrt{1-x^x})}\right) \sqrt{1 - \left(\ln(1+\tan \sqrt{x}) + e^{\operatorname{argsh}(\sqrt{1-x^x})}\right)^2}}\right).$