

Introduction à l'analyse**CUPGE****PLANCHE 4BIS EQUATIONS DIFFÉRENTIELLES**

Résoudre les équations différentielles suivantes :

1. $y'(x) + 2y(x) = x^2 - 2x + 3$;
2. $y'(x) + y(x) = xe^{-x}$;
3. $y'(x) - 2y(x) = \cos(x) + 2\sin(x)$;
4. $y'(x) + y(x) = \frac{1}{1+e^x}$;
5. $(1+x)y'(x) + y(x) = 1 + \ln(1+x)$ sur $] -1, +\infty[$;
6. $y'(x) - \frac{y(x)}{x} = x^2$ sur $]0, +\infty[$;
7. $y'(x) - 2xy(x) + (2x-1)e^x$;
8. $y'(x) - \frac{2y(x)}{x} = x^2$;
9. $y'(x) + \tan(x)y(x) = \sin(2x)$ sur $] -\frac{\pi}{2}, \frac{\pi}{2}[$;
10. $(1+x)y'(x) + xy(x) = x^2 - x + 1$ sur $] -1, +\infty[$;
11. $y''(x) - 2y'(x) + y(x) = x$;
12. $y''(x) - 4y'(x) + 3y(x) = (2x+1)e^{-x}$;
13. $y''(x) - 4y'(x) + 3y(x) = (2x+1)e^x$;
14. $y''(x) + 9y(x) = x + 1$;
15. $y''(x) + 6y'(x) + 9y(x) = x^2e^{2x}$;
16. $y''(x) - 2y'(x) + y(x) = \operatorname{ch}(x)$;
17. $y''(x) - 2y'(x) + 2y(x) = x \cos(x) \operatorname{ch}(x)$.

Solutions : dans ce qui suit, on donne directement les familles de solutions avec $A, B \in \mathbb{R}$.

1. $\left(x \mapsto \frac{x^2}{2} - \frac{3x}{2} + \frac{9}{4} + Ae^{-2x}\right);$
2. $\left(x \mapsto \left(\frac{x^2}{2} + A\right)e^{-x}\right);$
3. $\left(x \mapsto Ae^{2x} - \frac{1}{5}(3\sin(x) + 4\cos(x))\right);$
4. $\left(x \mapsto Ae^{-x} + \ln(1 + e^x)\right);$
5. $\left(x \mapsto \frac{A}{1+x} + \ln(1 + x)\right);$
6. $\left(x \mapsto Ax + \frac{x^3}{2}\right);$
7. $\left(x \mapsto Ae^{x^2} + e^x\right);$
8. $\left(x \mapsto Ax^2 + x^3\right);$
9. $\left(x \mapsto \cos(x)(A - 2\cos(x))\right);$
10. $\left(x \mapsto (x - 2) + A(x + 1)e^{-x}\right);$
11. $\left(x \mapsto (A + Bx)e^x + x + 2\right);$
12. $\left(x \mapsto \left(\frac{5}{16} + \frac{x}{4}\right)e^{-x} + Ae^x + Be^{3x}\right);$
13. $\left(x \mapsto \left(1 - \frac{x}{2}\right)xe^x + Ae^x + Be^{3x}\right);$
14. $\left(x \mapsto A\cos(3x) + B\sin(x) + \frac{1+x}{9}\right);$
15. $\left(x \mapsto \frac{1}{625}(25x^2 - 20x + 6)e^{2x} + (A + Bx)e^{-3x}\right);$
16. $\left(x \mapsto \left(A + Bx + \frac{x^2}{4}\right)e^x + \frac{1}{8}e^{-x}\right);$
17. $\left(x \mapsto \frac{1}{8}(x\cos(x) + x^2\sin(x) + A\cos(x) + B\sin(x))e^x + ((2x + 1)\cos(x) - 2(x + 1)\sin(x))e^{-x}\right).$