Université Aix-Marseille 2014–2015

LICENCE MATHS-INFO - **Analyse I**

COURS – QUELQUES DL EN 0 À CONNAÎTRE

1 Exponentielle et Logarithme

— $e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \cdots + \frac{x^n}{n!} + o(x^n)\,;$

— $\ln(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \cdots - \frac{x^n}{n} + o(x^n)\,;$

— $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots + (-1)^{n-1}\frac{x^n}{n} + o(x^n).$

2 Puissances

— $\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots + x^n + o(x^n)\,;$

— $\frac{1}{1+x} = 1 - x + x^2 - x^3 + \cdots + (-1)^n x^n + o(x^n)\,;$

— $\sqrt{1+x} = 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{16} - \frac{5x^4}{64} + \cdots + (-1)^{n-1}\frac{1\times3\times\cdots\times(2n-3)x^n}{2\times4\times\cdots\times2n} + o(x^n)\,;$

— $\frac{1}{\sqrt{1+x}} = 1 - \frac{x}{2} + \frac{3x^2}{8} - \frac{5x^3}{16} + \frac{35x^4}{128} + \cdots + (-1)^n\frac{1\times3\times\cdots\times(2n-1)x^n}{2\times4\times\cdots\times2n} + o(x^n)\,;$

— $(1+x)^\alpha = 1 + \alpha x + \frac{\alpha(\alpha-1)x^2}{2} + \frac{\alpha(\alpha-1)(\alpha-2)x^3}{6} + \cdots + \frac{\alpha(\alpha-1)\cdots(\alpha-n+1)x^n}{n!} + o(x^n).$

3 Fonctions trigonométriques et hyperboliques

— $\sin(x) = x - \frac{x^3}{6} + \cdots + (-1)^n\frac{x^{2n+1}}{(2n+1)!} + o(x^{2n+2})\,;$

— $\mathrm{sh}(x) = x + \frac{x^3}{6} + \cdots + \frac{x^{2n+1}}{(2n+1)!} + o(x^{2n+2})\,;$

— $\cos(x) = 1 - \frac{x^2}{2} + \frac{x^4}{24} - \cdots + (-1)^n\frac{x^{2n}}{(2n)!} + o(x^{2n+1})\,;$

— $\cos(x) = 1 - \frac{x^2}{2} + \frac{x^4}{24} - \cdots + (-1)^n\frac{x^{2n}}{(2n)!} + o(x^{2n+1})\,;$

— $\mathrm{ch}(x) = 1 + \frac{x^2}{2} + \frac{x^4}{24} + \cdots + \frac{x^{2n}}{(2n)!} + o(x^{2n+1})\,;$

— $\tan(x) = x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{17x^7}{315} + o(x^7)\,;$

— $\mathrm{th}(x) = x - \frac{x^3}{3} + \frac{2x^5}{15} - \frac{17x^7}{315} + o(x^7).$

4 Fonctions trigonométriques et hyperboliques réciproques

— $\arcsin(x) = x + \frac{x^3}{6} + \cdots + \frac{1\times 3\times\cdots(2n-1)x^{2n+1}}{(2\times 4\times\cdots 2n)(2n+1)} + o(x^{2n+2})$;

— $\operatorname{argsh}(x) = x - \frac{x^3}{6} + \cdots + (-1)^n \frac{1\times 3\times\cdots(2n-1)x^{2n+1}}{(2\times 4\times\cdots 2n)(2n+1)} + o(x^{2n+2})$;

— $\arccos(x) = \frac{\pi}{2} - x - \frac{x^3}{6} - \cdots - \frac{1\times 3\times\cdots(2n-1)x^{2n+1}}{(2\times 4\times\cdots 2n)(2n+1)} + o(x^{2n+2})$;

— $\arctan(x) = x - \frac{x^3}{3} + \cdots + (-1)^n \frac{x^{2n+1}}{2n+1} + o(x^{2n+2})$;

— $\operatorname{argth}(x) = x + \frac{x^3}{3} + \cdots + \frac{x^{2n+1}}{2n+1} + o(x^{2n+2})$.