Groupe de travail : O-minimality in Complex Geometry

October 2, 2018

Proposed seminars:

1. *Topic* : O-minimal structures (Antoine Etesse);

   Content : Definition of Structure; Definition of o-minimality; Two basic examples.

   Reference : [2] Chapter 1.2-1.3 and 1.6-1.7

   Remark : We might want to add 1.5, because it seems to help reading papers by logic specialists (such as Van den Dries, Miller, etc).

2. *Topic* : Semi-Algebraic Sets (Juliana Restrepo Velasquez);

   Content : Definition of Semi-Algebraic sets; Lojasiewicz sets; First Cell-decomposition type Theorem; Tarsky-Seidenbeg Theorem; (if time, Thom's Lemma).

   Reference : [2] Chapter 2.2 (if time 2.1, 2.3) and [1] Section 2.

   Remark : We might sacrifice Thom's Lemma, since we won't study real-algebraic geometry in details.

3. *Topic* : Sub-analytic Sets and $\mathbb{R}_{an}$ (Federico Lo Bianco);

   Content : Definition of semi-analytic sets; Osgord Example, Definition of Sub-analytic sets; Gabrielov-Complement Theorem; Definition of $\mathbb{R}_{an}$ by global sub-analytic sets; Definition of $\mathbb{R}_{an}$ by restricted analytic functions.

   Reference : [1] Section 3 and 4, [3].

   Remark : Definition of $\mathbb{R}_{an}$ by restricted analytic functions does not appear in the references above, but it should be easy to make the relation ourselves.

4. *Topic* : Example $\mathbb{R}_{exp}$, $\mathbb{R}_{an,exp}$ (David Trotman ?);

   Content : To be determined.

   Reference : [**?**]

   Remark : The proof seems logic intense – find a more modern geometrical version.

5. *Topic* : Cell-decomposition (Lorenzo Fantini);

   Content : Definition of Cells, (Continuous) Cell-decomposition, Application.

   Reference : [2] Chapter 3.2 (optional : [5]).

   Remark : We might enunciate results about C^k-cell decomposition (without a proof). If there is interest in this direction, we could make a further seminar, where we examine the necessary conditions for C^k, C^∞ and C^ω cell-decomposition, and we present counter-examples [5].

6. *Topic* : Definable-Chow Theorem (André Belotto); *Reference* : [6], [7]

7. *Topic* : Bi-algebraic geometry (Erwan Rousseau); *Content* : Ax-Lindemann-Weierstrass [8], holomorphic curves in Shimura varieties [9].

8. *Topic* : Pila-Wilkie (Xavier Roulleau);

   To be completed and divided in sections.

References

[1] E. Bierstone and P. Milman. Semianalytic and subanalytic sets. *Inst. Hautes Études Sci. Publ. Math.* No. 67 (1988), 5–42.

[2] L. van den Dries. *Tame topology and o-minimal structures,* vol. 248 of London Mathematical Society Lecture Note Series, Cambridge University Press, Cambridge, 1998.

[3] L. van den Dries. A generalization of the Tarski-Seidenberg theorem, and some nondefinability results. Bull. Amer. Math. Soc. (N.S.) 15 (1986), no. 2, 189âĂŞ193. 03C40 (03E47)

[4] L. van den Dries and C. Miller, On the real exponential field with restricted analytic functions. Israel J. Math. 85 (1994), no. 1-3, 19âĂŞ56.

[5] O. Le Gal and J-P. Rolin An o-minimal structure which does not admit C^∞ cellular decomposition. Ann. Inst. Fourier (Grenoble) 59 (2009), no. 2, 543-562.

[6] Y. Peterzil and S. Starchenko *Tame complex analysis and o-minimality,* Proceedings of the ICM, Hyderabad 2010.

[7] N. Mok, J. Pila and J. Tsimerman *Ax-Schanuel for Shimura varieties,* preprint arxiv:1711.02189.

[8] B. Klingler, E. Ullmo and A. Yafaev *The hyperbolic Ax-Lindemann-Weierstrass conjecture, Inst. Hautes Études Sci. Publ. Math.* No. 123 (2016), 333–360.

[9] *Holomorphic curves in compact Shimura varieties., Ann. Inst. Fourier (Grenoble)* 68 (2018), no. 2, 647–659