Recent advances in functional transcendence

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Functional transcendence results have in the last decade found a number of important applications to the algebraic and arithmetic geometry of varieties X admitting hyperbolic uniformizations : Pila and Zannier's new proof of the Manin-Mumford conjecture, the proof of the André-Oort conjecture for A_g, and the generic Shafarevich conjecture for hypersurfaces of Lawrence-Venkatesh, to name a few. The key insight (originally stemming from work of Pila and Zannier) is the use of o-minimal geometry to pass between the geometry of X and that of its uniformizing space.

The goal of this five part series will be to give a tour through the main elements of the proof of the Ax-Schanuel conjecture for variations of Hodge structures intended for non-experts. We will start by introducing the basic notions of o-minimal geometry with a view towards the two algebraization theorems of Pila-Wilkie and Peterzil-Starchenko. We will then show how these results are combined with local volume bounds in the style of Hwang-To to prove the Ax-Schanuel conjecture. 1. Overview 2. o-minimal geometry and algebraization 3. Ax–Lindemann in the flat case 4. Ax–Schanuel in the hyperbolic case 5. Volume bounds and further directions