Complex hyperbolicity of quotients of bounded symmetric domains

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Given a compactification of a quotient of a bounded symmetric domain, we are interested in the geometry of the entire curves it contains, as well as to the type of its subvarieties. To achieve this, we will first present a metric criterion for the positivity of the cotangent bundle of a complex manifold, based in particular on the work of J.-P. Demailly and of S. Boucksom. This criterion can be applied to a large class of varieties, going beyond the frame of quotients of bounded symmetric domains ; in a joint work with Y. Brunebarbe, we apply it to the case of manifolds supporting a complex variation of Hodge structures.

The previous criterion can be applied in many situations related to quotients of bounded symmetric domains. In the case of a ball quotient, we show that on a ramified cover of a toroidal compactification, étale on the inside part, and ramifying at orders higher than 7 on the boundary, there is no subvariety which is not of general type, and also not included in the boundary. In this setting, this gives an effective version of a theorem of Y. Brunebarbe. With E. Rousseau and B. Taji, we also apply these metric methods to give a criterion for the algebraic hyperbolicity of the compactifications of a quotient with cyclic singularities. Finally, we will explain how a slight extension of the previously described criterion permits to give a unified treatment of the algebraic and transcendental complex hyperbolicity for the compactification we consider. This gives new effective results for bounded symmetric domains other than the ball, refining previous theorems of Y. Brunebarbe and E. Rousseau.