

On the analytic class number formula for Selberg zeta functions

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The Selberg zeta function of a fuchsian group (of the first kind) was introduced by Selberg in analogy with the Riemann zeta function, in his formula for the trace of the resolvent kernel of the hyperbolic laplacian (a particular instance of the Selberg trace formula for a fuchsian group). In the analogy with the Riemann zeta function, the analytic class number formula would be an evaluation of the derivative at $s=1$ of the Selberg zeta function, and it seems missing in the litterature. In this talk I will describe this formula for any fuchsian group of the first kind, as a Riemann–Roch type formula. I will explain the arithmetic and geometric content in the simplest case of $PSL_2(Z)$, where the value involves logarithmic derivatives of L functions. These arise as arithmetic intersection numbers or Faltings heights of CM elliptic curves. This is joint work with Anna von Pippich.