Rational and Transcendental points on Riemann surfaces contained in a projective variety

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Let $(X; L)$ be a polarized smooth projective variety of dimension at least two defined over $\mathbb{Q}$. Let M be a Riemann surface contained in $X(\mathbb{C})$ (which may be non algebraic). Let U be an open set of Δ_1. The number $A(T)$ of points of logarithmic height less or equal then T contained in the image of U is, according to a classical theorem of Bombieri-Pila, bounded by $\exp(\epsilon T)$. Besides the fact that explicit examples show that this estimate is optimal, we will discuss many situations when $A(T)$ is bounded by a polynomial in T. An important class of these examples is when M is a leave of a smooth foliation.

We will also discuss the (mysterious) interaction within the behavior of $A(T)$ and the presence in the image of f of transcendental points (called of type S) which verify inequalities similar to the standard Liouville inequality : this interaction may be resumed in the following fact :

One transcendental point of type S in the image implies few rational points *and* many points of type S in the image, or equivalently, many rational points in the image imply *no* points of type S in the image.