

Survival of fragmentation processes with killing

Abstract:

We introduce homogenous fragmentation processes whose blocks are killed when the negative logarithm of their size at a certain time crosses a line with slope $c > 0$ starting from some point $x > 0$. Depending on c and x the process may survive or become extinct. In particular, for any fixed slope c we study the probability of survival as a function of the starting point x . In the case that extinction does not occur with probability 1, we give two qualitative results concerning the evolution of the process on survival. Our results show that the total number of fragments in the surviving process explodes and that the asymptotic exponential rate of decay of the largest fragment is the same as when the killing scheme is not in effect. The approach to prove these results is based on introducing related killed spectrally negative Lévy processes and makes use of two families of martingales. Our motivation for considering this killing mechanism mainly stems from its relation to the one-sided FKPP equation in the context of fragmentation processes. More precisely, we show that for certain values of an underlying parameter the abovementioned survival probability is the unique one-sided FKPP travelling wave solution and that there is no such solution otherwise. In the light of this uniqueness result we present some analytic properties of FKPP travelling waves.