Workshop LISA

A version of the Mumford's Theorem on regularity of normal complex surfaces in high dimension

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Abstract

In 1961, D. Mumford proved that a normal complex analytic surface X with simply connected link at 0 need to be smooth at 0. In the case $X \subset \mathbb{C}^3$ is a complex surface with an isolated singularity at 0, Mumford's result is equivalent to say that X is smooth at 0 if and only if X is a topological manifold at 0. However, this result does not hold true if $\dim X > 2$. In this talk, we prove a version of Mumford's result in high dimension. More precisely, if $X \subset \mathbb{C}^n$ is a LNE complex analytic set, we prove that the following statements are equivalent:

(1) X is a topological manifold at 0;

(2) X is smooth at 0.

No restriction on the dimension or codimension and no restriction on the singularity to be isolated is needed. In order to know, a set $X \subset \mathbb{R}^n$ is called Lipschitz normally embedded (LNE) if the identity map between X endowed with the inner distance and X endowed with the induced euclidean distance is a bi-Lipschitz homeomorphism.

Joint work with:
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Main references

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