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EDITORIAL

Spitsbergen volume

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This volume contains 20 papers related to the workshop *Frontiers of Rationality* that was held in Longyearbyen, Spitsbergen, in July 2014. Although the University of Svalbard in Longyearbyen is not a major mathematical center, we feel that the location was an ideal place for an international scientific meeting. Indeed, the two things that mathematicians like about their subject is that mathematics is a front edge of human knowledge, and that there are no national borders for mathematical studies. Spitsbergen is truly one of the edges of the populated land on Earth, and it is one of the places where people can come literally missing any borders.

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The history of Spitsbergen is interesting, and can be even called rich for such a lonely place. It was officially discovered by a Dutch explorer Willem Barentsz in June 1596, who gave the island its present name, which means "pointed mountains" in Dutch. However, people used to know this place under different names long before Barentsz. Russian pomors called it Grumant for centuries, and Norwegian seafarers claimed to visit it since the Middle Ages and gave the name Svalbard to the surrounding area. Until 1920, Spitsbergen was a neutral territory. Then it became part of Norway by the Spitsbergen treaty. Nowadays citizens of more than 40 countries who signed this treaty may settle in the island. Longyearbyen is the largest town in the area, but the second largest town is a coal-mining settlement Barentsburg, which is run by the company Arktikugol ("arctic coal" in Russian). It used to belong to the Soviet Union since 1932, and now belongs to Russia. It has surprisingly many facilities that can help organizing any major international event for such a small town: a scientific center run by the Russian Academy of Sciences, an Olympic size swimming pool, a big cinema hall, an excellent brewery, and even the second northernmost Lenin's monument in the world. Despite of all this, we decided in favor of Longyearbyen, where we enjoyed the hospitality of the University of Svalbard for a week.

The participants of the workshop were Hamid Ahmadinezhad and Tim Browning from Bristol, Florin Ambro from Bucharest, Ekaterina Amerik, Alexander Efimov, Sergey Gorchinskiy, Dmitry Kaledin, Viktor Kulikov, Vladimir Popov, Yuri Prokhorov, Victor Przyjalkowski, Marat Rovinsky, Constantin Shramov, Evgeny Smirnov and Andrey Trepalin from Moscow, Fedor Bogomolov from New York, Christian Böhning from Hamburg, Agnieszka Bodzenta-Skibińska and Jarosław Wiśniewski from Warsaw, Frédéric Campana from Nancy, Ivan Cheltsov and Hendrik Suess from Edinburgh, Joseph Cutrone from Baltimore, Kento Fujita and Masayuki Kawakita from Kyoto, Yoshinori Gongyo, Ilya Karzhemanov and Yujiro Kawamata from Tokyo, Daniel Grayson from Urbana, Vladimir Guletskiĭ from Liverpool, Dongseon Hwang from Seoul, Angelo Lopez from Rome, Frédéric Mangolte from Angers, Dimitri Markushevich from Lille, Anton Mellit from Trieste, Keiji Oguiso from Osaka, Takuzo Okada from Saga, Jihun Park from Pohang, Damiano Testa from Coventry, Anthony Várilly-Alvarado from Houston, and a Fields medalist of 2002 Vladimir Voevodsky from Princeton.

The location for the workshop was first proposed to us in 2013 by Pavel Iosad, a linguist at the University of Edinburgh. This was already the first sign to predict that the workshop will cover a broad variety of subjects. Overall, it was very successful, and after it 35 mathematicians contributed 20 papers to the Spitsbergen volume. Let us briefly describe their contribution.

In the paper *Cyclic covers and toroidal embeddings*, Florin Ambro presents an improved version of the cyclic covering trick, which works inside the category of toroidal embeddings.

In the paper *Birationally isotrivial fiber spaces*, Fedor Bogomolov, Christian Böhning, and Hans-Christian Graf von Bothmer prove that a family of varieties is birationally isotrivial provided that all the fibers are birational to each other.

In the paper *Counting rational points on the Cayley ruled cubic*, Régis de la Bretèche, Tim Browning, and Per Salberger count rational points of bounded height on a particular non-normal cubic surface in $\mathbb{P}^3$, which is commonly known as the

Cayley ruled cubic surface. This surface can be given by the homogenous equation

$$xyz - x^2t = y^3.$$

The authors obtain a precise asymptotic formula for the number of rational points of bounded height on it.

In the paper *Symplectic instanton bundles on* $\mathbb{P}^3$ *and 't Hooft instantons*, Ugo Bruzzo, Dimitri Markushevich, and Alexander Tikhomirov study the moduli space $I_{n,r}$ of rank $2r$ symplectic instanton vector bundles on $\mathbb{P}^3$ with $r \geqslant 2$ and second Chern class $n \geqslant r+1$ such that n and r have different parity. The authors introduce the notion of tame symplectic instantons by excluding a kind of pathological monads. They show that the locus $I^*_{n,r}$ of tame symplectic instantons is irreducible and has the expected dimension equal to

$$4n(r+1) - r(2r+1).$$

In the paper *Rational connectedness and order of non-degenerate meromorphic maps from* $\mathbb{C}^n$, Frédéric Campana and Jörg Winkelmann study relations between the existence of non-degenerate meromorphic maps from $\mathbb{C}^n$ to compact Kähler manifolds of dimension n and the geometry of X, more precisely its rational connectedness. The authors prove the following result: if the sheaf of holomorphic p-forms contains a pseudoeffective invertible subsheaf, then such a map must have order at least 2. An immediate corollary is that a compact Kähler manifold admitting such a map of order smaller than 2 is rationally connected.

In the paper *Quartic double solids with icosahedral symmetry*, Ivan Cheltsov, Victor Przyjalkowski and Constantin Shramov study (singular) Fano threefolds acted on by the icosahedral group $\mathfrak{A}_5$ that are double covers of $\mathbb{P}^3$ branched over (singular) quartic surfaces. The authors describe all such threefolds, study their rationality, and prove $\mathfrak{A}_5$-birational rigidity of some of them. As an application, they obtain an embedding of the group $\mathfrak{A}_5$ into the Cremona group $\mathrm{Bir}(\mathbb{P}^3)$ of rank 3 that is not conjugate to any of the three embeddings previously known.

In the paper *Around the Mukai conjecture for Fano manifolds*, Kento Fujita classifies Fano manifolds X such that

$$\rho_X(r_X - 1) \geqslant \dim X - 1$$

and either $\rho_X \leqslant 3$ or $\dim X \leqslant 5$ (or both). Here, ρ_X is the rank of the Picard group of the manifold X, and r_X is the largest integer such that $-K_X \sim r_X H$ for some Cartier divisor H on X.

In the paper *Real frontiers of fake planes*, Adrien Dubouloz and Frédéric Mangolte give many examples of *fake real planes,* that is topologically minimal complex surfaces with an anti-holomorphic involution whose real locus is diffeomorphic to the euclidean real plane $\mathbb{R}^2$. In the projective setting, $\mathbb{CP}^2$ is the only topologically minimal complexification of $\mathbb{RP}^2$. In fact there exist other smooth complex surfaces with the same homology as $\mathbb{CP}^2$, but none of them admits a real structure. Thus the existence of affine fake real planes was not expected.

In the paper *$\mathbb{A}^1$-connectivity on Chow monoids versus rational equivalence of algebraic cycles*, Vladimir Guletskiĭ studies the Chow group $CH_r(X)_0$ of degree 0 and dimension r algebraic cycles modulo rational equivalence on a projective variety X over an arbitrary field of characteristic zero $\mathbb{k}$. Choosing an r-cycle of minimal degree on X gives rise to a chain of embeddings of Chow schemes, whose colimit is the connective Chow monoid $C_r^\infty(X)$ of r-cycles on X. Guletskiĭ proves that the Chow group $CH_r(X)_0$ is isomorphic to the group of sections of the Nisnevich sheaf of $\mathbb{A}^1$-path connected components of the loop space of the classifying motivic space of the monoid $C_r^\infty(X)$ at $\mathrm{Spec}(\mathbb{k})$.

In the paper *Derived categories of toric varieties III*, Yujiro Kawamata studies interactions between derived categories of toric pairs and their birational maps. The author proves that any proper birational morphism between toric pairs that is a K-equivalence is a composition of flops. He also proves that derived McKay correspondence holds for all two-dimensional quotient singularities.

In the paper *On rigid plane curves*, Viktor Kulikov and Eugenii Shustin present new examples of plane curves whose collection of singularities determines them almost uniquely up to a projective transformation of the plane.

In the paper *Determinantal identities for flagged Schur and Schubert polynomials*, Grigory Merzon and Evgeny Smirnov prove new determinantal identities for a family of flagged Schur polynomials. As an application, the authors find determinantal expressions of Schubert polynomials for certain vexillary permutations.

In the paper *On automorphisms of the punctual Hilbert schemes of K3 surfaces*, Keiji Oguiso gives a sufficient condition for the punctual Hilbert scheme of length two of a K3 surface with finite automorphism group to have automorphism group of infinite order. As an application he shows that such K3 surfaces do exist.

In the paper *Examples of cylindrical Fano fourfolds*, Yuri Prokhorov and Mikhail Zaidenberg present four different families of smooth Fano fourfolds with Picard rank 1 that contain cylinders, i.e., Zariski open subsets of the form $Z \times \mathbb{C}^1$, where Z is a quasiprojective variety. These families are very explicit:

- the smooth intersections of two quadrics in $\mathbb{P}^6$;
- the del Pezzo fourfolds of degree 5;
- special Mukai fourfolds of genus 8;
- special Mukai fourfolds of genus 7.

By a theorem of Kishimoto, Prokhorov and Zaidenberg, the affine cones over these cylindrical fourfolds admit effective actions of the additive group $\mathbb{C}_+$.

In the paper *Birational splitting and algebraic group actions*, Vladimir Popov gives a characteristic free proof of the following classical result: every irreducible algebraic variety equipped with a nontrivial rational action of a connected linear algebraic group is (birationally) ruled.

In the paper *On the global log canonical threshold of Fano complete intersections*, Thomas Eckl and Aleksandr Pukhlikov study the α-invariant of Tian of some Fano complete intersections. Namely, they prove that the α-invariant of the generic Fano complete intersections in $\mathbb{P}^{M+k}$ of index 1 and codimension k is equal to 1 provided that

$$M \geqslant 3k + 4$$

and the highest degree of defining equations is at least 8. By the famous theorem of Tian, this implies that Fano complete intersections satisfying these assumptions are Kähler–Einstein.

In the paper *Flexible affine cones over del Pezzo surfaces of degree* 4, Jihun Park and Joonyeong Won prove that the affine cone over a smooth del Pezzo surface of degree 4 embedded by an arbitrary very ample divisor is flexible. Recall that an algebraic variety X is said to be flexible if the tangent space of X at each smooth point $P \in X$ is spanned by the tangent vectors to the H-orbits of the point P for all one-parameter unipotent groups H that act faithfully on X.

In the paper *Nonrational del Pezzo fibrations admitting an action of the Klein simple group*, Takuzo Okada proves the nonrationality of certain singular threefolds fibred into del Pezzo surfaces of degree 2. To give a simple description of these threefolds, put $V = \mathbb{P}^2 \times \mathbb{P}^1$. Then $\mathrm{PSL}_2(\mathbb{F}_7)$ faithfully acts on the second factor. Moreover, this action leaves invariant a unique quartic curve $\mathcal{C}$ in $\mathbb{P}^2$, which is usually called the *Klein quartic*. Fix a nonnegative integer n, and fix $2n$ different fibers $F_1, \ldots, F_{2n}$ of the natural projection $V \to \mathbb{P}^1$. Put $S = \mathcal{C} \times \mathbb{P}^1$, and put

$$R = S + F_1 + F_2 + \cdots + F_{2n}.$$

Let $\nu\colon X' \to V$ be a double cover ramified along R, and let $\pi'\colon X' \to \mathbb{P}^1$ be the composition of the double cover ν with the natural projection $V \to \mathbb{P}^1$. Then π' is a morphism whose general fiber is a smooth del Pezzo surface of degree 2, which is a double cover of $\mathbb{P}^2$ ramified over $\mathcal{C}$. Moreover, the morphism π' has exactly $2n$ bad fibers, which are preimages of the surfaces $F_1, \ldots, F_{2n}$ under ν. Denote these fibers (with reduced structure) by $\overline{F}_1, \ldots, \overline{F}_{2n}$. Then each $\overline{F}_i$ is isomorphic to $\mathbb{P}^2$. Moreover, the threefold X' is singular along a smooth curve in $\overline{F}_i$ isomorphic to the Klein quartic curve $\mathcal{C}$. Denote these curves by $\overline{C}_1, \ldots, \overline{C}_{2n}$. Then there exists a $\mathrm{PSL}_2(\mathbb{F}_7)$-equivariant commutative diagram:

$$\begin{array}{ccccc} & & Y & & \\ & \swarrow^{\alpha} & & \searrow^{\beta} & \\ X' & & & & X \\ & \searrow_{\pi} & & \swarrow_{\pi} & \\ & & \mathbb{P}^1. & & \end{array}$$

Here α is a blow up of the curves $\overline{C}_1, \ldots, \overline{C}_{2n}$, the morphism β is a contraction of the proper transforms on Y of the surfaces $\overline{F}_1, \ldots, \overline{F}_{2n}$, and π is a del Pezzo fibration. By construction, the threefold X is not Gorenstein. Indeed, it has $2n$ singular points that are images of the surfaces $\overline{F}_1, \ldots, \overline{F}_{2n}$. These singular points are quotient singularities of type $1/2\,(1, 1, 1)$. Nevertheless, if $n > 0$, then the del Pezzo fibration $\pi : X \to \mathbb{P}^1$ is a Mori fibre space. Okada proves that X is nonrational provided that

$n \geqslant 5$ and the fibers $F_1, \ldots, F_{2n}$ of the natural projection $V \to \mathbb{P}^1$ are chosen to be very general. To prove this result, he uses Kollár's method of reduction into positive characteristic.

In the paper *Quotients of cubic surfaces*, Andrey Trepalin studies quotients of smooth cubic surfaces by a finite group. Namely, let S be a smooth cubic surface in $\mathbb{P}^3$ that is defined over an arbitrary field $\Bbbk$ of characteristic zero. If $\Bbbk$ is algebraically closed, then S is known to be rational. This is no longer true if $\Bbbk$ is not algebraically closed. Indeed, if $\Bbbk = \mathbb{Q}$ and S is given by

$$2x^3 + 3y^3 + 5z^3 + 7t^3 = 0,$$

then S is not rational over $\Bbbk$ by the famous result of Manin and Segre. Suppose, in addition, that S is acted on by some finite group G. If $\Bbbk$ is algebraically closed, then the quotient S/G is rational. In the case when $\Bbbk$ is an arbitrary field of zero characteristic, Trepalin proves the following result: if S contains $\Bbbk$-points, the group G is nontrivial, and G is not a cyclic group of order 3 having no curves of fixed points, then the quotient S/G is rational over $\Bbbk$.

In the paper, *Galois groups of Mori trinomials and hyperelliptic curves with big monodromy*, Yuri Zarhin computes the Galois groups for a very nice class of polynomials over $\mathbb{Q}$ that was introduced by Shigefumi Mori in the seventies.

We hope that the reading of this volume will give the feeling of a large space as we experienced it when we were close to the North Pole!

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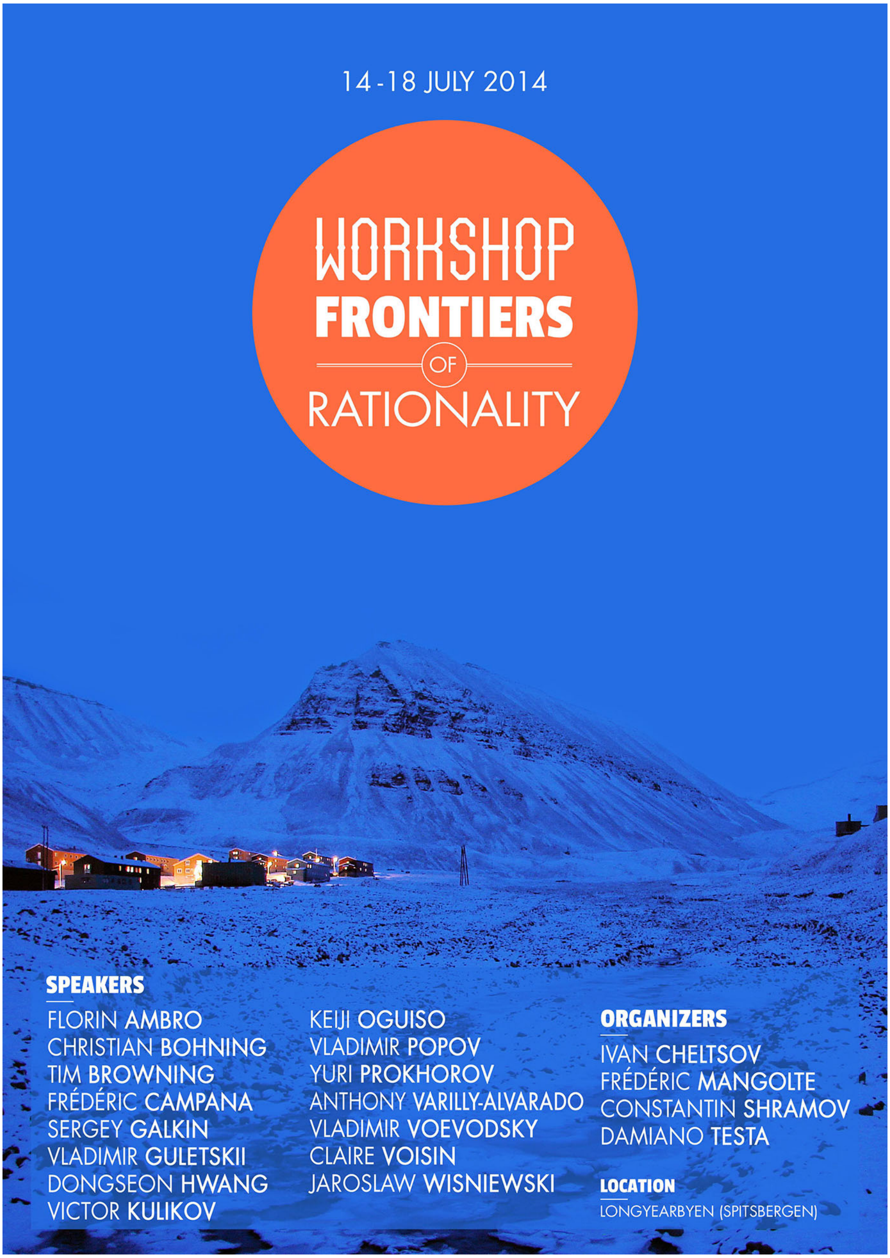
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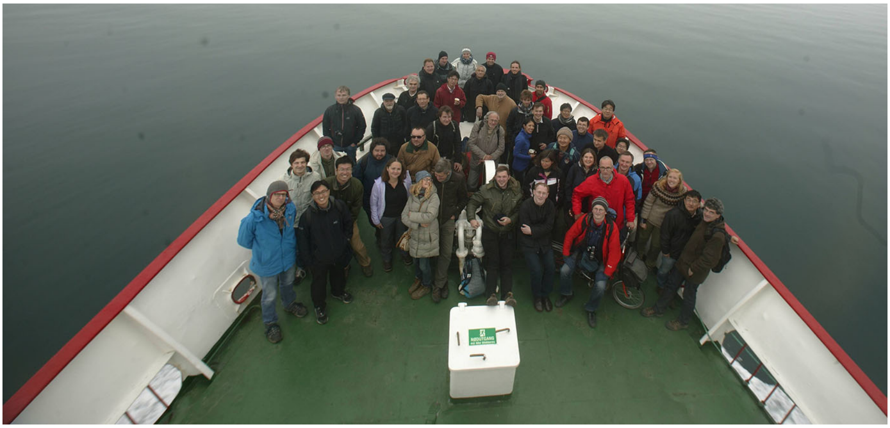

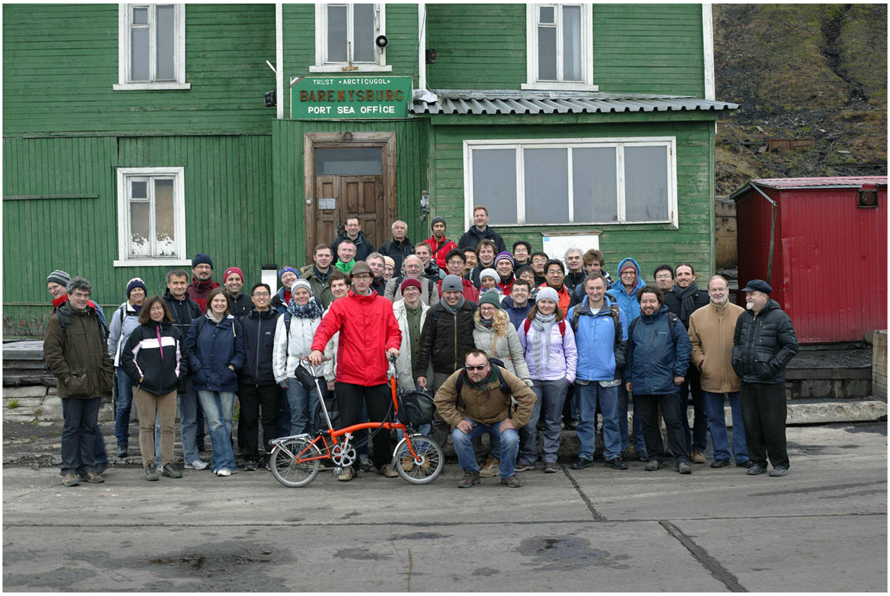
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