

Real and complex affine algebraic geometry

ANGERS – March 10-12 2011

Abstracts

Irreducibility criterion for quasi-ordinary polynomials

Abdallah Assi (Angers)

Let \mathbf{K} be an algebraically closed field of characteristic zero and let f be a nonzero polynomial of $\mathbf{K}[[x_1, \dots, x_n]][y]$. We say that f is a quasi-ordinary polynomial in y if the y -discriminant of f is of the form $x_1^{N_1} \dots x_n^{N_n} (a + \phi(x_1, \dots, x_n))$, where $a \in \mathbf{K}^*$ and $\phi(0, \dots, 0) = 0$. In this talk we give a criterion for a quasi-ordinary polynomial to be irreducible, and we give some local and global applications. In particular we introduce the notion of quasi-ordinary polynomials in $\mathbf{K}[x_1, \dots, x_n][y]$ and we generalize to these polynomials the theory of Abhyankar-Moh.

Generators and relations of the group of birational transformations over the real and complex number fields

Jérémy Blanc (Basel)

It is well-known that the Cremona group $\mathrm{Cr}_2(\mathbf{C})$ of birational transformations of the complex plane is generated by $\mathrm{PGL}(3, \mathbf{C})$ and by the standard quadratic transformation $(x, y, z) \dashrightarrow (1/x, 1/y, 1/z)$. Some relations are known, I will try to present a new one, which is quite simple.

I will then talk about the group $\mathrm{Cr}_2(\mathbf{R})$ of birational transformations of the real plane, which is NOT generated by $\mathrm{PGL}(3, \mathbf{R})$ and the standard quadratic transformation. I shall also talk about the subgroup of birational transformations which induce a diffeomorphism on the real points, and explain the generators/relations of this group.

Higher dimensional analogues of the Danilov-Gizatullin Theorem

Adrien Dubouloz (Dijon)

The Danilov-Gizatullin isomorphism theorem is a quite surprising result which asserts that the isomorphism type of the complement of an ample section C in a Hirzebruch surface $\mathbf{F}_n \rightarrow \mathbf{P}^1$ depends only on the self-intersection of C . In particular, it depends neither on the ambient surface nor on a particular section C . After giving a short and elementary proof of this result, we will consider the higher dimensional problem of characterizing isomorphism types of complements of ample sub-bundles in locally trivial \mathbf{P}^n -bundles over \mathbf{P}^1 .

Additive Group Invariants in Positive Characteristic

Emilie Dufresne (Basel)

Roberts, Freudenburg, and Daigle and Freudenburg have given the smallest counterexamples to Hilbert's fourteenth problem. Each arises as the ring of invariants of an additive group action on a polynomial ring over a field of characteristic zero, and thus, each corresponds to the kernel of a locally nilpotent derivation. In positive characteristic, additive group actions correspond to locally finite iterative higher derivations, a more restrictive notion. We set up characteristic-free analogs of the three examples mentioned above, and show that, contrary to characteristic zero, in every positive characteristic, the rings of invariant are finitely generated. (Joint work with Andreas Maurischat.)

Hypersurfaces algébriques réelles non singulières affines compactes et formes définies-positives

Johannes Huisman (Brest & Mittag-Leffler)

On met en évidence un lien entre les deux sujets mentionnés dans le titre de l'exposé, et on discutera de ses conséquences.

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Some \mathbf{A}^n -Fibrations are Fiber Bundles

Hanspeter Kraft (Basel)

A flat morphism $f: X \rightarrow Y$ is called an \mathbf{A}^n -fibration if all fibers are reduced and isomorphic to affine n -space \mathbf{A}^n . The morphism f is called a fiber bundle with fiber \mathbf{A}^n if it is locally trivial in the étale topology with fiber \mathbf{A}^n . In general, it is an open (and seemingly very difficult) question whether \mathbf{A}^n -fibrations are always fiber bundles. It is known in two "small" cases : (a) $n = 1$ and Y is normal (Kambayashi-Wright ; normality of Y is essential) (b) $n = 2$ and Y is a normal curve (Kaliman) In both cases, the bundle is locally trivial in Zariski-topology We will give a short unified proof for both results which also shows the obstructions for the obvious generalizations, e.g. the morphisms one gets from Venereau's polynomial.

Infinitely transitive actions on real affine suspensions

Karine Kuyumzhiyan (Moscow)

The action of a group G on a set X is called infinitely transitive if for every positive integer m , this action is m -transitive on X . An interesting class is formed by affine algebraic varieties X such that the group of algebraic automorphisms $\text{Aut}(X)$ acts on $\text{reg } X$ infinitely transitively. For the algebraically closed field of characteristic zero, Kaliman and Zaidenberg in 1999 and Arzhantsev-K-Zaidenberg in 2010 constructed a series of examples of varieties X with this property. One of these examples is a suspension over an affine variety Y already having this property. It was also shown that for the ground field $k = \mathbf{R}$ the main result holds under certain assumptions, in particular, Y and X need to be connected. In our work with F. Mangolte, we use the notion "infinite transitivity on each connected component" which has much more sense for real varieties. We show that this new property holds while passing to a suspension in the non-connected case under some mild restrictions. (Joint work with Frédéric Mangolte.)

Surfaces algébriques réelles infiniment homogènes

Frédéric Mangolte (Angers)

Après avoir décrit le groupe des difféomorphismes birationnels d'une surface algébrique réelle, je déterminerai les surfaces pour lesquelles ce groupe agit de manière infiniment transitive sur les points réels de la surface. (Travail en commun avec Jérémy Blanc.)

Les théorèmes de Kleiman et de Rosenlicht pour le groupe spécial d'automorphismes

Michel Zaidenberg (Grenoble)

Il s'agit du sous-groupe du groupe d'automorphismes d'une variété affine engendré par tous les sous-groupes unipotents à un paramètre. En général, c'est un groupe "algébrique-combinatoire" de dimension infinie. Cependant son action sur la variété manifeste une ressemblance étonnante avec une action d'un groupe algébrique.