

Real and Complex Affine Algebraic Geometry

La Rochelle (Birpol 2), May 29–31 2012

Tuesday, May 29

9h30–10h30 Frédéric Mangolte (Angers)

Approximating curves on real rational surfaces

It is known that every differentiable map from the circle to a rational variety $S^1 \rightarrow X$ can be approximated by an algebraic map $\mathbf{P}^1(\mathbf{R}) \rightarrow X$. In particular, any simple closed curve on a rational surface S can be approximated by a rational curve on S . Note that the usual result is about maps of rational curves, so the image may have some extra isolated points. As a consequence of our theorem on real algebraic automorphisms in the Cremona group, we get rid of these. Recall that by Comessatti's theorem, a real rational surface S is diffeomorphic to the sphere, the torus or to any nonorientable surface.

Main theorem: Let R be a closed topological surface and K be a simple closed curve on R . Assume that R is either nonorientable or of genus < 2 . Then the pair (R, K) is diffeomorphic to a pair (S, L) of a smooth rational curve L on a smooth rational surface S . Furthermore, if R is a rational surface, then K can be approximated in the C^∞ -topology by a smooth rational curve.

Furthermore, we give necessary and sufficient topological conditions for a simple closed curve on a rational surface S to be approximated by a (-1) -curve. Note that (-1) -curves are quite rigid objects, hence approximating by (-1) -curves is a subtle problem (joint work with Janos Kollár).

11h–12h Jérémy Blanc (Bâle)

Automorphisms of surfaces with dynamical degree > 1

In this talk, I will focus on the question, asked by E. Bedford, J. Déserti and J. Grivaux, which states the following: does there exist a birational map g of the projective plane such that fg is not conjugate to an automorphism of a projective variety, with dynamical degree > 1 , for any automorphism f of the projective plane. I will give an example of such map, which is of degree 6, and is just the extension of the automorphism $(x, y) \mapsto (x + (y + x^3)^2, y + x^3)$ of the affine plane. The proof uses the nice geometry of this map at infinity.

14h30–15h30 Julie Déserti (Paris VI, Bâle)

An introduction of holomorphic foliations of codimension one (Lecture 1)

I will try to present the basic notions, both local and global, using classical examples.

16h–17h Eric Edo (Nouméa)

Separability of wild automorphisms

We discuss the possibility of separating tame and wild automorphisms of \mathbf{C}^3 using weighted multidegree (joint work with M. Karas and S. Kuroda).

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Wednesday, May 30

9h30–10h30 Jean-Philippe Monnier (Angers)

Regulous functions

Let k be an integer. We study the ring of rational functions admitting an extension of class C^k to the real affine space. We establish several properties of this ring. In particular, we prove a strong Nullstellensatz. We also give a geometrical characterization of prime ideals of this ring in terms of their zero-locus (joint work with J. Huisman, F. Mangolte and G. Fichou).

11h–12h Alexei Kanel-Belov (Jerusalem)

$\text{Aut}(K[x_1, \dots, x_n])$ as ind-scheme and Konsevich conjecture

14h30–15h30 Julie Déserti (Paris VI, Bâle)

An introduction of holomorphic foliations of codimension one (Lecture 2)

16h–17h Michel Granger (Angers)

Partial normalizations of Coxeter arrangements and of their discriminants

In a recent paper in common with D. Mond and M. Schultze we define partial normalizations of Coxeter arrangements and of their discriminants. We shall begin with elementary examples like A_3 et B_3 , and then recall the classification of irreducible Coxeter arrangements, Chevalley's theorem about the ring of invariants polynomial and finally the notion of a free divisor in the sense of Saito. Coxeter arrangements and their discriminants are a well known example of such divisors. We will show how natural are the structural rings of the normalisations cited above. We shall conclude by giving an idea of the proof which relies on the existence of a structure of a Frobenius variety on the discriminant and we shall consider other natural situations where such a partial normalization might appear.

17h30–18h30 David Finston (Las Cruces, USA)

Additive Group Actions on Affine Spaces

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Thursday, May 31

9h30–10h30 Stéphane Lamy (Toulouse) On a theorem of Larsen and Lunts

The Grothendieck ring $K_0[\text{Var}]$ is the free ring generated by isomorphism classes of varieties modulo the "scissor relation" $[X \setminus Y] = [X] - [Y]$, where Y is any subvariety of a given variety X . The structure of this ring is still elusive; in particular an open conjecture is that $[X] = [Y]$ should imply that X and Y are piecewise isomorphic, i.e. there exist stratification of X and Y such that strata are pairwise isomorphic. A basic example is when we have a birational selfmap $X \dashrightarrow X$, hence by definition an isomorphism between open subsets U and V of X : we have $[X \setminus U] = [X \setminus V]$ in the Grothendieck ring. In a recent paper with Julien Sebag we prove that if $X = \mathbf{P}^3$ is the complex 3-dimensional projective space then indeed $[X \setminus U]$ and $[X \setminus V]$ are piecewise isomorphic, so the conjecture is true at least in this special case. Our proof relies in particular on a result of Larsen and Lunts (2004) which will be the main topic of my talk. The point is to show that there exists a canonical ring morphism from the Grothendieck ring to $\mathbf{Z}[SB]$, which is the ring freely generated by varieties under the "stably birational" equivalence relation; furthermore the kernel of this morphism is the principal ideal generated by the class of the affine line. I will try to give an idea of the tools needed for the (not so difficult) proof; notes (in french) will be made available on my webpage.

11h–12h Mikhail Zaidenberg (Grenoble) An example of computation of the groups SAut and Aut

We treat a classical example of an affine variety with an infinite dimensional special automorphism group. This group can be described explicitly, which leads to an explicit description of the whole automorphism group.