*Barême sur 35*

## Exercice 1

*Barême : 1 pt par question, dont 0,25 pour le domaine de dérivabilité*

1) $$\left(\cos(2x+3)e^{4x}\right)' \underset{(uv)'}{=} -2\sin(2x+3)e^{4x} + 4cos(2x+3)e^{4x}$$
$$= \boxed{(4\cos(2x+3) - 2\sin(2x+3))e^{4x}} \text{ sur } \mathbb{R}$$

2) $$\left(x^3\ln(2\sin x)\right)' \underset{(uv)'}{=} 3x^2\ln(2\sin x) + x^3 \times \frac{2\cos(x)}{2\sin(x)}$$
$$= \boxed{3x^2\ln(2\sin x) + x^3 \times \frac{\cos(x)}{\sin(x)}} \text{ sur } \bigcup_{k\in\mathbb{Z}} ]2k\pi;(2k+1)\pi[$$

3) $$\left(\frac{\tan(3x+4)}{x^2-1}\right)' \underset{\left(\frac{u}{v}\right)'}{=} \frac{3\times(1+\tan^2(3x+4))\times(x^2-1) - \tan(3x+4)\times 2x}{(x^2-1)^2}$$
$$= \boxed{\frac{3\times(x^2-1) + 3\tan^2(3x+4)\times(x^2-1) - 2x\times\tan(3x+4)}{(x^2-1)^2}}$$
$$\text{sur } \bigcup_{k\in\mathbb{Z}} \left]-\frac{4}{3} - \frac{\pi}{6} + \frac{k\pi}{3}; -\frac{4}{3} + \frac{\pi}{6} + \frac{k\pi}{3}\right[ \setminus \{-1;1\}$$

4) $$\left(\cos\left(\sqrt{1+x^2}\right)\right)' \underset{(g\circ f)'}{=} -\sin\left(\sqrt{1+x^2}\right) \times \frac{2x}{2\sqrt{1+x^2}}$$
$$= \boxed{-\frac{x\sin\left(\sqrt{1+x^2}\right)}{\sqrt{1+x^2}}} \text{ sur } \mathbb{R}$$

5) On commence par calculer :
$$(x^x)' = (\exp(x\ln x))' \underset{(e^u)'}{=} (\ln(x)+1)\times\exp(x\ln x) = (\ln(x)+1)\times x^x$$
$$\left(\ln\left(\frac{x^x-1}{x^x+1}\right)\right)' = \left(\ln\left(\frac{\exp(x\ln x)-1}{\exp(x\ln x)+1}\right)\right)'$$
$$\underset{\ln(u)'}{=} \frac{x^x+1}{x^x-1} \times \frac{(\ln(x)+1)x^x(x^x+1) - (x^x-1)(\ln(x)+1)x^x}{(x^x+1)^2}$$
$$= \boxed{\frac{x^x+1}{x^x-1} \times \frac{2x^x(\ln(x)+1)}{(x^x+1)^2}} \text{ sur } ]1,+\infty[$$

6) $$\left(\frac{1}{\sqrt{2+x}}\right)' = \left((2+x)^{-\frac{1}{2}}\right)' \underset{(u^n)'}{=} -\frac{1}{2}(2+x)^{-\frac{3}{2}} \times 1 = -\frac{1}{(4+2x)\sqrt{2+x}}$$
sur $]-2;+\infty[$

7) $$\left(\tan\left(\frac{2x}{1+x^2}\right)\right)' \underset{\tan(u)'}{=} \left(1+\tan^2\left(\frac{2x}{1+x^2}\right)\right) \times \frac{2(1+x^2) - 2x\times 2x}{(1+x^2)^2}$$
$$= \boxed{\left(1+\tan^2\left(\frac{2x}{1+x^2}\right)\right) \times \frac{2-2x^2}{(1+x^2)^2}}$$
$$\text{sur } \mathbb{R} \setminus \left\{x\in\mathbb{R} \mid \frac{2x}{1+x^2} \equiv \frac{\pi}{2}[\pi]\right\} = \mathbb{R}$$

8) $$\left(\sqrt{|1-x^2|}\right)' = \begin{cases} \dfrac{-2x}{2\sqrt{1-x^2}} = \boxed{-\dfrac{x}{\sqrt{1-x^2}}} & \text{si } x \in ]-1;1[ \\ \dfrac{2x}{2\sqrt{x^2-1}} = \boxed{\dfrac{x}{\sqrt{x^2-1}}} & \text{si } x \in ]-\infty;-1[\cup]1;+\infty1[ \end{cases}$$
sur $\mathbb{R} \setminus \{-1;1\}$

9) $$\left(\ln|x^2-3x+2|\right)' \underset{(\ln|u|)'}{=} \boxed{\frac{2x-3}{x^2-3x+2}} \text{ sur } \mathbb{R}\setminus\{1;2\}$$

10) $$\left(\sin(x^2-5x+1)\ln(2x+1)\right)' \underset{(uv)'}{=} \boxed{(2x-5)\cos(x^2-5x+1)\ln(2x+1)}$$
$$\boxed{+\sin(x^2-5x+1)\times\frac{2}{2x+1}}$$
$$\text{sur } \left]-\frac{1}{2};+\infty\right[.$$

## Exercice 2

*Barême : 1pt par question dont 0,25 pour le domaine de validité*

1) $$\int \frac{e^x}{e^x+1}dx \underset{\frac{u'}{u}}{=} \boxed{\ln(e^x+1)} \quad \text{sur } \mathbb{R}$$

2) $$\int \frac{1}{e^x+1}dx = \int \frac{e^{-x}}{(e^x+1)e^{-x}}dx = -\int \frac{-e^{-x}}{1+e^{-x}}dx \underset{\frac{u'}{u}}{=} \boxed{-\ln(1+e^{-x})} \quad \text{sur } \mathbb{R}$$

3) $$\int \frac{x+2}{x^2+4x+4}dx = \frac{1}{2}\int \frac{2x+4}{x^2+4x+4}dx \underset{\frac{u'}{u}}{=} \frac{1}{2}\ln|x^2+4x+4| = \frac{1}{2}\ln\left((x+2)^2\right)$$
$$= \boxed{\ln(x+2)} \quad \text{sur } \mathbb{R}\setminus\{-2\}$$

4) $$\int \cos(x)\sin^3(x)dx \underset{u'u^n}{=} \boxed{\frac{1}{4}\sin^4(x)} \quad \text{sur } \mathbb{R}$$

5) $$\int \frac{1}{\cos^2(-x+\frac{\pi}{7})}dx = -\int \frac{-1}{\cos^2(-x+\frac{\pi}{7})}dx \underset{u'f(u)}{=} \boxed{-\tan\left(-x+\frac{\pi}{7}\right)}$$
$$\text{sur } \bigcup_{k\in\mathbb{Z}} \left]-\frac{5\pi}{14}+k\pi; \frac{9\pi}{14}+k\pi\right[$$

6) $$\int \tan^2\left(4t-\frac{\pi}{5}\right)dt = \int \left(1+\tan^2\left(4t-\frac{\pi}{5}\right)\right)dt - \int 1dt$$
$$\underset{u'f(u)}{=} \boxed{\frac{1}{4}\tan\left(4t-\frac{\pi}{5}\right)-t}$$
$$\text{sur } \mathbb{R}\setminus\left(\frac{7\pi}{40}+\frac{\pi}{4}\mathbb{Z}\right)$$

7) $$\int 2^x dx = \int \exp(x\ln 2)dx = \boxed{\frac{1}{\ln 2}\times 2^x} \quad \text{sur } \mathbb{R}$$

8) $$\int \sin(\omega x+\phi)dx = \boxed{-\frac{1}{\omega}\cos(\omega x+\phi)} \quad \text{sur } \mathbb{R}$$

9) $$\int (2x+3)^4 dx = \frac{1}{2}\int 2(2x+3)^4 dx \underset{u'u^n}{=} \boxed{\frac{1}{10}(2x+3)^5} \quad \text{sur } \mathbb{R}$$

10) $$\int \frac{1}{\sqrt{3x+2}}dx = \frac{1}{3}\int \frac{3}{\sqrt{3x+2}}dx \underset{\frac{u'}{\sqrt{u}}}{=} \boxed{\frac{2}{3}\sqrt{3x+2}} \quad \text{sur } \left]-\frac{2}{3};+\infty\right[$$

11) $$\int \ln(3x-1)dx = \frac{1}{3}\int 3\ln(3x-1)dx \underset{u'f(u)}{=} \boxed{\frac{1}{3}\left[(3x-1)\ln(3x-1)-(3x-1)\right]}$$
$$\text{sur } \left]\frac{1}{3};+\infty\right[ \text{ car } \int \ln(x)dx = x\ln(x)-x.$$

12) $$\int \frac{x^2}{\sqrt{5+x^3}}dx \underset{\frac{u'}{\sqrt{u}}}{=} \boxed{\frac{2}{3}\sqrt{5+x^3}} \quad \text{sur } ]-\sqrt[3]{5};+\infty[.$$

13) $$\int \frac{1}{x\ln(x)}dx \underset{\frac{u'}{u}}{=} \boxed{\ln\left|\ln(x)\right|} \quad \text{sur } \mathbb{R}_+^*\setminus\{1\}$$

14) $$\int \frac{e^{\frac{1}{x}}}{x^2}dx \underset{u'e^u}{=} \boxed{-e^{\frac{1}{x}}} \quad \text{sur } \mathbb{R}^*$$

15) $$\int x^2\sqrt{1+x^3}dx = \frac{1}{3}\int 3x^2\sqrt{1+x^3}dx \underset{u'\sqrt{u}}{=} \boxed{\frac{2}{9}(1+x^3)\sqrt{1+x^3}}$$
$$\text{sur } [-1;+\infty[$$

16) $$\int \frac{x}{\cos^2(x^2)}dx = \frac{1}{2}\int \frac{2x}{\cos^2(x^2)}dx \underset{u'f(u)}{=} \boxed{\frac{1}{2}\tan(x^2)}$$
$$\text{sur } \mathbb{R}\setminus\left\{\pm\sqrt{\frac{\pi}{2}+k\pi} \mid k\in\mathbb{N}\right\}$$

17) $$\int \frac{\left(x+\sqrt{x^2+1}\right)^2}{\sqrt{x^2+1}}dx = \int \frac{x+\sqrt{x^2+1}}{\sqrt{x^2+1}}\times\left(x+\sqrt{x^2+1}\right)$$
$$= \int \left(1+\frac{x}{\sqrt{x^2+1}}\right)\times\left(x+\sqrt{x^2+1}\right)$$
$$\underset{u'u}{=} \boxed{\frac{1}{2}\left[\left(x+\sqrt{x^2+1}\right)^2\right]} \quad \text{sur } \mathbb{R}$$

## Exercice 3

*Barème : 2pt par question*

1.

$$\int (x^2+x+1)e^x dx = (x^2+x+1)e^x - \int (2x+1)e^x dx \quad PPP \left| \begin{array}{ll} u = x^2+x+1 & u' = 2x+1 \\ v = e^x & v' = e^x \end{array} \right.$$

$$= (x^2+x+1)e^x - \left((2x+1)e^x - \int 2e^x dx\right) \quad PPP \left| \begin{array}{ll} u = 2x+1 & u' = 2 \\ v = e^x & v' = e^x \end{array} \right.$$

$$= (x^2+x+1)e^x - (2x+1)e^x + 2e^x$$

$$= \boxed{(x^2-x+2)e^x} \quad \text{sur } \mathbb{R}$$

2.

$$\int e^{-x}\ln(1+e^x)dx = -e^{-x}\ln(1+e^x) - \int -e^{-x} \times \frac{e^x}{1+e^x}dx \quad PPP \left| \begin{array}{ll} u = \ln(1+e^x) & u' = \frac{e^x}{1+e^x} \\ v = -e^{-x} & v' = e^{-x} \end{array} \right.$$

$$= -e^{-x}\ln(1+e^x) + \int \frac{1}{1+e^x}dx$$

$$= -e^{-x}\ln(1+e^x) + \int \frac{e^{-x}}{(1+e^x)e^{-x}}dx$$

$$= -e^{-x}\ln(1+e^x) + \int \frac{e^{-x}}{1+e^{-x}}dx$$

$$= \boxed{-e^{-x}\ln(1+e^x) - \ln(1+e^{-x})}$$

que l'on peut simplifier :

$$= -e^{-x}\ln(e^x(1+e^{-x})) - \ln(1+e^{-x})$$

$$= -e^{-x}\left(\ln(e^x) + \ln(1+e^{-x})\right)) - \ln(1+e^{-x})$$

$$= \boxed{-xe^{-x} - (1+e^{-x})\ln(1+e^{-x})} \quad \text{sur } \mathbb{R}$$

3.

$$I = \int \cos(x)e^x dx = \cos(x)e^x - \int -\sin(x)e^x dx \quad PPP \left| \begin{array}{ll} u = \cos(x) & u' = -\sin(x) \\ v = e^x & v' = e^x \end{array} \right.$$

$$= \cos(x)e^x + \int \sin(x)e^x dx$$

$$= \cos(x)e^x + \sin(x)e^x - \int \cos(x)e^x dx \quad PPP \left| \begin{array}{ll} u = \sin(x) & u' = \cos(x) \\ v = e^x & v' = e^x \end{array} \right.$$

$$\implies 2I = \cos(x)e^x + \sin(x)e^x$$

$$\implies I = \frac{1}{2}e^x \times \sqrt{2}\cos\left(x - \frac{\pi}{4}\right)$$

$$= \boxed{\frac{\sqrt{2}}{2}e^x\cos\left(x-\frac{\pi}{4}\right)} \quad \text{sur } \mathbb{R}$$

4.

$$J = \int \cos(\ln(x))dx = \int x \times \frac{1}{x}\cos(\ln(x))dx$$

$$= x \times \sin(\ln(x)) - \int \sin(\ln(x))dx \quad PPP \left| \begin{array}{ll} u = x & u' = 1 \\ v = \sin(\ln(x)) & v' = \frac{1}{x}\cos(\ln(x)) \end{array} \right.$$

$$= x \times \sin(\ln(x)) - \int x \times \frac{1}{x}\sin(\ln(x))dx$$

$$= x \times \sin(\ln(x)) + x \times \cos(\ln(x)) + \int -\cos(\ln(x))dx \quad PPP \left| \begin{array}{ll} u = x & u' = 1 \\ v = -\cos(\ln(x)) & v' = \frac{1}{x}\sin(\ln(x)) \end{array} \right.$$

$$\implies 2J = x \times (\sin(\ln(x)) + \cos(\ln(x)))$$

$$\implies J = \frac{x}{2} \times (\sin(\ln(x)) + \cos(\ln(x)))$$

$$\implies J = \boxed{\frac{x}{\sqrt{2}} \times \cos\left(\ln(x) - \frac{\pi}{4}\right)} \quad \text{sur } \mathbb{R}_+^*$$