Exercice 4. On pose $t = \tan(\theta/2)$.

$$\begin{aligned}\frac{1}{1+t^2} &= \frac{1}{1+\frac{\sin^2(\theta/2)}{\cos^2(\theta/2)}}\\ &= \frac{1}{\frac{\cos^2(\theta/2)}{\cos^2(\theta/2)}+\frac{\sin^2(\theta/2)}{\cos^2(\theta/2)}}\\ &= \frac{\cos^2(\theta/2)}{\cos^2(\theta/2)+\sin^2(\theta/2)}\\ &= \cos^2(\theta/2)\end{aligned}$$

car $\cos^2 + \sin^2 = 1$.

$$\begin{aligned}\frac{1-t^2}{1+t^2} &= \frac{1-\tan^2(\theta/2)}{1+\tan^2(\theta/2)}\\ &= \frac{1-\frac{\sin^2(\theta/2)}{\cos^2(\theta/2)}}{1+\frac{\sin^2(\theta/2)}{\cos^2(\theta/2)}}\\ &= \frac{\frac{\cos^2(\theta/2)-\sin^2(\theta/2)}{\cos^2(\theta/2)}}{\frac{\cos^2(\theta/2)+\sin^2(\theta/2)}{\cos^2(\theta/2)}}\\ &= \cos^2(\theta/2)-\sin^2(\theta/2)\\ &= \cos(\theta)\end{aligned}$$

car $\cos(2a) = \cos^2(a) - \sin^2(a)$.

$$\begin{aligned}\frac{2t}{1+t^2} &= \frac{2\tan(\theta/2)}{1+\tan^2(\theta/2)}\\ &= \frac{2\times\frac{\sin(\theta/2)}{\cos(\theta/2)}}{1+\frac{\sin^2(\theta/2)}{\cos^2(\theta/2)}}\\ &= \frac{2\times\frac{\sin(\theta/2)}{\cos(\theta/2)}}{\frac{\cos^2(\theta/2)+\sin^2(\theta/2)}{\cos^2(\theta/2)}}\\ &= \cos^2(\theta/2)\times 2\times\frac{\sin(\theta/2)}{\cos(\theta/2)}\\ &= 2\sin(\theta/2)\cos(\theta/2)\\ &= \sin(\theta)\end{aligned}$$

car $\sin(2a) = 2\sin(a)\cos(a)$.