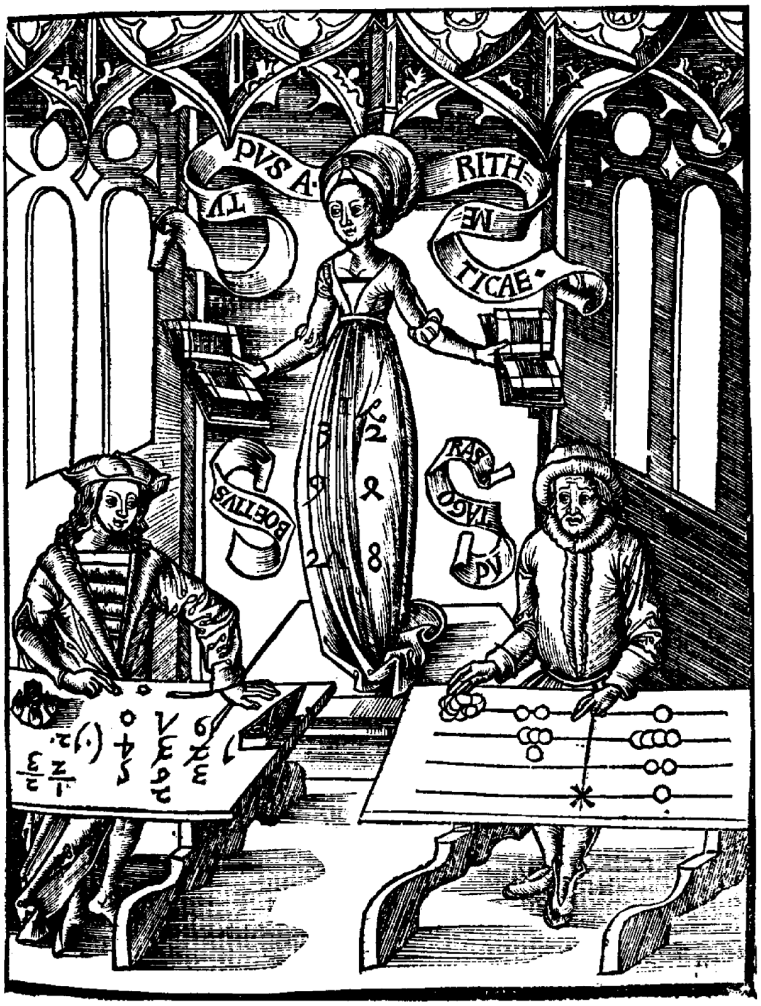
Cahier de calcul

— réponses —



Margarita philosophica (La perle philosophique), Gregor Reisch (1508)

Cette gravure, extraite d'un manuel d'université de l'époque, représente Arithmetica, allégorie des mathématiques, arbitrant une compétition entre Boèce, qui utilise les chiffres indo-arabes, et Pythagore, qui utilise un boulier.

Ce cahier de calcul a été écrit collectivement.

Coordination

Colas Bardavid

Équipe des participants

Vincent Bayle, Romain Basson, Olivier Bertrand, Ménard Bourgade, Julien Bureaux, Alain Camanes, Mathieu Charlot, Mathilde Colin de Verdière, Keven Commault, Miguel Concy, Rémy Eupherte, Hélène Gros, Audrey Hechner, Florian Hechner, Marie Hézard, Nicolas Laillet, Valérie Le Blanc, Thierry Limoges, Quang-Thai Ngo, Xavier Pellegrin, Fabien Pellegrini, Jean-Louis Pourtier, Valérie Robert, Jean-Pierre Técourt, Guillaume Tomasini, Marc Tenti

Le pictogramme de l'horloge a été créé par Ralf Schmitzer (The Noun Project).

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Le contenu a été adapté à la filière BCPST par Jean-Philippe Préaux.

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Sommaire

Fiche n° 1. Fractions

Réponses

1.1 a) ..... $\boxed{\frac{4}{5}}$

1.1 b) ..... $\boxed{2^5}$

1.1 c) ..... $\boxed{3}$

1.1 d) ..... $\boxed{-2 \times 3^{3k-2}}$

1.2 a) ..... $\boxed{\frac{1}{6}}$

1.2 b) ..... $\boxed{\frac{7}{15}}$

1.2 c) ..... $\boxed{9}$

1.2 d) ..... $\boxed{\frac{1}{9}}$

1.3 a) ..... $\boxed{247}$

1.3 b) ..... $\boxed{\frac{203}{24}}$

1.3 c) ..... $\boxed{\frac{-10}{3}}$

1.3 d) ..... $\boxed{1\ 000}$

1.4 ..... $\boxed{\frac{16}{35}}$

1.5 a) ..... $\boxed{2\ 022}$

1.5 b) ..... $\boxed{\frac{1}{2}}$

1.5 c) ..... $\boxed{1}$

1.5 d) ..... $\boxed{2}$

1.6 a) ..... $\boxed{\frac{-1}{n(n+1)^2}}$

1.6 b) ..... $\boxed{-\frac{ab}{a-b}}$

1.6 c) ..... $\boxed{\frac{3}{2}n}$

1.7 ..... $\boxed{\frac{n^3+n}{n+1}}$

1.8 a) ..... $\boxed{4+\frac{5}{6}}$

1.8 b) ..... $\boxed{1+\frac{1}{k-1}}$

1.8 c) ..... $\boxed{3+\frac{5}{x-2}}$

1.9 ..... $\boxed{2t}$

1.10 a) ..... $\boxed{\frac{3}{5} > \frac{5}{9}}$

1.10 b) ..... $\boxed{\frac{12}{11} > \frac{10}{12}}$

1.10 c) ..... $\boxed{\frac{125}{25} = \frac{105}{21}}$

1.11 ..... $\boxed{\text{Non}}$

1.12 ..... $\boxed{A > B}$

Fiche n° 2. Puissances

Réponses

2.1 a) ..... $\boxed{10^{8}}$

2.1 b) ..... $\boxed{10^{15}}$

2.1 c) ..... $\boxed{10^{2}}$

2.1 d) ..... $\boxed{10^{-2}}$

2.1 e) ..... $\boxed{10^{4}}$

2.1 f) ..... $\boxed{10^{-8}}$

2.2 a) ..... $\boxed{15^{4}}$

2.2 b) ..... $\boxed{5^{-6}}$

2.2 c) ..... $\boxed{2^{7}}$

2.2 d) ..... $\boxed{(-7)^{-2}}$

2.2 e) ..... $\boxed{3^{5}}$

2.2 f) ..... $\boxed{3^{28}}$

2.3 a) ..... $\boxed{2^{-4} \cdot 3^{-1}}$

2.3 b) ..... $\boxed{2^{21} \cdot 3}$

2.3 c) ..... $\boxed{2}$

2.3 d) ..... $\boxed{2^{38} \cdot 3^{26}}$

2.4 a) ..... $\boxed{8}$

2.4 b) ..... $\boxed{11}$

2.4 c) ..... $\boxed{3^{10}}$

2.4 d) ..... $\boxed{2^{6} \cdot 5}$

2.5 a) ..... $\boxed{\dfrac{x}{x+1}}$

2.5 b) ..... $\boxed{\dfrac{1}{x-2}}$

2.5 c) ..... $\boxed{\dfrac{2x}{x+1}}$

2.5 d) ..... $\boxed{\dfrac{2}{x-2}}$

Fiche n° 3. Calcul littéral

Réponses

3.1 a) $\boxed{8x^3 - 6x^2 + \frac{3}{2}x - \frac{1}{8}}$

3.1 b) $\boxed{x^5 - 2x^4 + x^3 - x^2 + 2x - 1}$

3.1 c) $\boxed{x^5 - x^3 + x^2 - 1}$

3.1 d) $\boxed{x^5 + 2x^4 + x^3 - x^2 - 2x - 1}$

3.1 e) $\boxed{x^5 - x^3 - x^2 + 1}$

3.1 f) $\boxed{x^4 + x^2 + 1}$

3.2 a) $\boxed{-2 + 12x - 17x^2 + 8x^3 - 3x^4}$

3.2 b) $\boxed{-28 + 21x}$

3.2 c) $\boxed{2 - x + x^3 - x^4 - x^5}$

3.2 d) $\boxed{-1 - 3x - 3x^2 + x^3}$

3.2 e) $\boxed{1 + x^4}$

3.2 f) $\boxed{1 + 2x + 3x^2 + 2x^3 + x^4}$

3.3 a) $\boxed{-6(6x + 7)}$

3.3 b) $\boxed{4(5x + 4)(-5x + 1)}$

3.3 c) $\boxed{2(3x - 4)(10x + 3)}$

3.3 d) $\boxed{-8(x + 1)(x + 16)}$

3.4 a) $\boxed{(x - 1)^2}$

3.4 b) $\boxed{(x + 2)^2}$

3.4 c) $\boxed{(x + 1)(x + 2)}$

3.4 d) $\boxed{3\left(x + \frac{7 - \sqrt{37}}{6}\right)\left(x + \frac{7 + \sqrt{37}}{6}\right)}$

3.4 e) $\boxed{2\left(x + \frac{3 - \sqrt{233}}{4}\right)\left(x + \frac{3 + \sqrt{233}}{4}\right)}$

3.4 f) $\boxed{-5(x - 1)\left(x - \frac{1}{5}\right)}$

3.5 a) $\boxed{(x + y - z)(x + y + z)}$

3.5 b) $\boxed{(14x + 3y)(-12x + 3y)}$

3.5 c) $\boxed{(x + 1)(y + 1)}$

3.5 d) $\boxed{(x - 1)(y - 1)}$

3.5 e) $\boxed{(x + y)(x + 1)^2}$

3.5 f) $\boxed{\left(a^2 + b^2\right)\left(y - 4x^2\right)\left(y + 4x^2\right)}$

3.6 a) $\boxed{(x - 1)(x + 1)\left(x^2 + 1\right)}$

3.6 b) $\boxed{-8\left(x^2 + 1\right)(x - 4)(x + 4)}$

3.6 c) $\boxed{\left(x^2 + x + 1\right)\left(x^2 - x + 1\right)}$

3.6 d) $\boxed{\left(a^2 + b^2\right)\left(c^2 + d^2\right)}$

3.6 e) $\boxed{\left(a^2 + b^2 + c^2 + d^2\right)\left(p^2 + q^2 + r^2 + s^2\right)}$

Fiche n° 4. Racines carrées

Réponses

4.1 a) ..... $\boxed{5}$

4.1 b) ..... $\boxed{\sqrt{3}-1}$

4.1 c) ..... $\boxed{-\sqrt{3}+2}$

4.1 d) ..... $\boxed{\sqrt{7}-2}$

4.1 e) ..... $\boxed{\pi-3}$

4.1 f) ..... $\boxed{|3-a|}$

4.2 a) ..... $\boxed{20}$

4.2 b) ..... $\boxed{9+4\sqrt{5}}$

4.2 c) ..... $\boxed{1+\sqrt{3}}$

4.2 d) ..... $\boxed{3+\sqrt{2}}$

4.2 e) ..... $\boxed{12\sqrt{7}}$

4.2 f) ..... $\boxed{12}$

4.2 g) ..... $\boxed{9-\frac{10}{3}\sqrt{2}}$

4.2 h) ..... $\boxed{10}$

4.3 a) .... $\boxed{2-\sqrt{2}-\sqrt{3}+\frac{1}{2}\sqrt{6}}$

4.3 b) ..... $\boxed{3-2\sqrt{2}}$

4.3 c) ..... $\boxed{1-\sqrt{10}+\sqrt{15}}$

4.3 d) .... $\boxed{\sqrt{15}+\sqrt{10}-\sqrt{6}-2}$

4.3 e) ..... $\boxed{-(\sqrt{2}+\sqrt{3})}$

4.3 f) .... $\boxed{-\frac{3+\sqrt{2}+\sqrt{3}+\sqrt{6}}{2}}$

4.3 g) ..... $\boxed{2\sqrt{2}}$

4.3 h) ..... $\boxed{50-25\sqrt{3}}$

4.4 ..... $\boxed{\frac{\sqrt{2}+2-\sqrt{6}}{4}}$

4.5 a) ..... $\boxed{\frac{x}{\sqrt{x-1}}}$

4.5 b) ..... $\boxed{x-\sqrt{x^2-1}}$

4.5 c) ..... $\boxed{1+\sqrt{x-1}}$

4.5 d) ..... $\boxed{\frac{1}{2}\frac{1}{x-1}}$

4.5 e) ..... $\boxed{\frac{x(x-2)}{(x-1)\sqrt{x-1}}}$

4.5 f) ..... $\boxed{-4(x-1)^2}$

4.6 a) ..... $\boxed{\sqrt{2}}$

4.6 b) ..... $\boxed{2\sqrt{2}}$

4.7 a) ..... $\boxed{-11+5\sqrt{5}}$

4.7 b) ..... $\boxed{1+\sqrt{2}}$

4.7 c) ..... $\boxed{1+\sqrt{2}}$

4.7 d) ..... $\boxed{\sqrt{3}}$

4.7 e) ..... $\boxed{1+\sqrt{5}}$

4.7 f) ..... $\boxed{\ln(1+\sqrt{2})}$

4.8 ..... $\boxed{1}$

Fiche n° 5. Expressions algébriques

Réponses

5.1 a) ..... $\boxed{7a^2+12a+7}$

5.1 b) ..... $\boxed{a^2-a-1}$

5.1 c) ..... $\boxed{4a^2-a-3}$

5.1 d) ..... $\boxed{-a^2+1}$

5.2 a) ..... $\boxed{8+6\mathrm{i}}$

5.2 b) ..... $\boxed{8-6\mathrm{i}}$

5.2 c) ..... $\boxed{18-26\mathrm{i}}$

5.2 d) ..... $\boxed{-9-46\mathrm{i}}$

5.3 a) ..... $\boxed{39-18\mathrm{i}}$

5.3 b) ..... $\boxed{2197}$

5.3 c) ..... $\boxed{-4+43\mathrm{i}\sqrt{5}}$

5.3 d) ..... $\boxed{1}$

5.4 a) ..... $\boxed{3}$

5.4 b) ..... $\boxed{1}$

5.4 c) ..... $\boxed{1}$

5.4 d) ..... $\boxed{0}$

5.4 e) ..... $\boxed{-1}$

5.4 f) ..... $\boxed{31}$

5.5 a) ..... $\boxed{a^2+2}$

5.5 b) ..... $\boxed{a^3+3a}$

5.5 c) ..... $\boxed{a^4+4a^2+2}$

5.6 a) ..... $\boxed{a^2-2b}$

5.6 b) ..... $\boxed{ab-3c}$

5.6 c) ..... $\boxed{a^3-3ab+3c}$

5.6 d) ..... $\boxed{ab-c}$

5.6 e) ..... $\boxed{ac}$

5.6 f) ..... $\boxed{-2ac+b^2}$

5.7 a) ..... $\boxed{a^2b-ac-2b^2}$

5.7 b) ... $\boxed{a^4-4a^2b+4ac+2b^2}$

5.7 c) ..... $\boxed{0}$

5.7 d) ..... $\boxed{1}$

5.7 e) ..... $\boxed{a}$

Fiche n° 6. Équations du second degré

Réponses

6.1 a) ..... $3, 3$

6.1 b) ..... $-1/3, -1/3$

6.1 c) ..... $2, -6$

6.1 d) ..... $2, 3$

6.1 e) ..... 0, donc 5

6.1 f) ..... 0, donc $-3/2$

6.1 g) ..... $\varnothing$

6.1 h) ..... 1 donc -5

6.1 i) ..... 1 donc $8/3$

6.1 j) ..... -1 donc $-19/5$

6.2 a) ..... $6, 7$

6.2 b) ..... $-3, -5$

6.2 c) ..... $-7, -11$

6.2 d) ..... $-3, 11$

6.2 e) ..... a, b

6.2 f) ..... $a - b, a + b$

6.3 a) ..... $2/3$

6.3 b) ..... $-2/7$

6.3 c) ..... $-1/m$

6.3 d) ..... $2m/(m + 3)$

6.4 a) ..... 1 donc $(a - b)/(b - c)$

6.4 b) ..... 1 donc $c(a - b)/(a(b - c))$

6.4 c) ..... m donc $-(m + a + b)$

6.4 d) ..... m donc $m(a - b)/(b - c)$

6.4 e) ..... m donc ab/m

6.4 f) ..... $a + b$ puis $2ab/(a + b)$.

6.5 a) ..... $x^2 - 22x + 117 = 0$

6.5 b) ..... $x^2 - 6x - 187 = 0$

6.5 c) ..... $x^2 - 4x + 1 = 0$

6.5 d) ..... $x^2 - 2mx + 3 = 0$

6.5 e) ..... $2x^2 - (4m + 1)x + (2m^2 + m - 15) = 0$

6.5 f) .... $m^2x^2 + (m - 2m^2)x + (m^2 - m - 2) = 0$

6.6 a) ..... $m = -3/4$ et $x = 3/4$

6.6 b) ... $m = -1$ et $x = -2$, ou $m = 7$ et $x = 2/3$

6.6 c) ..... $m = 1$ et $x = -1$ ou $m = -1$ et $x = 1$

6.7 a) ..... $a = 2$ et $b = 3$

6.7 b) ..... $a = -2$ et $b = 1$

6.7 c) ..... $a = -3$ et $b = 5$

6.7 d) ..... $a = 1/2$ et $b = 8$

6.7 e) ..... $a = 1$ et $b = 3\sqrt{7}$

6.8 a) ..... $]-\infty, 1] \cup [\sqrt{2}, +\infty[$

6.8 b) ..... $[-3, 5]$

6.8 c) ..... $]-\infty, -1] \cup [2/3, +\infty[$

6.8 d) ..... $]-\infty, -1/2[\cup [4, +\infty[$

Fiche n° 7. Exponentielle et Logarithme

Réponses

7.1 a) $\boxed{4\ln 2}$

7.1 b) $\boxed{9\ln 2}$

7.1 c) $\boxed{-3\ln 2}$

7.1 d) $\boxed{\frac{1}{2}\ln 2}$

7.1 e) $\boxed{3\ln 2}$

7.1 f) $\boxed{2\ln 2 + 2\ln 3}$

7.2 a) $\boxed{-\ln 3 - 2\ln 2}$

7.2 b) $\boxed{2\ln 3 - 2\ln 2}$

7.2 c) $\boxed{\ln 3 + 11\ln 2}$

7.2 d) $\boxed{3\ln 5 + 2\ln 2}$

7.2 e) $\boxed{-2\ln 5 + 4\ln 2}$

7.2 f) $\boxed{2\ln 5 - 2\ln 2}$

7.3 $\boxed{-2\ln 2 - 2\ln 5}$

7.4 a) $\boxed{\frac{25}{8}\ln(\sqrt{2}-1)}$

7.4 b) $\boxed{17 + 12\sqrt{2}}$

7.4 c) $\boxed{0}$

7.4 d) $\boxed{0}$

7.5 a) $\boxed{8}$

7.5 b) $\boxed{\frac{1}{2}}$

7.5 c) $\boxed{\frac{1}{3}}$

7.5 d) $\boxed{\frac{1}{9}}$

7.5 e) $\boxed{-\frac{1}{2}}$

7.5 f) $\boxed{\frac{3}{2}}$

7.6 a) $\boxed{-2}$

7.6 b) $\boxed{\frac{1}{\ln 2}}$

7.6 c) $\boxed{-17}$

7.6 d) $\boxed{1}$

7.6 e) $\boxed{-1}$

7.6 f) $\boxed{e}$

7.7 a) $\boxed{\text{impaire}}$

7.7 b) $\boxed{\text{impaire}}$

7.7 c) $\boxed{\text{impaire}}$

7.7 d) $\boxed{\text{impaire}}$

7.8 a) $\boxed{\mathbb{R}}$

7.8 b) $\boxed{\text{ok}}$

7.8 c) $\boxed{1}$

7.8 d) $\boxed{\text{-1}}$

7.9 a) $\boxed{x + \ln 2}$

7.9 b) $\boxed{\frac{e^x}{\sqrt{1+x}}}$

7.9 c) $\boxed{\ln|x-1|}$

7.9 d) $\boxed{-\frac{1}{1+x}}$

7.9 e) $\boxed{e^{x\ln(1+x)}}$

7.10 a) $\boxed{x \geqslant \frac{\ln 12 + 5}{3}}$

7.10 b) $\boxed{x \in [0,1]}$

7.10 c) $\boxed{x \geqslant \frac{2}{e}}$

7.10 d) $\boxed{x \geqslant -\frac{1}{12}}$

7.10 e) $\boxed{\varnothing}$

7.10 f) $\boxed{\frac{-13-\sqrt{273}}{2}}$

Fiche nº 8. Trigonométrie

Réponses

8.1 a) ..... $\boxed{0}$

8.1 b) ..... $\boxed{0}$

8.1 c) ..... $\boxed{-1-\sqrt{3}}$

8.1 d) ..... $\boxed{-\frac{1}{2}}$

8.2 a) ..... $\boxed{0}$

8.2 b) ..... $\boxed{-\sin x}$

8.2 c) ..... $\boxed{2\cos x}$

8.2 d) ..... $\boxed{-2\cos x}$

8.3 a) ..... $\boxed{\frac{\sqrt{6}-\sqrt{2}}{4}}$

8.3 b) ..... $\boxed{\frac{\sqrt{6}+\sqrt{2}}{4}}$

8.3 c) ..... $\boxed{\frac{\sqrt{6}-\sqrt{2}}{4}}$

8.3 d) ..... $\boxed{\frac{\sqrt{3}-1}{\sqrt{3}+1}}$

8.4 a) ..... $\boxed{-\sin x}$

8.4 b) ..... $\boxed{\frac{1}{\cos x}}$

8.4 c) ..... $\boxed{0}$

8.4 d) ..... $\boxed{4\cos^3 x - 3\cos x}$

8.5 a) ..... $\boxed{\frac{\sqrt{2+\sqrt{2}}}{2}}$

8.5 b) ..... $\boxed{\frac{\sqrt{2-\sqrt{2}}}{2}}$

8.6 a) ..... $\boxed{\tan x}$

8.6 b) ..... $\boxed{2}$

8.6 c) ..... $\boxed{8\cos^4 x - 8\cos^2 x + 1}$

8.7 a) ..... $\boxed{\left\{\frac{\pi}{3}, \frac{5\pi}{3}\right\}}$

8.7 a) ..... $\boxed{\left\{-\frac{\pi}{3}, \frac{\pi}{3}\right\}}$

8.7 a) ..... $\boxed{\left\{\frac{\pi}{3}+2k\pi, k\in\mathbb{Z}\right\} \cup \left\{-\frac{\pi}{3}+2k\pi, k\in\mathbb{Z}\right\}}$

8.7 b) ..... $\boxed{\left\{\frac{4\pi}{3}, \frac{5\pi}{3}\right\}}$

8.7 b) ..... $\boxed{\left\{\frac{-2\pi}{3}, \frac{-\pi}{3}\right\}}$

8.7 b) ..... $\boxed{\left\{\frac{4\pi}{3}+2k\pi, k\in\mathbb{Z}\right\} \cup \left\{\frac{5\pi}{3}+2k\pi, k\in\mathbb{Z}\right\}}$

8.7 c) ..... $\boxed{\left\{\frac{7\pi}{6}, \frac{11\pi}{6}\right\}}$

8.7 c) ..... $\boxed{\left\{-\frac{5\pi}{6}, -\frac{\pi}{6}\right\}}$

8.7 c) ..... $\boxed{\left\{\frac{7\pi}{6}+2k\pi, k\in\mathbb{Z}\right\} \cup \left\{\frac{11\pi}{6}+2k\pi, k\in\mathbb{Z}\right\}}$

8.7 d) ..... $\boxed{\left\{\frac{\pi}{4}, \frac{5\pi}{4}\right\}}$

8.7 d) ..... $\boxed{\left\{-\frac{3\pi}{4}, \frac{\pi}{4}\right\}}$

8.7 d) ..... $\boxed{\left\{\frac{\pi}{4}+k\pi, k\in\mathbb{Z}\right\}}$

8.7 e) ..... $\boxed{\left\{\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}\frac{7\pi}{4}\right\}}$

8.7 e) ..... $\boxed{\left\{-\frac{3\pi}{4}, -\frac{\pi}{4}, \frac{\pi}{4}, \frac{3\pi}{4}\right\}}$

8.7 e) ..... $\boxed{\left\{\frac{\pi}{4}+k\frac{\pi}{2}, k\in\mathbb{Z}\right\}}$

8.7 f) ..... $\boxed{\left\{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}\right\}}$

8.7 f) ..... $\boxed{\left\{-\frac{5\pi}{6}, -\frac{\pi}{6}, \frac{\pi}{6}, \frac{5\pi}{6}\right\}}$

8.7 f) ..... $\boxed{\left\{\frac{\pi}{6}+k\pi, k\in\mathbb{Z}\right\} \cup \left\{\frac{5\pi}{6}+k\pi, k\in\mathbb{Z}\right\}}$

8.7 g) ..... $\boxed{\left\{\frac{\pi}{12}, \frac{11\pi}{12}, \frac{13\pi}{12}, \frac{23\pi}{12}\right\}}$

8.7 g) ..... $\boxed{\left\{-\frac{11\pi}{12}, -\frac{\pi}{12}, \frac{\pi}{12}, \frac{11\pi}{12}\right\}}$

8.7 g) ..... $\boxed{\left\{\frac{\pi}{12}+k\pi, k\in\mathbb{Z}\right\} \cup \left\{\frac{11\pi}{12}+k\pi, k\in\mathbb{Z}\right\}}$

8.7 h) ..... $\boxed{\left\{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}\right\}}$

8.7 h) ..... $\boxed{\left\{-\frac{\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}\right\}}$

8.7 h) $\left\{\frac{\pi}{6}+k\frac{2\pi}{3}, k\in\mathbb{Z}\right\}$

8.7 i) $\left\{\frac{\pi}{7}, \frac{13\pi}{7}\right\}$

8.7 i) $\left\{-\frac{\pi}{7}, \frac{\pi}{7}\right\}$

8.7 i) $\left\{\frac{\pi}{7}+2k\pi, k\in\mathbb{Z}\right\}\cup\left\{-\frac{\pi}{7}+2k\pi, k\in\mathbb{Z}\right\}$

8.7 j) $\left\{\frac{5\pi}{14}, \frac{9\pi}{14}\right\}$

8.7 j) $\left\{\frac{5\pi}{14}, \frac{9\pi}{14}\right\}$

8.7 j) $\left\{\frac{5\pi}{14}+2k\pi, k\in\mathbb{Z}\right\}\cup\left\{\frac{9\pi}{14}+2k\pi, k\in\mathbb{Z}\right\}$

8.8 a) $\left[0, \frac{3\pi}{4}\right]\cup\left[\frac{5\pi}{4}, 2\pi\right]$

8.8 a) $\left[-\frac{3\pi}{4}, \frac{3\pi}{4}\right]$

8.8 b) $\left[\frac{\pi}{3}, \frac{5\pi}{3}\right]$

8.8 b) $\left[-\pi, -\frac{\pi}{3}\right]\cup\left[\frac{\pi}{3}, \pi\right]$

8.8 c) $\left[0, \frac{\pi}{6}\right]\cup\left[\frac{5\pi}{6}, 2\pi\right]$

8.8 c) $\left[-\pi, \frac{\pi}{6}\right]\cup\left[\frac{5\pi}{6}, \pi\right]$

8.8 d) $\left[0, \frac{\pi}{6}\right]\cup\left[\frac{5\pi}{6}, \frac{7\pi}{6}\right]\cup\left[\frac{11\pi}{6}, 2\pi\right]$

8.8 d) $\left[-\pi, -\frac{5\pi}{6}\right]\cup\left[-\frac{\pi}{6}, \frac{\pi}{6}\right]\cup\left[\frac{5\pi}{6}, \pi\right]$

8.8 e) $\left[\frac{\pi}{4}, \frac{\pi}{2}\right[\cup\left[\frac{5\pi}{4}, \frac{3\pi}{2}\right[$

8.8 e) $\left[-\frac{3\pi}{4}, -\frac{\pi}{2}\right[\cup\left[\frac{\pi}{4}, \frac{\pi}{2}\right[$

8.8 f) $\left[\frac{\pi}{4}, \frac{\pi}{2}\right[\cup\left]\frac{\pi}{2}, \frac{3\pi}{4}\right]\cup\left[\frac{5\pi}{4}, \frac{3\pi}{2}\right[\cup\left]\frac{3\pi}{2}, \frac{7\pi}{4}\right]$

8.8 f) $\left[-\frac{3\pi}{4}, -\frac{\pi}{2}\right[\cup\left]-\frac{\pi}{2}, -\frac{\pi}{4}\right]\cup\left[\frac{\pi}{4}, \frac{\pi}{2}\right[\cup\left]\frac{\pi}{2}, \frac{3\pi}{4}\right]$

8.8 g) $\left[0, \frac{3\pi}{4}\right]\cup\left[\frac{7\pi}{4}, 2\pi\right]$

8.8 g) $\left[-\frac{\pi}{4}, \frac{3\pi}{4}\right]$

8.8 h) $\left[0, \frac{3\pi}{8}\right]\cup\left[\frac{7\pi}{8}, \frac{11\pi}{8}\right]\cup\left[\frac{15\pi}{8}, 2\pi\right]$

8.8 h) $\left[-\pi, -\frac{5\pi}{8}\right]\cup\left[-\frac{\pi}{8}, \frac{3\pi}{8}\right]\cup\left[\frac{7\pi}{8}, \pi\right]$

Fiche n° 9. Dérivation

Réponses

9.1 a) ..... $\boxed{6x^2 + 2x - 11}$

9.1 b) ..... $\boxed{5x^4 - 6x^2 + 4x - 15}$

9.1 c) ..... $\boxed{(2x^2 - 2x + 10)\exp(2x)}$

9.1 d) ..... $\boxed{(6x - 1)\ln(x - 2) + \dfrac{3x^2 - x}{x - 2}}$

9.2 a) ..... $\boxed{5(x^2 - 5x)^4(2x - 5)}$

9.2 b) ..... $\boxed{4(2x^3 + 4x - 1)(3x^2 + 2)}$

9.2 c) ..... $\boxed{8\cos^2(x) - 6\cos(x)\sin(x) - 4}$

9.2 d) ..... $\boxed{-3(3\cos(x) - \sin(x))^2(3\sin(x) + \cos(x))}$

9.3 a) ..... $\boxed{\dfrac{2x}{x^2 + 1}}$

9.3 b) ..... $\boxed{\dfrac{1}{x\ln(x)}}$

9.3 c) ..... $\boxed{(-2x^2 + 3x - 1)\exp(x^2 + x)}$

9.3 d) ..... $\boxed{6\cos(2x)\exp(3\sin(2x))}$

9.4 a) ..... $\boxed{\dfrac{6x}{(x^2 + 1)^2}\cos\left(\dfrac{2x^2 - 1}{x^2 + 1}\right)}$

9.4 b) ..... $\boxed{\dfrac{2x^2 + 2x - 8}{(x^2 + 4)^2}\sin\left(\dfrac{2x + 1}{x^2 + 4}\right)}$

9.4 c) ..... $\boxed{\dfrac{\cos(x)}{2\sqrt{\sin(x)}}}$

9.4 d) ..... $\boxed{\dfrac{\cos(\sqrt{x})}{2\sqrt{x}}}$

9.5 a) ..... $\boxed{\dfrac{(2x + 3)(2\sin(x) + 3) - (x^2 + 3x) \times 2\cos(x)}{(2\sin(x) + 3)^2}}$

9.5 b) ..... $\boxed{\dfrac{2 - 3x}{2\sqrt{x}(3x + 2)^2}}$

9.5 c) ..... $\boxed{-2\dfrac{(x^2 + 1)\sin(2x + 1) + x\cos(2x + 1)}{(x^2 + 1)^2}}$

9.5 d) ..... $\boxed{\dfrac{(4x + 3)\ln(x) - 2x - 3}{(\ln(x))^2}}$

9.6 a) ..... $\boxed{2x\sin\left(\dfrac{1}{x}\right) - \cos\left(\dfrac{1}{x}\right)}$

9.6 b) ..... $\boxed{\dfrac{9}{(9 - x^2)\sqrt{9 - x^2}}}$

9.6 c) ..... $\boxed{\dfrac{1}{1 - x^2}}$

9.6 d) ..... $\boxed{\dfrac{x\cos(x) - \sin(x)}{x\sin(x)}}$

9.7 a) ..... $\boxed{\dfrac{10x - 5}{(3 - x)^2(2 + x)^2}}$

9.7 b) ..... $\boxed{\dfrac{2}{x + 1}\left(x + \dfrac{1 + \sqrt{3}}{2}\right)\left(x + \dfrac{1 - \sqrt{3}}{2}\right)}$

9.7 c) ..... $\boxed{\dfrac{2x^2 + 2x + 5}{(x + 2)(x - 1)^2}}$

9.7 d) ..... $\boxed{\dfrac{x^2}{(x + 1)^2}}$

9.7 e) ..... $\boxed{\dfrac{2}{x(1 - \ln(x))^2}}$

Fiche n° 10. Primitives

Réponses

10.1 a) $\boxed{\ln|t+1|}$

10.1 b) $\boxed{-\dfrac{3}{t+2}}$

10.1 c) $\boxed{-\dfrac{3}{2(t+2)^2}}$

10.1 d) $\boxed{-\dfrac{\cos(4t)}{4}}$

10.2 a) $\boxed{\dfrac{2}{3}(1+t)^{\frac{3}{2}}-\dfrac{3}{4}t^{\frac{4}{3}}}$

10.2 b) $\boxed{\dfrac{1}{2}\mathrm{e}^{2t+1}}$

10.2 c) $\boxed{\dfrac{1}{3}\mathrm{Arctan}(3t)}$

10.3 a) $\boxed{\dfrac{2}{3}\ln|1+t^3|}$

10.3 b) $\boxed{\dfrac{1}{6}(1+2t^2)^{\frac{3}{2}}}$

10.3 c) $\boxed{-\sqrt{1-t^2}}$

10.3 d) $\boxed{\dfrac{3}{4}(1+7t^2)^{\frac{2}{3}}}$

10.3 e) $\boxed{\dfrac{1}{6}\ln(1+3t^2)}$

10.3 f) $\boxed{-\dfrac{1}{(1+3t^2)^2}}$

10.4 a) $\boxed{\dfrac{1}{4}\ln^4 t}$

10.4 b) $\boxed{2\sqrt{\ln t}}$

10.4 c) $\boxed{\dfrac{2}{(3-\mathrm{e}^{2t})^2}}$

10.4 d) $\boxed{-\dfrac{2}{3t^{\frac{3}{2}}}}$

10.4 e) $\boxed{\ln|1-\mathrm{e}^{-t}+\mathrm{e}^{t}|}$

10.4 f) $\boxed{-\mathrm{e}^{\frac{1}{t}}}$

10.5 a) $\boxed{-\dfrac{1}{3}\cos^3 t}$

10.5 b) $\boxed{\mathrm{e}^{\sin t}}$

10.5 c) $\boxed{-\ln|\cos t|}$

10.5 d) $\boxed{-\ln|1-\sin t|}$

10.5 e) $\boxed{-2\cos\sqrt{t}}$

10.5 f) $\boxed{\dfrac{1}{\pi}\sin(\pi\ln t)}$

10.5 g) $\boxed{\tan t - t}$

10.5 h) $\boxed{\dfrac{1}{2}\tan^2 t+\ln|\cos t|}$

10.5 i) $\boxed{\dfrac{1}{4}\tan^4 t}$

10.5 j) $\boxed{2\sqrt{\tan t}}$

10.5 k) $\boxed{-\dfrac{1}{\tan t}}$

10.5 l) $\boxed{\dfrac{1}{2}\dfrac{1}{(1-\sin t)^2}}$

10.5 m) $\boxed{\dfrac{1}{2}\mathrm{Arctan}(2t)}$

10.5 n) $\boxed{\mathrm{Arctan}(\mathrm{e}^t)}$

10.6 a) $\boxed{\dfrac{t}{2}+\dfrac{\sin(2t)}{4}}$

10.6 b) $\boxed{-\dfrac{\cos(4t)}{8}-\dfrac{\cos(2t)}{4}}$

10.6 c) $\boxed{-\cos t+\dfrac{1}{3}\cos^3 t}$

10.6 d) $\boxed{\ln(1+\sin^2 t)}$

10.6 e) $\boxed{\ln|\tan t|}$

10.6 f) $\boxed{-\mathrm{cotan}t+\tan t}$

10.6 g) $\boxed{\dfrac{1}{4}\ln|\tan 2t|}$

10.7 a) $\boxed{t+\ln t-\dfrac{1}{t}}$

10.7 b) $\boxed{\ln t-\dfrac{1}{2t^2}}$

10.7 c) $\boxed{t+\dfrac{t^3}{3}+\dfrac{t^5}{5}}$

10.7 d) $\boxed{t-\dfrac{t^2}{2}+\dfrac{t^3}{3}}$

10.7 e) $\boxed{t-2\ln|t+1|}$

10.7 f) ..... $t - \dfrac{t^2}{2} + \dfrac{t^3}{3} - \ln|t+1|$

10.7 g) ..... $\dfrac{1}{2}\ln(1+t^2) - \text{Arctan}(t)$

10.7 h) ..... $\ln|t+1| + \dfrac{1}{t+1}$

10.8 a) ..... $2(t-1)$ puis $\dfrac{1}{3}t^3 - t^2 + 5t$

10.8 b) ..... $-\dfrac{1}{t^2}\left(\dfrac{2}{t}+1\right)$ puis $-\dfrac{1}{t} + \ln|t|$

10.8 c) ..... $\dfrac{1}{2\sqrt{t}} + \dfrac{3}{t^4}$ puis $\dfrac{2}{3}t^{\frac{3}{2}} + \dfrac{1}{2t^2}$

10.8 d) ..... $-\dfrac{4}{t^5} - \dfrac{3}{2}\dfrac{1}{t^{5/2}}$ puis $-\dfrac{1}{3}\dfrac{1}{t^3} - \dfrac{2}{\sqrt{t}}$

10.8 e) ..... $2\mathrm{e}^{2t} - 3\mathrm{e}^{-3t}$ puis $\dfrac{1}{2}\mathrm{e}^{2t} - \dfrac{1}{3}\mathrm{e}^{-3t}$

10.8 f) ..... $3\mathrm{e}^{3t-2}$ puis $\dfrac{1}{3}\mathrm{e}^{3t-2}$

10.8 g) ..... $-\dfrac{t(t^3+2)}{(t-1)^2(t^2+t+1)^2}$ puis $\dfrac{1}{3}\ln(|t^3-1|)$

10.8 h) .. $-\dfrac{3t^2-2t-3}{(t^2+1)^2}$ puis $\dfrac{3}{2}\ln(t^2+1) - \text{Arctan}(t)$

10.8 i) ..... $\cos t(3\cos^2 t - 2)$ puis $-\dfrac{1}{3}\cos^3 t$

10.8 j) ..... $-\dfrac{2t\sin\frac{1}{t} + \cos\frac{1}{t}}{t^4}$ puis $\cos\dfrac{1}{t}$

10.8 k) ..... $\dfrac{2\mathrm{e}^t}{(2+\mathrm{e}^t)^2}$ puis $\ln(2+\mathrm{e}^t)$

10.8 l) ..... $\dfrac{2\cos t + 3}{(2+3\cos t)^2}$ puis $-\dfrac{1}{3}\ln|2+3\cos t|$

10.8 m) ..... $\dfrac{1}{(1-t^2)^{3/2}}$ puis $-\sqrt{1-t^2}$

10.8 n) ..... $2\dfrac{3\cos^2 t - 1}{(1+\cos^2 t)^2}$ puis $-\ln(1+\cos^2(t))$

10.8 o) ..... $(1-2t^2)\mathrm{e}^{-t^2}$ puis $-\dfrac{1}{2}\mathrm{e}^{-t^2}$

10.8 p) ..... $\dfrac{\ln t - 2}{t^2}$ puis $\ln t - \dfrac{1}{2}\ln^2 t$

10.8 q) ..... $-\dfrac{1+\ln t}{t^2\ln^2 t}$ puis $\ln|\ln t|$

10.8 r) ..... $\dfrac{\cos\ln t - \sin\ln t}{t^2}$ puis $-\cos(\ln t))$

10.8 s) ..... $-\dfrac{\mathrm{e}^t(\mathrm{e}^{2t}-1)}{(1+\mathrm{e}^{2t})^2})$ puis $\text{Arctan}(\mathrm{e}^t)$

Fiche n° 11. Calcul d'intégrales

Réponses

11.1 a) ..... $\boxed{\text{Positif}}$

11.1 b) ..... $\boxed{\text{Négatif}}$

11.1 c) ..... $\boxed{\text{Positif}}$

11.2 a) ..... $\boxed{14}$

11.2 b) ..... $\boxed{50}$

11.2 c) ..... $\boxed{\frac{147}{2}}$

11.2 d) ..... $\boxed{-54}$

11.2 e) ..... $\boxed{0}$

11.2 f) ..... $\boxed{\frac{5}{2}}$

11.3 a) ..... $\boxed{8}$

11.3 b) ..... $\boxed{-2}$

11.3 c) ..... $\boxed{\frac{8}{3}}$

11.3 d) ..... $\boxed{0}$

11.3 e) ..... $\boxed{-\frac{1}{30}}$

11.3 f) ..... $\boxed{-\frac{2}{101}}$

11.4 a) ..... $\boxed{0}$

11.4 b) ..... $\boxed{1}$

11.4 c) ..... $\boxed{\frac{1}{2}}$

11.4 d) ..... $\boxed{18}$

11.4 e) ..... $\boxed{\mathrm{e}^2 - \mathrm{e}^{-3}}$

11.4 f) ..... $\boxed{-\ln 3}$

11.5 a) ..... $\boxed{78}$

11.5 b) ..... $\boxed{2(\mathrm{e}^3 - 1)}$

11.5 c) . $\boxed{\frac{1}{\pi}\ln\left(1+\frac{\pi}{2}\right)}$

11.5 d) ..... $\boxed{\frac{\sqrt{2}}{6}}$

11.5 e) ..... $\boxed{6}$

11.5 f) ..... $\boxed{\frac{1}{2} - \frac{\sqrt{3}}{2}}$

11.6 a) ..... $\boxed{0}$

11.6 b) ..... $\boxed{0}$

11.6 c) ..... $\boxed{\ln\left(\frac{2}{\sqrt{3}}\right)}$

11.6 d) ..... $\boxed{\frac{1}{384}}$

11.6 e) ... $\boxed{\frac{1}{2}\left(1-\frac{1}{\mathrm{e}}\right)}$

11.6 f) ..... $\boxed{\frac{7}{48}}$

11.7 a) .... $\boxed{\frac{1}{2} - \frac{1}{\mathrm{e}+1}}$

11.7 b) ..... $\boxed{\frac{17}{2}}$

11.7 c) ..... $\boxed{\mathrm{e}^2}$

11.7 d) ..... $\boxed{3\mathrm{e} - 4}$

11.7 e) ..... $\boxed{-\frac{1}{3}}$

11.7 f) ..... $\boxed{\frac{5}{8}}$

11.8 a) ..... $\boxed{\frac{\pi}{4}}$

11.8 b) ..... $\boxed{\frac{99}{\ln 10}}$

11.8 c) ..... $\boxed{\frac{2}{3}}$

11.8 d) ..... $\boxed{\frac{2\pi}{9}}$

Fiche n° 12. Intégration par parties

Réponses

12.1 a) $\boxed{\frac{\pi}{2} - 1}$

12.1 b) $\boxed{4}$

12.1 c) $\boxed{\frac{(\ln(2))^2 2^{\ln(2)} - 2\ln(2) - 2^{\ln(2)} + 2}{(\ln(2))^2}}$

12.1 d) $\boxed{1}$

12.1 e) $\boxed{2\ln 2 - \frac{3}{4}}$

12.1 f) $\boxed{\ln(2) - 2 + \frac{\pi}{2}}$

12.1 g) $\boxed{\frac{\pi}{4} - \frac{1}{2}}$

12.1 h) $\boxed{-\frac{2\sqrt{2}}{3} + \frac{4}{3}}$

12.1 i) $\boxed{\frac{4}{3}\sqrt{2}\ln(2) - \frac{8}{9}\sqrt{2} + \frac{4}{9}}$

12.1 j) $\boxed{\frac{\pi}{4} - \frac{1}{2}\ln 2 - \frac{\pi^2}{32}}$

12.2 a) $\boxed{\begin{cases} \mathbb{R} \to \mathbb{R} \\ x \mapsto (-x+2)\mathrm{e}^x \end{cases}}$

12.2 b) $\boxed{\begin{cases} \mathbb{R}_+^* \to \mathbb{R} \\ x \mapsto -\dfrac{1+\ln x}{x} \end{cases}}$

12.2 c) $\boxed{\begin{cases} \mathbb{R} \to \mathbb{R} \\ x \mapsto x\arctan(x) - \dfrac{1}{2}\ln(1+x^2) \end{cases}}$

12.3 a) $\boxed{\frac{5}{2} - \mathrm{e}^2}$

12.3 b) $\boxed{\frac{\mathrm{e}^{\frac{\pi}{2}} + 1}{2}}$

12.4 a) $\boxed{\begin{cases} \mathbb{R}_+^* \to \mathbb{R} \\ x \mapsto x\ln^2 x - 2x\ln x + 2x \end{cases}}$

12.4 b) $\boxed{\begin{cases} \mathbb{R}_+^* \to \mathbb{R} \\ x \mapsto x^3\left(\dfrac{1}{3}\ln^2 x - \dfrac{2}{9}\ln x + \dfrac{2}{27}\right) \end{cases}}$

Fiche n° 13. Changements de variable

Réponses

13.1 a) $\boxed{\frac{\pi}{2}}$

13.1 b) $\boxed{\frac{\pi}{6}}$

13.1 c) $\boxed{\frac{1}{4}}$

13.1 d) $\boxed{\frac{1}{12}}$

13.1 e) $\boxed{2\ln\left(\frac{3}{2}\right)}$

13.2 a) $\boxed{\frac{\pi}{3\sqrt{3}}}$

13.2 b) $\boxed{\frac{1}{2}\ln\left(\frac{2\mathrm{e}+1}{3}\right)}$

13.2 c) $\boxed{\frac{\pi}{2}}$

13.2 d) $\boxed{\frac{1}{4}+\frac{\pi}{8}}$

13.2 e) $\boxed{\frac{\pi}{12}}$

13.2 f) $\boxed{\frac{1}{2}\ln\frac{5}{2}}$

13.3 a) $\boxed{2\mathrm{e}^2}$

13.3 b) $\boxed{-2((\sqrt{3}-1)\ln(\sqrt{3}-1)-4+2\sqrt{3}}$

13.4 a) $\boxed{\left\{\begin{array}{rcl}\left]0,\frac{\pi}{2}\right[& \to & \mathbb{R}\\ x & \mapsto & \tan x+\ln\tan(x)\end{array}\right.}$

13.4 b) $\boxed{\left\{\begin{array}{rcl}\mathbb{R}_+^* & \to & \mathbb{R}\\ x & \mapsto & 2\arctan\left(\sqrt{\mathrm{e}^x-1}\right)\end{array}\right.}$

13.4 c) $\boxed{\left\{\begin{array}{rcl}\mathbb{R}_+^* & \to & \mathbb{R}\\ x & \mapsto & \frac{3}{2}\ln(x^{\frac{2}{3}}+1)\end{array}\right.}$

13.4 d) $\boxed{\left\{\begin{array}{rcl}]1,+\infty[& \to & \mathbb{R}\\ x & \mapsto & \arctan\sqrt{x^2-1}\end{array}\right.}$

Fiche n° 14. Systèmes linéaires

Réponses

14.1 a) ..... $\boxed{\{(3,1)\}}$

14.1 b) ..... $\boxed{\{(7,2)\}}$

14.1 c) ..... $\boxed{\left\{(\frac{1}{3},\frac{2}{3})\right\}}$

14.1 d) ..... $\boxed{\left\{(\frac{\sqrt{2}}{3},\frac{\sqrt{2}}{2})\right\}}$

14.2 a) ..... $\boxed{\left\{(1-\frac{a}{4},\frac{-1}{2}+\frac{3}{8}a)\right\}}$

14.2 b) ..... $\boxed{(2,-3)}$

14.2 c) ..... $\boxed{\left\{(\frac{1}{13}a+\frac{5}{13}a^2,\frac{2}{13}a-\frac{3}{13}a^2)\right\}}$

14.2 d) ..... $\boxed{(a-2a^2,a+a^2)}$

14.3 a) ..... $\boxed{\{(1+z,-z,z);z\in\mathbb{R}\}}$

14.3 b) ..... $\boxed{\{(1,y,3+2y);y\in\mathbb{R}\}}$

14.3 c) ..... $\boxed{\left\{(\frac{13}{6}-\frac{5}{3}z,-\frac{1}{3}+\frac{4}{3}z,z);z\in\mathbb{R}\right\}}$

14.3 d) ..... $\boxed{\left\{(x,\frac{-5}{12}-\frac{3}{2}x,\frac{-25}{24}-\frac{7}{4}x);x\in\mathbb{R}\right\}}$

14.4 a) ..... $\boxed{\{(2,-1,3)\}}$

14.4 b) ..... $\boxed{\{(-1,4,2)\}}$

14.4 c) ..... $\boxed{\varnothing}$

14.4 d) ..... $\boxed{\left\{(-\frac{2}{7}-z,\frac{-3}{7},z)\,;\,z\in\mathbb{R}\right\}}$

14.5 a) ..... $\boxed{\{(1,1/2,1/2)\}}$

14.5 b) ..... $\boxed{\varnothing}$

14.5 c) ..... $\boxed{\{(5z,1-4z,z)\,;\,z\in\mathbb{R}\}}$

14.5 d) ..... $\boxed{\left\{(1,\frac{1}{a+2},\frac{1}{a+2})\right\}}$

14.6 a) ..... $\boxed{\{(5,3,-1)\}}$

14.6 b) ..... $\boxed{\varnothing}$

14.6 c) .... $\boxed{\left\{(\frac{a^2+a-1}{a^3-1}c,\frac{a^2-a-1}{a^3-1}c,\frac{-a^2+a+1}{a^3-1}c)\right\}}$

14.7 a) ..... $\boxed{\{(0,0,0)\}}$

14.7 b) ..... $\boxed{\{(x,y,-x-y)\,;\,(x,y)\in\mathbb{R}^2\}}$

14.7 c) ..... $\boxed{\{(x,x,x)\,;\,x\in\mathbb{R}\}}$

Fiche n° 15. Nombres complexes

Réponses

15.1 a) ..... $\boxed{4+32\mathrm{i}}$

15.1 b) ..... $\boxed{13-\mathrm{i}}$

15.1 c) ..... $\boxed{7-24\mathrm{i}}$

15.1 d) ..... $\boxed{5}$

15.1 e) .. $\boxed{-119+120\mathrm{i}}$

15.1 f) ..... $\boxed{\frac{3}{10}+\frac{1}{10}\mathrm{i}}$

15.1 g) ..... $\boxed{\frac{4}{29}-\frac{19}{29}\mathrm{i}}$

15.1 h) ..... $\boxed{\frac{1}{2}-\frac{\sqrt{3}}{2}\mathrm{i}}$

15.2 a) ..... $\boxed{12}$

15.2 b) ..... $\boxed{8\mathrm{e}^{\mathrm{i}\pi}}$

15.2 c) ..... $\boxed{\sqrt{3}\mathrm{e}^{\mathrm{i}\frac{\pi}{2}}}$

15.2 d) ..... $\boxed{2\mathrm{e}^{-\mathrm{i}\frac{\pi}{2}}}$

15.2 e) ..... $\boxed{2e^{\mathrm{i}\frac{8\pi}{5}}}$

15.2 f) ..... $\boxed{5\mathrm{e}^{-\frac{\pi}{4}\mathrm{i}}}$

15.2 g) ..... $\boxed{10\mathrm{e}^{-\frac{2\pi}{3}\mathrm{i}}}$

15.2 h) $\boxed{2\cos\left(\frac{\pi}{12}\right)\mathrm{e}^{\mathrm{i}\frac{\pi}{4}}}$

15.3 a) ..... $\boxed{1}$

15.3 b) ... $\boxed{\frac{1}{\sqrt{2}}+\mathrm{i}\frac{1}{\sqrt{2}}}$

15.3 c) .. $\boxed{-\frac{1}{\sqrt{2}}-\mathrm{i}\frac{1}{\sqrt{2}}}$

Fiche n° 16. Trigonométrie et nombres complexes

Réponses

16.1 a) ..... $\boxed{\frac{1}{4}\cos(3x)+\frac{3}{4}\cos(x)}$

16.1 b) ..... $\boxed{-\frac{1}{4}\cos(4x)+\frac{1}{2}\cos(2x)-\frac{1}{4}}$

16.1 c) ... $\boxed{-\frac{1}{8}\cos(6x)+\frac{1}{4}\cos(4x)-\frac{3}{8}\cos(2x)+\frac{1}{4}}$

16.1 d) ... $\boxed{-\frac{\sin(9x)}{8}+\frac{3\sin(5x)}{8}-\frac{\sin(3x)}{8}-\frac{3\sin(x)}{8}}$

16.1 e) .... $\boxed{\frac{\cos(9x)}{8}+\frac{3\cos(5x)}{8}+\frac{\cos(3x)}{8}+\frac{3\cos(x)}{8}}$

16.1 f) ..... $\boxed{-\frac{1}{4}\sin(11x)+\frac{1}{4}\sin(5x)+\frac{1}{2}\sin(3x)}$

16.2 a) ..... $\boxed{2\cos\left(\frac{\pi}{12}\right)\mathrm{e}^{\mathrm{i}\frac{\pi}{12}}}$

16.2 b) ..... $\boxed{\left(-2\cos\left(\frac{7\pi}{12}\right)\right)\mathrm{e}^{-\mathrm{i}\frac{5\pi}{12}}}$

16.2 c) ..... $\boxed{2\sin\left(\frac{\pi}{12}\right)\mathrm{e}^{\frac{-7\mathrm{i}\pi}{12}}}$

16.2 d) ..... $\boxed{2\cos\left(\frac{5\pi}{12}\right)\mathrm{e}^{\frac{5\mathrm{i}\pi}{12}}}$

16.2 e) ..... $\boxed{2\cos\left(\frac{\pi}{12}\right)\mathrm{e}^{\mathrm{i}\frac{13\pi}{12}}}$

16.2 f) ..... $\boxed{2\sin\left(\frac{\pi}{24}\right)\mathrm{e}^{-\mathrm{i}\frac{11\pi}{24}}}$

16.2 g) ..... $\boxed{\frac{\cos\left(\frac{\pi}{12}\right)}{\sin\left(\frac{\pi}{24}\right)}\mathrm{e}^{\frac{13\mathrm{i}\pi}{24}}}$

16.2 h) ..... $\boxed{2^{27}\cos^{27}\left(\frac{\pi}{12}\right)\mathrm{e}^{\mathrm{i}\frac{\pi}{4}}}$

16.3 a) ..... $\boxed{2\cos\left(\frac{\pi}{12}\right)\mathrm{e}^{\mathrm{i}\frac{5\pi}{12}}}$

16.3 b) ..... $\boxed{2\sin\left(\frac{\pi}{12}\right)\mathrm{e}^{\mathrm{i}\frac{11\pi}{12}}}$

16.4 a) ..... $\boxed{4\cos^3(x)-3\cos(x)}$

16.4 b) ..... $\boxed{4\cos^3(x)\sin(x)-4\cos(x)\sin^3(x)}$

16.5 a) ..... $\boxed{2\cos(2x)\cos(x)}$

16.5 b) ..... $\boxed{2\cos(4x)\sin(x)}$

16.5 c) ..... $\boxed{2\sin(x)\sin(2x)}$

16.5 d) ..... $\boxed{2\sin(4x)\cos(x)}$

16.6 a) ..... $\boxed{\frac{\sin\left(\frac{3x}{2}\right)\sin(2x)}{\sin\left(\frac{x}{2}\right)}}$

16.6 b) ..... $\boxed{\frac{\sin(8x)}{2\sin(x)}}$

16.6 c) ..... $\boxed{0}$

Fiche n° 17. Sommes et produits

Réponses

17.1 a) ..... $\boxed{n(n+2)}$

17.1 b) ..... $\boxed{\dfrac{7(n+1)(n+4)}{2}}$

17.1 c) ..... $\boxed{\dfrac{n(5n+1)}{2}}$

17.1 d) ..... $\boxed{\dfrac{(n-2)(n-7)}{6}}$

17.2 a) ..... $\boxed{\dfrac{n(n+1)(n+2)}{3}}$

17.2 b) ... $\boxed{n(n+1)(n^2+n+4)}$

17.2 c) ..... $\boxed{\dfrac{9}{2}(3^{n-2}-1)}$

17.2 d) ..... $\boxed{5^{n+1}\dfrac{1-\left(\frac{2}{5}\right)^{n+1}}{3}}$

17.2 e) ... $\boxed{\dfrac{7}{6}(7^n-1)+n(n+4)}$

17.2 f) ..... $\boxed{\dfrac{n+1}{2n}}$

17.3 a) ..... $\boxed{2^{q-p+1}}$

17.3 b) ..... $\boxed{3^{\frac{n(n+1)}{2}}}$

17.3 c) ..... $\boxed{5^n(n!)^{\frac{3}{2}}}$

17.3 d) ..... $\boxed{0}$

17.4 a) ..... $\boxed{\dfrac{n(n+1)}{2}}$

17.4 b) ..... $\boxed{0}$

17.4 c) ..... $\boxed{n2^{n+1}+2(1-2^n)}$

17.4 d) ..... $\boxed{\dfrac{n^2(n+1)^2}{4}}$

17.5 a) ..... $\boxed{(n+2)^3-2^3}$

17.5 b) ..... $\boxed{\ln(n+1)}$

17.5 c) ..... $\boxed{1-\dfrac{1}{(n+1)!}}$

17.5 d) ..... $\boxed{(n+1)!-1}$

17.6 a) ..... $\boxed{n+1}$

17.6 b) ..... $\boxed{1-4n^2}$

17.6 c) ..... $\boxed{\dfrac{1}{n}}$

17.6 d) ..... $\boxed{\dfrac{n+1}{2n}}$

17.7 a) ..... $\boxed{1-\dfrac{1}{n+1}}$

17.7 b) ..... $\boxed{\dfrac{1}{2}-\dfrac{1}{n+3}}$

17.8 a) ..... $\boxed{2n^2+n}$

17.8 b) ..... $\boxed{\dfrac{n(3n+1)}{2}}$

17.9 a) ..... $\boxed{\dfrac{n^2(n+1)}{2}}$

17.9 b) ..... $\boxed{\dfrac{n(n+3)}{4}}$

17.9 c) ..... $\boxed{\dfrac{n(n^2-1)}{2}}$

17.9 d) .. $\boxed{\dfrac{n(n+1)(7n^2+13n+4)}{12}}$

17.9 e) ..... $\boxed{\dfrac{n(n+1)}{2}\ln(n!)}$

17.9 f) ..... $\boxed{\dfrac{n(n+1)(4n-1)}{6}}$

Fiche n° 18. Coefficients binomiaux

Réponses

18.1 a) ..... $\boxed{10\ 100}$

18.1 b) ..... $\boxed{720}$

18.1 c) ..... $\boxed{\dfrac{1}{30}}$

18.1 d) ..... $\boxed{15}$

18.1 e) ..... $\boxed{56}$

18.1 f) ..... $\boxed{140}$

18.2 a) ..... $\boxed{\dfrac{9!}{5!}}$

18.2 b) ..... $\boxed{\binom{9}{4}}$

18.2 c) ..... $\boxed{2^n \times n!}$

18.2 d) ..... $\boxed{\dfrac{(2n+1)!}{2^n \times n!}}$

18.3 a) ..... $\boxed{\dfrac{n(n-1)}{2}}$

18.3 b) ..... $\boxed{\dfrac{n(n-1)(n-2)}{6}}$

18.3 c) ..... $\boxed{\dfrac{k+1}{n-k}}$

18.3 d) ..... $\boxed{(n+2)(n+1)}$

18.3 e) ..... $\boxed{\dfrac{1}{(n+1)!}}$

18.3 f) ..... $\boxed{\dfrac{n! \times (n-3)}{2^{2n+2}}}$

18.4 a) ..... $\boxed{\dfrac{(n+1)^3}{n \times (n+2)!}}$

18.4 b) ..... $\boxed{\dfrac{3(3n+2)(3n+1)}{a^3(n+1)^2}}$

18.5 a) ..... $\boxed{3^n}$

18.5 b) ..... $\boxed{0}$

18.5 c) ..... $\boxed{6^n}$

18.5 d) ..... $\boxed{12 \times 15^n}$

18.6 a) ..... $\boxed{2 \times \sum_{p=0}^{\lfloor \frac{n}{2} \rfloor} \binom{n}{2p}}$

18.6 b) ..... $\boxed{2^{n-1}}$

18.7 a) ..... $\boxed{2^n}$

18.7 b) ..... $\boxed{n2^{n-1}}$

18.7 c) ..... $\boxed{n(n+1)2^{n-2}}$

18.7 d) ..... $\boxed{\dfrac{2^{n+1}-1}{n+1}}$

18.8 a) ..... $\boxed{\binom{2n}{n}}$

18.8 b) ..... $\boxed{\sum_{k=0}^{n} \binom{n}{k}^2}$

18.8 c) ..... $\boxed{\binom{2n}{n}}$

Fiche n° 19. Manipulation des fonctions usuelles

Réponses

19.1 a) ..... $\boxed{\frac{\pi}{6}}$

19.1 b) ..... $\boxed{\frac{\pi}{4}}$

19.2 a) ..... $\boxed{\frac{\ln(2)}{\ln(3)}}$

19.2 b) ..... $\boxed{1}$

19.2 c) ..... $\boxed{-\frac{\ln(3)}{\ln(2)}}$

19.2 d) ..... $\boxed{\frac{\ln(4)}{\ln(20/3)}}$

19.3 a) ..... $\boxed{\frac{\ln\left(\frac{\sqrt{17}-1}{2}\right)}{\ln(2)}}$

19.3 b) ..... $\boxed{\left\{0; \frac{1}{2}\right\}}$

19.3 c) ..... $\boxed{1 - \frac{\ln(2)}{\ln(3)}}$

19.3 d) ..... $\boxed{\frac{\ln\left(\frac{\sqrt{5}-1}{2}\right)}{\ln(3)}}$

19.4 a) ... $\boxed{x \mapsto \ln(2) \times 2^x + 2x}$

19.4 b). $\boxed{x \mapsto \frac{15^x \ln(3/5) + 3^x \ln(3)}{(5^x+1)^2}}$

19.4 c)..... $\boxed{x \mapsto (\ln(x)+1)x^x}$

Fiche n° 20. Suites numériques

Réponses

20.1 a) ..... $\boxed{\frac{12}{5}}$

20.1 b) ..... $\boxed{8}$

20.1 c) ..... $\boxed{\frac{(2n+5)\cdot 2^{n+3}}{5}}$

20.1 d) ..... $\boxed{\frac{3(2n+1)\cdot 2^{3n+2}}{5}}$

20.2 a) ..... $\boxed{13}$

20.2 b) ..... $\boxed{29}$

20.3 a) ..... $\boxed{2^{\frac{1}{8}}}$

20.3 b) ..... $\boxed{2^{\frac{1}{64}}}$

20.4 a) ..... $\boxed{2}$

20.4 b) ..... $\boxed{2}$

20.5 a) ..... $\boxed{2n\ln(n)}$

20.5 b) ..... $\boxed{4n\ln(2n)}$

20.6 a) ..... $\boxed{21}$

20.6 b) ..... $\boxed{10\ 000}$

20.6 c) ..... $\boxed{2\ 001}$

20.6 d) ..... $\boxed{10\ 201}$

20.7 a) ..... $\boxed{\frac{17}{24}}$

20.7 b) ..... $\boxed{\frac{1}{24}}$

20.8 a) ..... $\boxed{\frac{3}{512}}$

20.8 b) ..... $\boxed{\frac{3069}{512}}$

20.8 c) ..... $\boxed{\frac{3}{1\ 024}}$

20.8 d) ..... $\boxed{\frac{6141}{1024}}$

20.9 a) ..... $\boxed{\frac{\pi\sqrt{5}}{5}}$

20.9 b) ..... $\boxed{\frac{11\sqrt{5}}{25}}$

20.10 a) ..... $\boxed{3^n+(-2)^n}$

20.10 b) ..... $\boxed{211}$

20.11 a) .. $\boxed{\frac{(1+\sqrt{2})^n-(1-\sqrt{2})^n}{2}}$

20.11 b) ..... $\boxed{2\sqrt{2}}$

20.12 a) ..... $\boxed{257}$

20.12 b) ..... $\boxed{65\ 537}$

20.12 c) ..... $\boxed{F_n}$

20.12 d) ..... $\boxed{F_{n+1}-2}$

20.12 e) ..... $\boxed{F_{n+1}+2^{2^n+1}}$

20.12 f) ..... $\boxed{F_{n+2}}$

Fiche n° 21. Développements limités

Réponses

21.1 a) $\boxed{3x - x^2 + \frac{x^3}{2} - \frac{x^4}{2} + \underset{x\to 0}{\mathrm{o}}(x^4)}$

21.1 b) $\boxed{x - \frac{3}{2}x^2 + \frac{11}{6}x^3 - \frac{25}{12}x^4 + \underset{x\to 0}{\mathrm{o}}(x^4)}$

21.1 c) $\boxed{x + x^2 + \frac{x^3}{3} - \frac{x^5}{30} - \frac{x^6}{90} + \underset{x\to 0}{\mathrm{o}}(x^6)}$

21.2 a) $\boxed{\mathrm{e} - \frac{\mathrm{e}x}{2} + \frac{11\mathrm{e}x^2}{24} - \frac{7\mathrm{e}x^3}{16} + \frac{2447\mathrm{e}x^4}{5760} + \underset{x\to 0}{\mathrm{O}}(x^5)}$

21.2 b) $\boxed{1 - \frac{1}{4}x^2 - \frac{1}{96}x^4 - \frac{19}{5760}x^6 + \underset{x\to 0}{\mathrm{O}}(x^7)}$

21.2 c) $\boxed{1 - x + \frac{3}{2}(x-1)^2 + \underset{x\to 1}{\mathrm{o}}\big((x-1)^2\big)}$

21.3 a) $\boxed{1 - \frac{3\pi^2}{8}\left(x - \frac{\pi}{3}\right)^2 + \underset{x\to \frac{\pi}{3}}{\mathrm{o}}\left(\left(x - \frac{\pi}{3}\right)^2\right)}$

21.3 b) $\boxed{1 + 2\left(x - \frac{\pi}{4}\right) + 2\left(x - \frac{\pi}{4}\right)^2 + \frac{8}{3}\left(x - \frac{\pi}{4}\right)^3 + \underset{x\to \frac{\pi}{4}}{\mathrm{O}}\left(\left(x - \frac{\pi}{4}\right)^4\right)}$

21.3 c) $\boxed{-1 + \frac{\pi^2}{8}\left(x - \frac{\pi}{2}\right)^4 - \frac{\pi^2}{48}\left(x - \frac{\pi}{2}\right)^6 + \underset{x\to \frac{\pi}{2}}{\mathrm{o}}\left(\left(x - \frac{\pi}{2}\right)^7\right)}$

21.4 a) $\boxed{-\frac{1}{2x} + \frac{1}{12} - \frac{1}{720}x^2 + \underset{x\to 0}{\mathrm{o}}(x^2)}$

21.4 b) $\boxed{\frac{1}{x^2} - \frac{1}{x^3} + \frac{5}{6x^4} - \frac{5}{6x^5} + \underset{x\to +\infty}{\mathrm{O}}\left(\frac{1}{x^6}\right)}$

21.4 c) $\boxed{-\ln(x) + 1 - \frac{1}{2x} + \frac{1}{3x^2} - \frac{1}{4x^3} + \underset{x\to +\infty}{\mathrm{o}}\left(\frac{1}{x^3}\right)}$

21.4 d) $\boxed{\mathrm{e}^{-\frac{1}{2}}\left(\mathrm{e}^x + \frac{\mathrm{e}^x}{3x} - \frac{7\mathrm{e}^x}{36x^2}\right) + \underset{x\to +\infty}{\mathrm{o}}\left(\frac{\mathrm{e}^x}{x^2}\right)}$

Fiche n° 22. Calcul matriciel

Réponses

22.1 a) $\begin{pmatrix} 1 & -3 & -1 \\ 3 & 3 & 4 \\ 9 & -7 & 3 \end{pmatrix}$

22.1 b) $\begin{pmatrix} -2 & -6 & -5 \\ 15 & -1 & 11 \\ 18 & -26 & -1 \end{pmatrix}$

22.1 c) 17 (matrice 1×1)

22.1 d) $\begin{pmatrix} 1 & 7 & -2 \\ 2 & 14 & -4 \\ -1 & -7 & 2 \end{pmatrix}$

22.1 e) $\begin{pmatrix} -1 \\ 3 \\ -1 \end{pmatrix}$

22.1 f) $\begin{pmatrix} -5 & 15 & 3 \end{pmatrix}$

22.1 g) $\begin{pmatrix} 5 & 4 \\ 4 & 5 \end{pmatrix}$

22.1 h) $\begin{pmatrix} 5 & 3 & -1 & 1 \\ 4 & 3 & 1 & 2 \end{pmatrix}$

22.1 i) $\begin{pmatrix} 1 & 7 & -2 \\ 7 & 49 & -14 \\ -2 & -14 & 4 \end{pmatrix}$

22.2 a) $\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$

22.2 b) $\begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix}$

22.2 c) $\begin{pmatrix} 1 & k \\ 0 & 1 \end{pmatrix}$

22.2 d) $\begin{pmatrix} 4 & 5 \\ 0 & 9 \end{pmatrix}$

22.2 e) $\begin{pmatrix} 8 & 19 \\ 0 & 27 \end{pmatrix}$

22.2 f) $\begin{pmatrix} 2^k & 3^k - 2^k \\ 0 & 3^k \end{pmatrix}$

22.2 g) $\begin{pmatrix} \cos(2\theta) & -\sin(2\theta) \\ \sin(2\theta) & \cos(2\theta) \end{pmatrix}$

22.2 h) $\begin{pmatrix} \cos(3\theta) & -\sin(3\theta) \\ \sin(3\theta) & \cos(3\theta) \end{pmatrix}$

22.2 i) $\begin{pmatrix} \cos(k\theta) & -\sin(k\theta) \\ \sin(k\theta) & \cos(k\theta) \end{pmatrix}$

22.2 j) $\begin{pmatrix} n & \cdots & n \\ \vdots & (n) & \vdots \\ n & \cdots & n \end{pmatrix}$

22.2 k) $\begin{pmatrix} n^2 & \cdots & n^2 \\ \vdots & (n^2) & \vdots \\ n^2 & \cdots & n^2 \end{pmatrix}$

22.2 l) $n^{k-1}D$

22.3 a) $2 \times 3^{j-i} \times 5^{i-1}$

22.3 b) $2^{i+1}3^{j-i}(2^n - 1)$

22.3 c) $2 \times 3^{i+j}\left(1 - \left(\frac{2}{3}\right)^n\right)$

22.3 d) $\binom{i-1}{j} + \binom{i-1}{j-2}$

22.4 a) $2^{i-j}\binom{i-1}{j-1}$

22.4 b) $(1-\delta_{i,1})(\delta_{i-1,j+1} + \delta_{i,j})$
$+(1-\delta_{i,n})(\delta_{i,j} + \delta_{i+1,j-1})$

22.5 a) $\frac{1}{2(\pi - e)}\begin{pmatrix} 2 & -e \\ -2 & \pi \end{pmatrix}$

22.5 b) $\frac{1}{3}\begin{pmatrix} 1 & -1-2\mathrm{i} \\ 1 & -1+\mathrm{i} \end{pmatrix}$

22.5 c) $\frac{1}{2}\begin{pmatrix} 5 & 2 & -1 \\ 3 & 2 & -1 \\ -6 & -2 & 2 \end{pmatrix}$

22.5 d) $\frac{1}{4\pi}\begin{pmatrix} 0 & 4 & 0 \\ 0 & -2 & -2 \\ 2 & -1 & 1 \end{pmatrix}$

22.5 e) $\frac{1}{8}\begin{pmatrix} 8 & 4 & -2 \\ -16 & -6 & 7 \\ 0 & -2 & 1 \end{pmatrix}$

22.5 f) $\frac{1}{6}\begin{pmatrix} -2 & 2 & 2 \\ 1 & -1 & 2 \\ 4 & 2 & -4 \end{pmatrix}$

22.5 g) ..... $\boxed{\frac{1}{2}\begin{pmatrix} 4 & -2 & 2 & 0 \\ 8 & -6 & 4 & 2 \\ -7 & 5 & -3 & -1 \\ -5 & 3 & -1 & -1 \end{pmatrix}}$

22.5 h) ..... $\boxed{\text{Non inversible !}}$

22.5 i) ..... $\boxed{\frac{1}{2}\begin{pmatrix} 0 & -1 & 0 & -1 \\ 1 & 1 & 0 & 0 \\ -1 & 0 & -1 & 0 \\ 0 & 0 & 1 & -1 \end{pmatrix}}$

22.6 a) ..... $\boxed{\lambda \neq 1}$

22.6 b) ..... $\boxed{\frac{1}{1-\lambda}\begin{pmatrix} -4 & -1 & 3 \\ 2\lambda+2 & \lambda & -2\lambda-1 \\ \lambda-1 & 0 & 1-\lambda \end{pmatrix}}$

22.6 c) ..... $\boxed{\lambda \neq 1}$

22.6 d) ..... $\boxed{\frac{1}{1-\lambda}\begin{pmatrix} -1-\lambda+\lambda^2 & 1-\lambda & 2-\lambda \\ 1 & 0 & -1 \\ 1-\lambda^2 & \lambda-1 & \lambda-1 \end{pmatrix}}$

Fiche n° 23. Algèbre linéaire

Réponses

23.1 a) $\boxed{(3,-1)}$

23.1 b) $\boxed{(-1,3)}$

23.1 c) $\boxed{(9/11, 2/11)}$

23.1 d) $\boxed{(-2, 4/5, 11/5)}$

23.1 e) $\boxed{(-1, 1/2, 1/2)}$

23.1 f) $\boxed{(0,2,4,1)}$

23.1 g) $\boxed{(1/2, -\sqrt{3}/2)}$

23.2 a) $\boxed{2}$

23.2 b) $\boxed{1}$

23.2 c) $\boxed{1}$

23.2 d) $\boxed{2}$

23.2 e) $\boxed{2}$

23.2 f) $\boxed{1}$

23.3 a) $\boxed{2}$

23.3 b) $\boxed{2}$

23.3 c) $\boxed{3}$

23.3 d) $\boxed{4}$

23.4 a) $\boxed{\begin{pmatrix} 1 & 1 \\ 3 & -5 \end{pmatrix}}$

23.4 b) $\boxed{\begin{pmatrix} -5 & 3 \\ 1 & 1 \end{pmatrix}}$

23.4 c) $\boxed{\frac{1}{2}\begin{pmatrix} -19 & -43 \\ 9 & 21 \end{pmatrix}}$

23.4 d) $\boxed{\begin{pmatrix} 1 & 0 & 1 \\ 3 & -1 & 1 \\ 0 & 1 & 1 \end{pmatrix}}$

23.4 e) $\boxed{\begin{pmatrix} 1 & 2 & 4 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{pmatrix}}$

23.5 a) $\boxed{\begin{pmatrix} -1 & -1 & 1 \\ 4 & 15 & 0 \end{pmatrix}}$

23.5 b) $\boxed{\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}}$

Fiche n° 24. Équations différentielles

Réponses

24.1 a) $\boxed{x \mapsto 56\mathrm{e}^{12x}}$

24.1 b) $\boxed{x \mapsto 6\mathrm{e}^{x} - 1}$

24.1 c) $\boxed{x \mapsto \dfrac{8\mathrm{e}^{3x} - 5}{3}}$

24.1 d) $\boxed{x \mapsto 9\mathrm{e}^{2x} - 6}$

24.2 a) $\boxed{x \mapsto \mathrm{e}^{(6-x)/5}}$

24.2 b) $\boxed{x \mapsto 1 - 2\mathrm{e}^{-2x/7+2}}$

24.2 c) $\boxed{x \mapsto \left(\dfrac{6}{\sqrt{5}} + \pi\right)\mathrm{e}^{\sqrt{5}x} - \dfrac{6}{\sqrt{5}}}$

24.2 d) $\boxed{x \mapsto \left(12 + \dfrac{2\mathrm{e}}{\pi}\right)\mathrm{e}^{\pi x - \pi^2} - \dfrac{2\mathrm{e}}{\pi}}$

24.3 a) $\boxed{x \mapsto \mathrm{e}^{2x}}$

24.3 b) $\boxed{x \mapsto \mathrm{e}^{x}}$

24.3 c) $\boxed{x \mapsto 2\mathrm{e}^{2x} - \mathrm{e}^{x}}$

24.3 d) $\boxed{x \mapsto (2 - 3\mathrm{i})\mathrm{e}^{x} + (3\mathrm{i} - 1)\mathrm{e}^{2x}}$

24.4 a) $\boxed{x \mapsto \mathrm{e}^{x}}$

24.4 b) $\boxed{x \mapsto 7\mathrm{e}^{-x} - 5\mathrm{e}^{-2x}}$

24.4 c) $\boxed{x \mapsto \dfrac{4}{3}\mathrm{e}^{x} - \dfrac{1}{3}\mathrm{e}^{-2x}}$

24.4 d) $\boxed{x \mapsto (2 - x)\mathrm{e}^{x}}$

24.4 e) $\boxed{x \mapsto (2 - x)\mathrm{e}^{2-2x}}$

24.5 a) $\boxed{x \mapsto \cos x + 2\sin x}$

24.5 b) $\boxed{x \mapsto \mathrm{e}^{-x/2}\left(\cos\dfrac{\sqrt{3}x}{2} - \dfrac{1}{\sqrt{3}}\sin\dfrac{\sqrt{3}x}{2}\right)}$

24.5 c) $\boxed{x \mapsto \mathrm{e}^{-x}\sin(x)}$

24.5 d) $\boxed{x \mapsto \mathrm{e}^{x}\left(\dfrac{-1+\mathrm{i}}{2}\mathrm{e}^{2\mathrm{i}x} + \dfrac{1+\mathrm{i}}{2}\mathrm{e}^{-2\mathrm{i}x}\right)}$

Fiche n° 25. Fonctions de deux variables

Réponses

25.1 a) $\boxed{]0,+\infty[\times[0,+\infty[}$

25.1 b) $\boxed{\{(x,y)\in\mathbb{R}^2,\ y\geqslant 0\}\backslash\{(0,0)\}}$

25.1 c) $\boxed{\varnothing}$

25.2 a) $\boxed{\dfrac{\partial f}{\partial x}(x,y)=2x+y \text{ et } \dfrac{\partial f}{\partial y}(x,y)=5y^4+x}$

25.2 b) $\boxed{\dfrac{\partial f}{\partial x}(x,y)=2y\cos(2xy-y) \text{ et } \dfrac{\partial f}{\partial y}(x,y)=(2x-1)\cos(2xy-y)}$

25.2 c) $\boxed{\dfrac{\partial f}{\partial x}(x,y)=(2xy,2x) \text{ et } \dfrac{\partial f}{\partial y}(x,y)=(x^2,-2y)}$

25.2 d) $\boxed{\dfrac{\partial f}{\partial x}(x,y)=\dfrac{2}{1+(2x+y)^2} \text{ et } \dfrac{\partial f}{\partial y}(x,y)=\dfrac{1}{1+(2x+y)^2}}$

25.3 a) $\boxed{\dfrac{\partial f}{\partial x}(x,y)=-\sin(x-y) \text{ et } \dfrac{\partial f}{\partial y}(x,y)=\sin(x-y)}$

25.3 b) $\boxed{\dfrac{\partial f}{\partial x}(x,y)=\cos(\mathrm{e}^{xy})-xy\sin(\mathrm{e}^{xy})\,\mathrm{e}^{xy} \text{ et } \dfrac{\partial f}{\partial y}(x,y)=-x^2\sin(\mathrm{e}^{xy})\,\mathrm{e}^{xy}}$

25.3 c) $\boxed{\dfrac{\partial f}{\partial x}(x,y)=y\,x^{y-1} \text{ et } \dfrac{\partial f}{\partial y}(x,y)=x^y\ln x}$

25.3 d) $\boxed{\dfrac{\partial f}{\partial x}(x,y)=\begin{cases}\dfrac{y^2(y^2-x^2)}{(x^2+y^2)^2} & \text{si } (x,y)\neq(0,0)\\ 0 & \text{sinon}\end{cases} \text{ et } \dfrac{\partial f}{\partial y}(x,y)=\begin{cases}\dfrac{2x^3y}{(x^2+y^2)^2} & \text{si } (x,y)\neq(0,0)\\ 0 & \text{sinon}\end{cases}}$

25.4 a) $\boxed{\sin(2t)}$

25.4 b) $\boxed{\dfrac{2\mathrm{e}^{4t}+\mathrm{e}^{-2t}}{\sqrt{\mathrm{e}^{4t}-\mathrm{e}^{-2t}}}}$

25.4 c) $\boxed{-72\cos(4t)-46\sin(4t)}$

25.5 a) $\boxed{\dfrac{\partial(f\circ\varphi)}{\partial u}(u,v)=\dfrac{1}{2}\dfrac{\partial f}{\partial x}\left(\dfrac{u+v}{2},\dfrac{v-u}{2c}\right)-\dfrac{1}{2c}\dfrac{\partial f}{\partial y}\left(\dfrac{u+v}{2},\dfrac{v-u}{2c}\right)}$

25.5 a) $\boxed{\dfrac{\partial(f\circ\varphi)}{\partial v}(u,v)=\dfrac{1}{2}\dfrac{\partial f}{\partial x}\left(\dfrac{u+v}{2},\dfrac{v-u}{2c}\right)+\dfrac{1}{2c}\dfrac{\partial f}{\partial y}\left(\dfrac{u+v}{2},\dfrac{v-u}{2c}\right)}$

25.5 b) $\boxed{\dfrac{\partial(f\circ\varphi)}{\partial r}(r,\theta)=\cos\theta\dfrac{\partial f}{\partial x}(r\cos\theta,r\sin\theta)+\sin\theta\dfrac{\partial f}{\partial y}(r\cos\theta,r\sin\theta)}$

25.5 b) $\boxed{\dfrac{\partial(f\circ\varphi)}{\partial\theta}(r,\theta)=-r\sin\theta\dfrac{\partial f}{\partial x}(r\cos\theta,r\sin\theta)+r\cos\theta\dfrac{\partial f}{\partial y}(r\cos\theta,r\sin\theta)}$