SIGEST

The SIGEST article in this issue is "Are Adaptive Galerkin Schemes Dissipative?" by Rodrigo M. Pereira, Natacha Nguyen van yen, Kai Schneider, and Marie Farge.

"Although this may seem a paradox, all exact science is dominated by the idea of approximation." With this quote from Bertrand Russell from 1931 commences this issue's SIGEST article. Indeed, approximation is at the core of mathematics associated to studying partial differential equations (PDEs) with the idea of approximating the solution to the continuous equation with a finite number of modes. The finite element method for PDEs is a prime exemplar of such an approximation, and much research has been dedicated to getting this approximation as accurate and computationally efficient as possible. In this context, adaptive finite element methods and especially Galerkin methods are often the method of choice. Here, typically, when used for solving evolutionary PDEs the number of modes in the Galerkin scheme is fixed over time. In this article, the authors consider adaptive Galerkin schemes in which the number of modes can change over time, and they introduce a mathematical framework for studying evolutionary PDEs discretized with these schemes. In particular, they show that the associated projection operators, i.e., the operators that project the continuous solution onto the finite-dimensional finite element spaces, are discontinuous and introduce energy dissipation. That this is a significant result is demonstrated by studying adaptive Galerkin schemes for the time evolution of the inviscid Burgers equation in 1D and the incompressible Euler equations in 2D and 3D. They show that adaptive wavelet schemes regularize the solution of the Galerkin truncated equations and yield convergence towards the exact dissipative solution for the inviscid Burgers equation. Also for the Euler equations this regularizing effect can be numerically observed though no exact reference solutions are available in this case. This motivates, in particular, adaptive wavelet Galerkin schemes for nonlinear hyperbolic conservation laws and leave their systematic study for this class of PDEs for an interesting future work.

For the SIGEST article the authors have expanded their original *Multiscale Modeling & Simulation* article by providing a more comprehensive discussion on adaptive Galerkin methods fit for a general mathematical audience. They have also added a new section on continuous wavelet analysis of the inviscid Burgers equation, analyzing its time evolution, and added an illustration for the development of thermal resonances in wavelet space.

Overall, adaptive Galerkin methods and their mathematical properties will be of interest to a wide range of applied mathematicians who study PDE models, and also to applied analysts and numerical analysts who wish to simulate PDEs numerically.

The Editors