Research internship proposal: Revisiting *"A semantic measure of the execution time in Linear Logic"* using Taylor expansion

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In their 2011 paper *A semantic measure of the execution time in Linear Logic*, de Carvalho, Pagani and Tortora de Falco explain how to the relational semantics of Linear Logic proof nets allows to obtain exact bounds on the number of cut elimination steps leading to a normal form.

This might seem surprising, since the semantics is invariant under cut elimination: if M reduces to M' by eliminating a cut, then M and M' have the same semantics. So the semantics of M gives no information on the number of cut elimination steps that one can perform starting from M.

Nonetheless, the authors showed that, starting from two proof nets M and N in normal form, together with a choice of a conclusion c of M and a conclusion d of N, the relational semantics of M and N allows to compute the maximum number of cut elimination steps starting from the net obtained by putting M and N side by side, and by cutting c with d (following two possible strategies: top level reduction and stratified reduction).

We propose to revisit these results by leveraging a powerful and relatively new tool for the study of proof nets: the Taylor expansion of proof nets into resource nets, as introduced by Ehrhard and Regnier. This operation associates to each proof net a set of multilinear, strongly normalizing approximants, in a way that is compatible with cut elimination.

Indeed, Taylor expansion is strongly related with relational semantics: for instance, it has already been used by de Carvalho to prove the injectivity of the relational semantics on cut free proof nets. In a sense the elements of the Taylor expansion of a net represent *experiments* on this net (*i.e.* particular annotations of the links of this net with elements of the relational model), while the relational semantics only collects the results of these experiments (the annotations of the conclusions). And this notion of experiment was instrumental in the results of de Carvalho, Pagani and Tortora de Falco. Recasting their work in the setting of Taylor expansion, will hopefully shed new light on those important results.

Practical information

Location: Institut de Mathématiques de Marseille, Luminy site (South of Marseille city centre, near the calanques)

Team: Logique de la Programmation

Expected abilities: At least some knowledge about linear logic (even if only as a proof system); and possibly some experience with denotational semantics. Discovering Taylor expansion of proof nets be the first phase of the internship.

Funding: If necessary, some funding is available from the lab (subject to a local competition).