Strong Normalizability as a Finiteness Structure via the Taylor Expansion of λ-terms

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Everything is in the title

$$(\forall M \in \Lambda_+)\ M \in \mathsf{SN} \iff \mathcal{T}(M) \in \mathfrak{F}$$

We characterize the strong normalizability (SN)
of (non-deterministic) λ-terms (Λ_+)
as a finiteness structure ($\mathfrak{F}$)
via Taylor expansion ($\mathcal{T}$).

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A prime aged idea (Girard, '80s, before LL)

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Finitness spaces (Ehrhard, early 2000's)

Reformulate q.s. in a linear logic setting using standard algebra:

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  $[\![A]\!] \subseteq \mathbf{k}^{|A|}$ + some additional structure
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Differentiation of λ-terms (Ehrhard-Regnier 2003-2004)

So we can *differentiate* λ-terms, and compute their Taylor expansion!
And one can mimick that in the syntax:

- differential λ-calculus
- a finitary fragment: resource λ-calculus
  = the target of Taylor expansion

Resource λ-calculus

Resource terms

$$\begin{array}{rcll} \Delta & \ni & s, t, \ldots & ::= \quad x \mid \lambda x.s \mid \langle s \rangle\ \bar{t} \\ \Delta^! & \ni & \bar{s}, \bar{t}, \ldots & ::= \quad [s_1, \ldots, s_n] \end{array}$$

Meaning: $\langle s \rangle\ [s_1, \ldots, s_n] = (Ds)_0 \cdot (s_1, \ldots, s_n)$

Resource reduction

$$\langle \lambda x.s \rangle\ \bar{t} \rightarrow_\rho \partial_x s \cdot \bar{t} \quad \text{(anywhere)}$$

$$\partial_x s \cdot \bar{t} = \begin{cases} \sum_{f \in \mathfrak{S}_n} s\left[t_{f(1)}, \ldots, t_{f(n)}/x_1, \ldots, x_n\right] & \text{if } \deg_x(s) = \#\bar{t} = n \\ 0 & \text{otherwise} \end{cases}$$

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linearity: $\lambda x.0 = 0$, $\langle s\rangle\ [t_1 + t_2, u] = \langle s\rangle\ [t_1, u] + \langle s\rangle\ [t_2, u]$, …

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- Resource reduction preserves free variables, is size-decreasing, strongly confluent and normalizing.

Taylor expansion of λ-terms

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Taylor expansion: $\vec{\mathcal{T}}\,(M) \in \mathbf{Q}^{+\Delta}$

$$\vec{\mathcal{T}}\,((M)\,N) = \sum_{n\in\mathbf{N}} \frac{1}{n!}\left\langle \vec{\mathcal{T}}\,(M)\right\rangle \vec{\mathcal{T}}\,(N)^n$$

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$$\vec{\mathcal{T}}(x) = x \qquad \vec{\mathcal{T}}(\lambda x.M) = \lambda x.\vec{\mathcal{T}}(M)$$

Theorem (Ehrhard-Regnier, CiE 2006)

If $M \in \Lambda$, then $\vec{\mathcal{T}}(M)$ normalizes to $\vec{\mathcal{T}}(\mathsf{BT}(M))$.

Moral

In the ordinary λ-calculus $\mathsf{BT}(M) \simeq \mathsf{NF}(\vec{\mathcal{T}}(M))$.

Normalizing Taylor expansions

But how can $\vec{\mathcal{T}}(M)$ even normalize?

We want to set

$$\mathsf{NF}\left(\vec{\mathcal{T}}(M)\right) = \sum_{s \in \Delta} \vec{\mathcal{T}}(M)_s . \mathsf{NF}(s)$$

$\rightsquigarrow$ infinite sums (and in general we might consider all kinds of coefficients)
$\rightsquigarrow$ convergence?

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Theorem (Ehrhard-Regnier 2004, published in TCS in 2008)

Write $\mathcal{T}(M) = \left|\vec{\mathcal{T}}(M)\right|$. *Then for all* $t \in \Delta$, *there is at most one* $s \in \mathcal{T}(M)$ *such that* $\mathsf{NF}(s)_t \neq 0$.

Proof.

λ-terms are uniform: their finitary approximants are pairwise coherent. $\square$

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This fails in general

$\mathsf{NF}\left(\sum_{n \in \mathbf{N}} \langle \lambda x.x \rangle^n [y]\right) = ?$ $\qquad$ $\langle \lambda x.x \rangle^n [y] = \langle \lambda x.x \rangle \left[\langle \lambda x.x \rangle \left[\cdots [y] \cdots\right]\right]$

A minimalistic non-uniform calculus

$$\Lambda_+ \ni M, N, \ldots ::= x \mid \lambda x.M \mid (M)\,N \mid M + N$$

$$(\lambda x.M)\,N \to_\beta M\,[N/x]$$

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Taylor expansion in a non uniform setting

$$\vec{\mathcal{T}}\,(M + N) = \vec{\mathcal{T}}\,(M) + \vec{\mathcal{T}}\,(N)$$

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$$\vec{\mathcal{T}}\,(M + N) = \vec{\mathcal{T}}\,(M) + \vec{\mathcal{T}}\,(N)$$

Then $\mathsf{NF}\left(\vec{\mathcal{T}}\,(\infty_M)\right) = ?$

Finiteness structures to the rescue

The main artifact of Ehrhard's finiteness spaces:

Definition

- If $a, a' \subseteq A$, write $a \perp a'$ iff $a \cap a'$ is finite.
- If $\mathfrak{S} \subseteq \mathfrak{P}(A)$, let $\mathfrak{S}^\perp := \{a' \subseteq A;\ \forall a \in \mathfrak{S},\ a \perp a'\}$.
- A finiteness structure is any $\mathfrak{F} = \mathfrak{S}^\perp$.

When is $\vec{\mathcal{T}}(M)$ normalizable?

- Write $s \geq t$ if $s \to_\rho^* t + \cdots$.
- Let $\uparrow t = \{s \in \Delta;\ s \geq t\}$.
- $\vec{\mathcal{T}}(M)$ is normalizable iff for all normal $t \in \Delta$, $\mathcal{T}(M) \perp \uparrow t$.
- $\{\uparrow t\ ;\ t \text{ normal } \in \Delta\}^\perp$ is the finiteness structure of (supports of) normalizable vectors.

Typed terms have a finitary Taylor expansion

Let system F_+ be system F plus $\dfrac{\Gamma \vdash M : A \qquad \Gamma \vdash N : A}{\Gamma \vdash M + N : A}$.

Theorem (Ehrhard, LICS 2010)

If $M \in \Lambda_+$ is typable in system F_+, then $\mathcal{T}(M) \in \{\uparrow t \,;\, t \in \Delta\}^\perp$.

Proof.

Manage sets of resource terms as if they were λ-terms, and follow the usual reducibility technique, associating a finiteness structure $\mathfrak{Fin}(A) \subseteq \{\uparrow t \,;\, t \in \Delta\}^\perp$ with each type A.

$\square$

Our results

- Typability in F can be relaxed to strong normalizability.
- Then the implication

$$M \in \mathsf{SN} \Rightarrow \mathcal{T}(M) \in \{\uparrow t \ ; \ t \in \Delta\}^{\perp}$$

  can be reversed...
- provided the finiteness $\{\uparrow t \ ; \ t \in \Delta\}^{\perp}$ is refined to a tighter one.

$M \in \mathsf{SN} \Rightarrow \mathcal{T}(M) \in \{\uparrow t \,;\, t \in \Delta\}^{\perp}$

In the ordinary λ-calculus:

- SN = typability in system D (simple types + $\cap$)
- "any" proof by reducibility for simple types is valid for D

So we:

- introduce a system D_+ of intersection types for non uniform terms (this needs some care)
- prove that $M \in \mathsf{SN}$ implies $\Gamma \vdash M : A$ in D_+
- adapt Ehrhard's proof to D_+

$\mathcal{T}(M) \in \{\uparrow t\ ;\ t \in \Delta\}^{\perp} \Rightarrow M \in \mathsf{SN}$

Finiteness prevents loops…

Consider $\delta_n = \lambda x. \langle x \rangle\ [x^n]$; then for all $n \in \mathbf{N}$, $\mathcal{T}(\Omega) \ni \langle \delta_n \rangle\ [\delta_0, \delta_0, \delta_1 \ldots, \delta_{n-1}] \geq \langle \delta_0 \rangle\ [] \to_\rho 0$. Hence $\mathcal{T}(\Omega) \notin \{\uparrow t\ ;\ t \in \Delta\}^{\perp}$.

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. . . but not divergence

Let $\Delta_3 := \lambda x. (x)\, x\, x$ and $\Omega_3 := (\Delta_3)\, \Delta_3$, then $\mathcal{T}(\Omega_3) \perp \uparrow s$ for all s.

$\mathcal{T}(M) \in \{\uparrow t \; ; \; t \in \Delta\}^{\perp} \not\Rightarrow M \in \mathsf{SN}$

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However, writing $\ell(M)$ for the only linear term $\in \mathcal{T}(M)$:

$$\mathcal{T}(\Omega_3) \not\subseteq \bigcup_{\Omega_3 \to_{\beta}^{*} M} \uparrow \ell(M)$$

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Fix: add more tests

- Consider a structure $\mathfrak{S} \subseteq \mathfrak{P}(\Delta)$ and let $\mathfrak{F}_{\mathfrak{S}} = \{\uparrow a \,;\, a \in \mathfrak{S}\}^{\perp}$ with $\uparrow a = \bigcup_{s \in a} \uparrow s$.
- Idea: $\mathfrak{S}$ is a set of *tests*: M passes the test $a \in \mathfrak{S}$ if $\mathcal{T}(M) \perp \uparrow a$.
- Ehrhard's finiteness is $\mathfrak{F}_{\mathfrak{P}_f(\Delta)}$: we need to consider infinite tests.
- Of course, not all $\mathfrak{S}$ are acceptable, otherwise we reject too many terms (consider $\mathfrak{S} = \mathfrak{P}(\Delta)$).

Glueing everything together

- We can adapt the reducibility proof and show that $M \in \mathsf{SN} \Rightarrow \mathcal{T}(M) \in \mathfrak{F}_{\mathfrak{S}}$ provided $\mathfrak{S}$ satisfies:
  - for all $n \in \mathbf{N}$, for all $a \in \mathfrak{S}$, $\{s \in a;\ \mathrm{height}(s) = n\}$ is finite.
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Example

$\mathfrak{B} = \{a \subseteq \Delta;\ \#(a) \text{ is bounded}\}$ where $\#(a) = \{\#(s);\ s \in a\}$ and $\#(s)$ is the maximum size of a bag of arguments in s.

Theorem (Pagani-Tasson-V.)

The following are equivalent:

- $M \in \mathsf{SN}$;
- $\mathcal{T}(M) \in \mathfrak{F}_{\mathfrak{B}}$.

Moreover, in this case, $\vec{\mathcal{T}}(M)$ is normalizable.

Conclusion

We are happy.

We have established a nice and novel characterization of SN.

~~Conclusion~~ No, wait!

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Are we?

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Yes we are: plenty of future work!

- Our machinery is modular enough that it can be adapted to weak- and head-normalizability *(WIP with Pagani and Tasson)*.
- That $\vec{\mathcal{T}}(\mathsf{NF}(M)) = \mathsf{NF}(\vec{\mathcal{T}}(M))$ follows from the fact that we can track β-reduction through Taylor expansion *(WIP)*.
- Towards a semantically founded notion of Böhm trees for various non uniform settings (quantitative non-determinism, probabilistic stuff, *etc.*).

The end

Thanks for your attention.

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Questions?