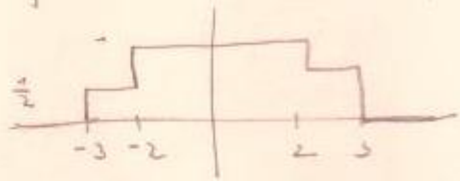
1) a) $\frac{1}{2}\left(\delta(\nu-\frac{1}{2})+\delta(\nu+\frac{1}{2})\right)$ 0,5

b) $H(\nu) = \frac{1}{2}\left(\delta(\nu-\frac{1}{2})+\delta(\nu+\frac{1}{2})\right) * \Pi_5(\nu)$

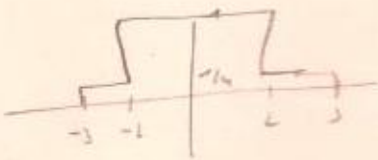
$= \frac{1}{2}\left(\Pi_5(\nu-\frac{1}{2})+\Pi_5(\nu+\frac{1}{2})\right)$ 1,5

c) 

1

d) 

1,5

2) a) 

1

b) $E = \frac{1}{4} + 4\times 1 + \frac{1}{4} = 4+\frac{1}{2} = \frac{9}{2}$ 1

3) a) Det a PMF 0,5

b) $T=\frac{2}{5}$ $f=\frac{5}{2}$ 1

c) $X(\nu) = \delta + \frac{1}{2}\delta_{5/2} - \frac{1}{2}\delta_{-5/2} + \delta_5 + \delta_{-5} - \frac{\delta_{10}}{2} - \frac{\delta_{-10}}{2} + \frac{3}{2j}\left(\delta_{10}-\delta_{-10}\right)$ 1

d) $Y(\nu) = H(\nu)X(\nu)$

$= H(0)\delta + \frac{H(5/2)}{2}\left(\delta_{5/2}+\delta_{-5/2}\right) + 0$

$= \delta + \frac{1}{4}\left(\delta_{5/2}+\delta_{-5/2}\right)$ 1,5

e) $y(t) = 1 + \frac{1}{2}\cos 5\pi t$ 1

f) $\langle x^2\rangle = 1+\frac{1}{2}(15) = 17/2$

$\langle y^2\rangle = 1+\frac{1}{8} = \frac{9}{8}$ 1

Ex 2

1) $\partial_x = 2x - \frac{y^2}{4x^2}$

$\partial_y = \frac{y}{2x} - 1$ 2

2 $\partial^2_{xx} = 2 + \frac{y^2}{2x^3}$

$\partial^2_{yy} = \frac{1}{2x}$

$\partial^2_{xy} = -\frac{y}{2x^2}$ 2

2) $\begin{cases} 2x - \frac{y^2}{4x^2} = 0 \\ \frac{y}{2x} = 1 \end{cases} \Leftrightarrow \begin{cases} y = 2x \\ 2x - 1 = 0 \end{cases}$

$\Leftrightarrow \begin{cases} x = \frac{1}{2} \\ y = 1 \end{cases}$

3) $\Delta = \frac{y^2}{4x^4} - \frac{1}{2x}\left(2 + \frac{y^2}{2x^3}\right)$

$= -\frac{1}{x} + \frac{y^2-y^2}{4x^4} = -2 < 0$

$\partial^2_{yy} = \frac{1}{2} > 0 \Rightarrow$ Min local.

1 4) a) $\frac{(y-2x)^2}{4x} + \left(x-\frac{1}{2}\right)^2 = \frac{y^2}{4x} + x^2 - y + \frac{1}{4}$

1 b) $f(x;y) - f\left(\frac{1}{2}, 1\right) = f(x,y) + \frac{1}{4} = \frac{(y-2x)^2}{4x} + \left(x-\frac{1}{2}\right)^2 \geqslant 0$ car $x > 0$

Ex 3

1 1) $D_g = \mathbb{R}^2$ $\quad D_f = \mathbb{R} \times]-1; +\infty[$

2,5 2)

$$\boxed{\begin{aligned} \partial_x f &= \ln(1+y) \\ \partial_y f &= \frac{x}{1+y} \end{aligned}}$$

$\partial_x g = 2y e^{2x+y}$

$\partial_y g = e^{2x+y} + y e^{2x+y} = (1+y) e^{2x+y}$

$$\boxed{\begin{aligned} \partial^2_{xx} &= 4y e^{2x+y} \\ \partial^2_{yy} &= (2+y) e^{2x+y} \\ \partial^2_{xy} &= 2(1+y) e^{2x+y} \end{aligned}}$$