



## Conference Kinetic Equations CIRM, November 10<sup>th</sup> to 14<sup>th</sup>, 2014

### Titles and Abstracts

**Kazuo Aoki, Kyoto University**

Title: A numerical study of the Taylor-Couette problem for a vapor-gas mixture.

Abstract: We consider steady flows of a mixture of a vapor and a non-condensable gas between two coaxial cylinders, the rotating inner cylinder and the resting outer cylinder, in the near continuum regime. More specifically, the vapor is the gas phase of the material forming the cylinders, so that it may evaporate or condense on the surfaces of the cylinders, whereas the non-condensable gas neither evaporates nor condenses there.

As studied in our previous paper [Y. Yoshida and K. Aoki, Phys. Fluids 18, 087103 (2006)], the simple flow of axial symmetry and axial uniformity, which may be called the cylindrical Couette flow, exhibits a nontrivial flow bifurcation in the near continuum regime, which is caused by a weak radial flow attributed to evaporation and condensation on the cylinder surfaces. Therefore, if we release the constraint of axial uniformity, we expect more complex bifurcation nature because of the interaction between the bifurcated cylindrical Couette flow and the so-called Taylor-vortex flow.

In the present study, we investigate this problem numerically on the basis of kinetic theory using the DSMC method. To be more specific, we investigate the steady and axisymmetric flow of the mixture in an annular domain bounded by the rotating inner and resting outer cylinders and by the top and bottom boundaries with specular reflection. As the result, it is shown that at high-speed rotation of the inner cylinder, three types of flows, a one-dimensional cylindrical Couette flow, a Taylor-vortex flow with a single vortex, and a Taylor-vortex flow with double vortices, can exist stably for a range of parameters. It is also found that, compared with

the ordinary cylindrical Couette flow (of a single non-condensable gas), the cylindrical Couette flow of the mixture of the vapor and non-condensable gas is more stabilized by the effect of weak radial flow caused by evaporation and condensation.

**Leif Arkeryd, Chalmers University of Technology, Göteborg**

Title: On low temperature kinetic theory; spin diffusion, anyons, Bose Einstein condensates.

Abstract: To illustrate specifically quantum behaviours, the talk will consider three typical problems for non-linear kinetic models evolving through pair collisions at temperatures not far from absolute zero. Based on those examples, a number of differences between quantum and classical Boltzmann theory is discussed in more general terms.

**Anton Arnold, Technische Universität, Vienna**

Title: Entropy method for hypocoercive & non-symmetric Fokker-Planck equations with linear drift.

Abstract: In the last 15 years the entropy method became an invaluable tool for analyzing the large-time behavior (in particular the convergence to a steady state) for wide classes of PDEs: starting from linear Fokker-Planck equations, to various dissipative kinetic models, and up to several quasi-linear equations. For Fokker-Planck equations, the essence of the method is to first derive a differential inequality between the first and second time derivative of the relative entropy, and then between the entropy dissipation and the entropy.

For degenerate parabolic equations, the entropy dissipation may vanish for states other than the equilibrium. Hence, the standard entropy method does not carry over. For hypocoercive Fokker-Planck equations, we first establish a condition that is equivalent to the existence of a unique normalised steady state. By introducing an auxiliary functional (of entropy dissipation type) we prove the exponential decay of the solution towards the steady state in relative entropy. Originally, this requires the initial condition to lie in a weighted  $H^1$ -space. But exploiting the parabolic/hypocoercive regularization, this requirement can be reduced to a finite relative entropy. Finally, we show that the obtained rate is indeed sharp (both for the

logarithmic and quadratic entropy). Applying this new method to (non-degenerate) non-symmetric Fokker-Planck equations yields better global decay rates, compared to the results obtained from the standard entropy method.

**Véronique Bagland, Blaise Pascal University, Clermont-Ferrand**

Title : The Boltzmann equation for ballistic annihilation.

Abstract: We consider a modified Boltzmann equation describing probabilistic ballistic annihilation in a spatially homogeneous setting. Such a model describes a system of hard-spheres such that, whenever two particles meet, they either annihilate with probability  $\alpha \in (0, 1)$  or they undergo an elastic collision with probability  $1 - \alpha$ . For such a model, the number of particles, the linear momentum and the kinetic energy are not conserved. Physicists expect that any solution to the Boltzmann equation for ballistic annihilation should approach for large times a self-similar solution. We shall investigate here the existence of such self-similar solutions.

**Claude Bardos, Paris 7-Denis Diderot University**

Title : Semi Classical limit and Vlasov Dirac Benney Equation.

Abstract: This is a report on a work in progress initiated with Anne Nouri and continuing with Nicolas Besse. The Dirac in the above name refers to the fact that in this equation the usual Coulomb potential is replaced by the Dirac mass. This singularity leads to instabilities which depend on the initial data. The analysis of such instabilities contributes to the understanding of the semi classical limit.

**Julien Barré, Nice Sophia-Antipolis University**

Title : From Vlasov-Poisson-Fokker-Planck to incompressible Euler equations.

Abstract: Vlasov-Poisson-Fokker-Planck equations provide a simplified model for a cloud of cold atoms in a Magneto Optical Trap. The *strong field*, or *quasi-neutral* regime, where the repulsive interaction dominates, is often relevant for experiments. Motivated by this example and more generally by trapped non neutral plasmas,

we study this quasi-neutral limit, and show under certain conditions the convergence of the solution of Vlasov-Poisson-Fokker-Planck equations to the solution of incompressible Euler equation.

For an infinite or periodic system, this limit has already been studied by Y. Brenier and N. Masmoudi. New difficulties arise here from the Fokker-Planck operator, and especially from the boundary conditions (Joint work with D. Chiron, T. Goudon et N. Masmoudi.).

**Niclas Bernhoff, Karlstad University**

Title: Boundary layers for discrete quantum kinetic equations.

Abstract: In this talk we consider different general discrete quantum kinetic equations and see that results for boundary layers for the discrete Boltzmann equation can be applied for them. In fact, we make suitable linearizations around equilibrium states and obtain systems, which have a similar structure as corresponding systems for the discrete Boltzmann equation. Equations we will consider are general discrete versions of a kinetic equation for excitations interacting with a Bose-Einstein condensate, the Uehling-Uhlenbeck (or Nordheim-Boltzmann) equation, and the anyon Boltzmann equation.

**Alexander V. Bobylev, Keldysh Institute (Moscow) and Karlstad University**

Title: On the structure of the Chapman-Enskog expansion.

Abstract: We analyse the classical Chapman-Enskog expansion for the Boltzmann equation. We show that this expansion should be considered as an intermediate step in constructing the asymptotics of solutions to the Boltzmann equation for small Knudsen numbers. It is explained that the way of transformation of the expansion is quite clear for the linearized case. Difficulties of the nonlinear problem are also briefly discussed.

**Stéphane Brull, Bordeaux 1 University**

Title: Asymptotic-preserving scheme for the Fokker-Planck-Landau-Maxwell system

in the quasi-neutral regime.

Abstract: This work deals with the numerical resolution of the Fokker-Planck-Maxwell system in the quasi-neutral regime. In this regime the stiffness of the stability constraints of classic schemes causes huge calculation times. That is why, we introduce a new stable numerical scheme consistent with the transitional and limit models. Such schemes are called Asymptotic-Preserving (AP) schemes in literature. This new scheme is able to handle the quasi-neutrality limit regime without any restrictions on time and space steps. This approach can be easily applied to angular moment models by using a moments extraction. Finally, two physically relevant numerical test cases are presented for the Asymptotic-Preserving scheme in different regimes. The first one shows the efficiency of the Asymptotic-Preserving scheme in the quasi-neutral regime whereas the second one on the contrary corresponds to a regime where electromagnetic effects are predominant.

**Kleber Carrapatoso, Paris Dauphine University**

Title: Rate of convergence to equilibrium for the Landau equation.

Abstract: We present in this talk some results concerning the rate of convergence to equilibrium for the spatially homogeneous Landau equation with hard and (moderately) soft potentials. We will show an exponential decay to equilibrium with optimal rate given by the spectral gap of the associated linearised operator.

**José Antonio Carrillo, Imperial College, London**

Title: Swarming models with repulsive-attractive effects.

Abstract: I will discuss several aspects of models for collective behavior with attraction and repulsion as main effects. Questions related to nonlinear stability of patterns, hydrodynamic equations, and related kinetic models.

**Thomas Cartier-Michaud, CEA Cadarache**

Title: Filamentation in a kinetic Rayleigh Bénard instability: plasma trapped ion turbulence.

Abstract: A minimum model of plasma turbulence in a kinetic framework is presented. It is based on trapped ion turbulence, gyro and bounce averaged, and implemented in the versatile and efficient code TERESA. Zonal flow - streamer interplay are readily shown to be key players that govern the confinement properties of the model. The parameter space of the model is explored with brute force numerics. A generic result is either a streamer dominated pattern with large transport, or a staircase temperature profile with very marked corrugations and quenched transport. A case with off-axis heating is found to exhibit quasi-periodic relaxation events relevant to investigate dynamical turbulence self-organization. The strong shearing associated to the steep gradients of the staircase like profiles leads to strong filamentation that are difficult to handle numerically. Rather than letting the numerical scheme govern the flux to the mesh sub-scales, we introduce a filtering technique and analyse its impact on the overall simulation. Specific attention is given to the spectrum of the fluctuations from the Zonal flow scale to the dissipation scale.

**Bruno Despres, Paris 6 University**

Title: Symmetrization of Vlasov-Poisson and Vlasov-Ampère equations.

Abstract: We detail the eigenstructure of the linearized Vlasov-Ampère equation and construct an integro-differential operator which is related to the eigenstructure. It gives a representation formula for the electric field which is known since Morrison and Pfirsch (92') for the linear Vlasov-Poisson equation, and yields sharp estimates for the linear Landau damping with low regularity data. We apply the technique to a problem with a dependence to the Debye length and show weaker damping for small Debye length. A nonlinear variant of the main quadratic framework, based on Antonov's energy (61') is finally discussed.

**Miguel Escobedo, Universidad del Pais Vasco, Bilbao**

Title: Convergence to equilibrium of a linearized kinetic equation for phonons.

Abstract: We consider a kinetic equation that arises in the kinetic theory of condensed Bose gases. That equation contains a collision integral describing splitting of an excitation into two others in the presence of the condensate. We shall study

the linearised equation around an equilibrium in the small momentum and low temperature limit, for which the collision frequency is neither bounded from below nor from above. We prove the existence and uniqueness of solutions that satisfy the conservation of the energy. We show that each of these solutions converge to the corresponding stationary state, algebraically as time tends to infinity.

**Raffaele Esposito, L'Aquila University**

Title: Stationary solutions to the Boltzmann equation and their hydrodynamic limit.

Abstract: In a two- or three- dimensional domain of arbitrary shape I consider a rarefied gas described by the Boltzmann equation with diffuse reflection boundary conditions modelling the contact with thermal wall where a temperature profile is given. I describe recent results showing, for fixed Knudsen numbers, the existence, uniqueness, stability and positivity and regularity properties of the stationary solution under the assumption that the temperatures at the boundary are close to the homogeneous one. The asymptotic when the Knudsen number is small (hydrodynamic limit) requires a more accurate analysis. I review the few results based on special geometries and describe the difficulties for a general geometry. In particular, I consider the low Mach numbers regime, whose asymptotic behaviour is given by the stationary Navier-Stokes-Fourier system. I discuss the difficulty of the approach based on bulk + boundary layer expansion and show how to avoid the boundary layer expansion.

**Damien Estève, CEA Cadarache**

Title: On impurity transport and collisional exchange.

**Francis Filbet, Lyon 1 University**

Title: On the numerical simulation of the Vlasov-Poisson model with an external magnetic field.

Abstract: We present different numerical methods for the simulation of the Vlasov

equation in the presence of an external magnetic field. Reduced and high dimensional systems are considered. Hence, numerical simulations come to illustrate both stabilities and instabilities : diocotron instability, convergence to equilibrium with polynomial decay rate, ITG instability.

**Emmanuel Frénod, Université de Bretagne-Sud**

Title: An exponential integrator for the 4D-Vlasov-Poisson system with strong magnetic field.

Abstract: With the aim of solving in a four dimensional phase space a multi-scale Vlasov-Poisson system, we propose in a Particle-In-Cell framework a robust time-stepping method that works uniformly when the small parameter vanishes. As an exponential integrator, the scheme is able to use large time steps with respect to the typical size of the solutions fast oscillations. In addition, we show numerically that the method has accurate long time behaviour and that it is asymptotic preserving with respect to the limiting Guiding Center system.

**Philippe Ghendrih, CEA Cadarache**

Title: Around fusion and Landau collisional model.

**François Golse, Ecole Polytechnique**

Title: The Boltzmann equation over  $\mathbf{R}^3$ : dispersion vs dissipation.

Abstract: Solutions of the Boltzmann equation set in the spatial domain  $\mathbf{R}^3$  are subject to the dispersive effect of the advection operator, and to the dissipation effect of the collision integral, expressed in terms of the entropy production integral in the Boltzmann H Theorem. Dispersion offsets the dissipative effect of collisions by increasing the rarefaction effect. On the other hand, relaxation to equilibrium due to the dissipative effect of collisions occurs at an exponential rate, and is therefore much faster than dispersion, which occurs at polynomial rate. The talk analyzes the long time behavior of solutions of the Boltzmann equation in the vicinity of global Maxwellian solutions, and establishes the existence of a scattering regime (In collaboration with C. Bardos, I.M. Gamba and C.D. Levermore).



**Stéphane Junca, Nice Sophia-Antipolis University**

Title: A continuous model for ratings : a Kinetic model with a mean field derivation.

Abstract: The Elo rating system is a method for calculating the relative skill levels of players in two-player games such as chess (Wikipedia). This system is widely used to rank online games players and sport teams for instance. When the players are numerous and interact a lot we derive a new continuous model: a kinetic equation with a mean field velocity.

An important issue for the validity of the rating system is the asymptotic behavior of the ratings for large time. The idealistic case when all players are compared yields an exponential rate to the expected rating independently of the initial rating. The realistic and complex case with only local interactions has several equilibria. The convergence holds with no rate to an equilibrium depending on the initial ratings (Joint work with Pierre-Emmanuel Jabin).

**Mohammed Lemou, Rennes 1 University**

Title: Quantitative stability inequalities for Vlasov-Poisson and 2D-Euler systems

Abstract: In this talk, we will first present a new Hardy-Littlewood type inequality for generalized rearrangement of functions, discuss its motivations and give the main lines of the proof. We then show how this inequality provides quantitative stability results of steady states to evolution systems that essentially preserve the rearrangements and some energy functional. Some known stability results for kinetic and 2D-Euler equations are essentially based on compactness arguments, and the goal of this work is to make completely quantitative these stability results, avoiding the use of compactness tools. In particular we first derive a quantitative stability result for a large class of steady state solutions to the gravitational Vlasov-Poisson systems; more precisely we show that this type of inequality implies the control of the L1 norm of the perturbation by the relative Hamiltonian and the rearrangements. In fact, the non linear stability in this context has already been obtained in the recent past, but the proof was based in a crucial way on some compactness arguments, which by construction provides no quantitative control of the perturbation. We also

investigate other applications of this inequality to the context of plasma physics kinetic equations and 2D-Euler systems.

**Tai-Ping Liu, Taipei University**

Title: Boundary Phenomena for Some Kinetic Equations.

Abstract: We will report on the recent progresses on the boundary effects for some kinetic models. The boundary effects include the boundary singularity, initial-boundary layer, thermal creep, and equilibrating effects of the boundary. Examples for the full Boltzmann equation, linearized Boltzmann equation, and free molecular flows will be given to illustrate these phenomena.

**Jani Lukkarinen, Helsinki University**

Title: Kinetic theory of the Hubbard model: a matrix-valued Boltzmann equation with a twist.

Abstract: In this joint work with Peng Mei and Herbert Spohn we consider the standard Hubbard model which is a simplified model for the evolution of fermions on a lattice with an on-site quartic interaction. In the kinetic scaling limit, space and time scaled by  $\lambda^2$  where  $\lambda$  is the strength of the interaction term, we obtain a matrix-valued Boltzmann equation. The Hubbard model conserves energy and the total spin, including its direction, and since the nonlinear interaction term couples two different spin-components, it will vanish if one of the components vanishes. These properties are reflected in several novel features of the Boltzmann equation: it has an additional effective Hamiltonian term describing a rotation of the spin-basis on the kinetic scale, as well as special *degenerate* solutions which have no collisions and do not equilibrate.

**Phil Morrison, Texas University**

Title: Lifting, a method for constructing consistent kinetic theories with electromagnetic interaction.

Abstract: Starting from ordinary differential equations that describe orbits in given electromagnetic-like fields it is shown how to construct consistent partial differential equations that describe transport with self-consistent interaction. General expressions are given for the polarization and magnetization, and their corresponding sources that enter Maxwell's equations. Thus, general constitutive relations for describing electromagnetism in media, which are in general nonlinear, are obtained. Examples, including kinetic theories based on guiding center motion and other systems of interest in plasma physics, will be given.

**Claudia Norcini, CEA Cadarache**

Title: Comparison of numerical codes and work with GYSELA in this perspective.

**Chiara Saffirio, Zurich University**

Title: From the Hartree-Fock dynamics to the Vlasov equation.

Abstract: We will discuss the convergence (in the semiclassical limit) of a solution to the Hartree-Fock equation towards an operator, whose Wigner transform is a solution to the Vlasov equation. We will consider both cases of positive and zero temperature. The results we will present are part of a project in collaboration with N. Benedikter, M. Porta and B. Schlein.

**Christian Schmeiser, University of Vienna**

Title: Existence and stability of stationary solutions of kinetic chemotaxis models.

Abstract: Kinetic models for chemotaxis typically describe velocity jump processes of biological cells, where the jumps are biased towards large values of a chemoattractant density. For a linear model with prescribed chemoattractant distribution, existence and dynamic stability of aggregated steady states has been proven, using hypocoercivity methods (joint work with V. Calvez and G. Raoul). For a nonlinear model also including chemoattractant production by the cells, formal computations

with moments indicate a dichotomy between dispersion and aggregation. Some results on the existence of aggregated steady states could be proven (joint work with A. Nouri).

**Jacques Schneider, Toulon University**

Title: On a well-posed simulation model for multicomponent dilute reacting gases.

Abstract: We aim to present a relaxation model that can be used in real simulations of dilute multicomponent reacting gases. Our approach is limited to moderately non-equilibrium reacting gases but goes beyond the range of validity of the corresponding Navier-Stokes equations [1]. Relaxation times for the internal energy modes are assumed to be smaller than the chemistry characteristic times as well as the characteristic times of macroscopic processes. We consider the semi-classical approach with only one variable for the internal energy modes. The strategy is the same as in [2]. That is a sum of operators for respectively the mechanical and chemical processes. The mechanical operator(s) is the "natural" extension to polyatomic gases of the method of moment relaxations presented in [3] [4]. The chemical ones feature production rates at thermal equilibrium. The whole model is shown to satisfy the same properties as the Boltzmann equation: conservations and entropy production. Moreover null entropy production states are characterized by vanishing chemical production rates. We also study the hydrodynamic limit in the slow chemistry regime. Finally we show that the whole set of parameters that are used in the derivation of the model can be calculated by softwares such as EGlib [5] or STANJAN [6].

### References

- [1] V. Giovangigli, *Multicomponent flow modeling*, Birkhauser, Boston-Basel-Berlin, 1998.
- [2] S. Brull, Jacques Schneider, *Derivation of a BGK model for reacting gas mixtures*, Comm. Math. Sciences, Vol 12, No 7, 1199-1223, 2014.
- [3] S. Brull, J. Schneider, *A new approach of the Ellipsoidal Statistical Model*, Cont. Mech. Thermodyn. 20, no. 2, 63-74, 2008.
- [4] S. Brull, V. Pavan, J. Schneider, *Derivation of BGK models for mixtures*, Eur. J. Mechanics (B-Fluids), 33, 74-86, 2012.
- [5] A. Ern, V. Giovangigli, *EGlibServer and User's Manual*,

<http://www.cmap.polytechnique.fr/www.eglib/>

[6] B. Reynolds, *The Element Potential Method for Chemical Equilibrium Analysis*, <http://www.stanford.edu/>

**Sergio Simonella, Technische Universität München**

Title: The correlation error in the Boltzmann-Grad limit.

Abstract: In this talk I will give a survey of the recent progress regarding the derivation of the Boltzmann equation from Newtonian particle systems. Focusing on quantitative results, I shall introduce a set of functions (correlation errors) measuring the deviations in time from the statistical independence of particles (propagation of chaos), and discuss their behaviour in the low-density limit of a hard sphere system.

**Walter Strauss, Brown University**

Title: Stability Analysis of a Hot Plasma in a Torus.

Abstract: We model a collisionless plasma by the relativistic Vlasov-Maxwell system. There are many equilibria, of which some are stable and some unstable. In this talk I will present recent work with TOAN NGUYEN where we allow a boundary, namely the surface of a torus. The particles reflect specularly while the surface acts as a perfect conductor. While these are not the physical boundary conditions for a tokamak of course, our analysis is a first step toward understanding the effect of toroidal geometry on the rigorous stability theory of plasmas. We provide sharp criteria for the stability of equilibria under the assumption that the particle distributions and the electromagnetic fields depend only on the cross-sectional variables of the torus.

**Shigeru Takata, Kyoto University**

Title: Grazing collision effect of non-cutoff potentials in a toy kinetic equation.

Abstract: Consider the behavior of a rarefied gas whose initial state is such that the velocity distribution function (VDF) has a discontinuity. Though decaying, the discontinuity propagates in time for  $t > 0$ . A similar propagation of the discontinuity on the boundary into a gas occurs in steady problems around convex bodies. These phenomena are well understood both theoretically and numerically.

**Ariane Trescases, ENS Cachan**

Title: Regularity of the Boltzmann Equation with Diffuse Boundary Conditions.

Abstract: The Boltzmann Equation models the evolution of a rarefied gas. Taking into account the collisions of the particles of gas against a physical body leads to considering boundary conditions. We present new results of regularity for the Boltzmann Equation in a bounded domain with diffuse boundary conditions. In the case of a convex domain, we prove a result of propagation of Sobolev regularity.