Energy Economics 49 (2015) 540–549

Contents lists available at ScienceDirect

Energy Economics

journal homepage: www.elsevier.com/locate/eneco

Analyzing volatility spillovers and hedging between oil and stock markets: Evidence from wavelet analysis

R. Khalfaoui [a,*], M. Boutahar [a,b], H. Boubaker [b]

[a] IMM, Campus de Luminy, Case 907, 13288 Marseille cedex 09, France
[b] IPAG LAB-IPAG Business School, 184, Boulevard Saint-Germain, 75006 Paris, France

ARTICLE INFO

Article history:
Received 6 September 2013
Received in revised form 26 March 2015
Accepted 28 March 2015
Available online 10 April 2015

JEL classification:
C5
C58
G1
G110
G180
G15

Keywords:
Discrete wavelet analysis
Wavelet coherence
BEKK-GARCH
Volatility spillovers
Hedge ratio
Crude oil prices
Stock prices

ABSTRACT

This paper examines the linkage of crude oil market (WTI) and stock markets of the G-7 countries. We study the mean and volatility spillovers of oil and stock market prices over various time horizons. We propose a new approach incorporating both multivariate GARCH models and wavelet analysis: wavelet-based MGARCH approach. We combine a bivariate GARCH-BEKK model with wavelet multiresolution analysis in order to capture the multiscale features of mean and volatility spillovers between time series. For optimal portfolio allocation decisions, we analyze the multiscale behavior of hedge ratio. Empirical results show strong evidence of significant volatility spillovers between oil and stock markets, as well as time-varying correlations for various market pairs. However, results of wavelet coherence indicate that in most, the WTI market was leading. In addition, it is stated that the decomposed volatility spillovers permit investors to adapt their hedging strategies.

1. Introduction

Multivariate generalized conditional heteroskedasticity (MGARCH) models have been widely used in the empirical studies to estimate the volatility spillover effects among different markets. In this study, we used bivariate GARCH models to simultaneously estimate the mean and conditional variance of oil and stock market prices. We employ a BEKK representation of the multivariate GARCH model, which has been widely used in order to study the international linkage of multiple markets. Specifically, we adopt a bivariate GARCH(1,1)-BEKK model which allows us to study the volatility transmission between global oil market and stock markets of G-7 countries. It is important for energy and financial researchers, market participants and policy makers to understand the volatility spillover effects from one market to another.

While previous works have examined the mean and volatility transmission between oil markets and stock markets using the return time series and a full period framework, in this paper we develop a new approach which examines the volatility spillover of oil and stock market prices on level prices (without computing returns of the original series) and the multiscale behavior, in order to study the spillover effects from oil markets and the G-7 stock markets at different time horizons. We combine the multivariate GARCH models and wavelet multiresolution analysis. In our work we study the spillover effects using level prices.

The link between oil markets and stock markets has been investigated by a large number of researchers. Recently, there has been an increasing interest in modeling the equity volatility and analyzing the volatility transmission mechanism that exists across major financial markets. Nevertheless, studies focusing on relationship between major developed financial markets and global crude oil market are limited. Some papers have explicitly examined the transmission of mean and volatility across oil and financial markets, for instance Malik and Hammoudeh (2007) studied the volatility and shock transmission between US equity market, global crude oil market, and equity markets of major rich Gulf countries

* Corresponding author. Tel.: +33 4 91 82 92 38; fax: +33 4 91 82 93 56.
E-mail addresses: r.kalfaoui@gmail.com (R. Khalfaoui), mohammed.boutahar@univ-amu.fr (M. Boutahar), heniboubaker@gmail.com (H. Boubaker).

http://dx.doi.org/10.1016/j.eneco.2015.03.023
0140-9883/

(Saudi Arabia, Kuwait, and Bahrain). Arouri and Nguyen (2010) studied the relationship between oil price changes and stock returns at the disaggregated sector level in Europe by investigating their short term linkages over the last turbulent decade using different econometric techniques. Singh et al. (2010) examined the temporal volatility spillovers between developed and emerging stock markets using VAR–GARCH models. Zhang and Wei (2010) tested the relation between the crude oil and gold markets from January 2000 to March 2008. Arouri et al. (2011) analyzed the return links and volatility transmission between oil and stock markets in the Gulf Cooperation Council (GCC) countries. Filis et al. (2011) studied the time-varying correlation between stock market prices and oil prices for oil-importing and oil-exporting countries based on a DCC–GARCH–GJR approach. Vo (2011) models the volatility of stock and oil futures markets using the multivariate stochastic volatility structure. Du et al. (2011) tested the factors that have a potential influence on the volatility of crude oil prices and the relationship between this volatility and agricultural commodity markets. Kumar et al. (2012) argued that the variation in the indices of clean energy stocks is explained by past movements in oil prices, the stock prices of high technology firms and interest rates. Sadorsky (2012) modeled the volatility dynamics between oil and the stock prices of clean energy and technology companies using the dynamic conditional correlation MGARCH models and Awartani and Maghyereh (2013) investigated return and volatility spillover effects between oil and equities in the GCC countries during the period from 2004 to 2012.

A number of previous studies dealing with wavelet filter are applied in financial literature. For instance, see Gencay et al. (2002, 2005) proposal of a new approach based on wavelet analysis to estimate the systematic risk of some stock market indices, Fernandez (2006) estimated the beta in capital asset pricing model (CAPM) at different time-scales in order to study the impact of time scaling on the computation of value-at-risk. Kim and In (2007) studied the relationship between changes in stock prices and bond yields in the G7 countries. Rua and Nunes (2009) analyzed the co-movement among international stock market returns by developing a new approach based on wavelet analysis. He et al. (2009) analyzed the crude oil prices using wavelet analysis and artificial neural network technique. Masih et al. (2010) analyzed stocks in emerging Gulf Cooperation Council (GCC) equity markets by wavelet analysis. Gallegati (2010) tested the financial market contagion using a wavelet-based approach. Boubaker and Boutahar (2011) focused on modeling the conditional mean and conditional variance of exchange rates. They estimated the GARMA–FIGARCH model using the wavelet-based maximum likelihood estimator. Sun and Meinl (2012) proposed a new filtering algorithm based on MODWT to decompose pattern and noises and Fernández-Macho (2012) analyzed the correlation and the cross-correlation of the Eurozone stock market returns on a scale-by-scale basis.

In this study we show strong evidence of GARCH(1,1)–BEKK model to analyze the volatility spillover effects of oil market and the G-7 developed stock markets. The results show evidence of significant volatility spillovers between oil and stock markets, as well as time-varying correlations for various market pairs.

In addition, we extend our methodology to portfolio diversification strategies, which is the most important objective of market participants and market makers. We analyze the hedge ratio and hedging effectiveness across different time horizons. We define the wavelet hedge ratio and wavelet portfolio allocations. The results of wavelet hedging showed that hedge ratios are different across scales and the investor can easily understand the decision strategy by choosing the minimum portfolio risk.

The rest of the paper is organized as follows: Section 2 presents the wavelet methodology. Section 3 describes the wavelet-based MGARCH approach to study the mean and volatility spillovers among oil and stock markets. Discussion of the empirical results is given in Section 4. Section 5 concludes and the last section reports the appendix.

2. Wavelet approach

2.1. Multiresolution analysis

Wavelet theory is a powerful mathematical tool for time series analysis. It has attracted an increasing interest of economists in the last years. It provides a time–frequency representation of a time series X_t (in our study, X_t is the daily oil spot price and daily stock market index), and it can be used to analyze non-stationary time series, which are very common in finance and economics, given the continuous presence of abrupt changes and volatility. Recently, this methodology has received great interest in the financial literature including those of Gencay et al. (2005), Kim and In (2007), Rua and Nunes (2009), He et al. (2009), Genest et al. (2009), Masih et al. (2010), Rua (2010), He et al. (2012) and Jammazi (2012).

The wavelet transform uses multiresolution analysis by which different frequencies are analyzed with different resolutions.[1] For these multiresolution analyses, few conditions must be satisfied: let $L^2(\mathbb{R})$ be the space of square-integrable functions. Now consider a sequence of closed sub-spaces $\{W_k\}_{k=j}^{\infty}$ (relative to the detail spaces of the series) and V_j (relative to the approximation of the series) of $L^2(\mathbb{R})$, such that $V_j \subset V_{j+1}$ and $\cap_j V_j = 0$, and $\cup_j V_j = L^2(\mathbb{R})$, which indicates that all integrable functions should be included at the highest resolution. Moreover, we say that V_j is a multiresolution if it satisfies the following conditions:

i. dilation invariance: $X(t) \in V_j \Leftrightarrow X(2t) \in V_j$.
ii. translation invariance: $X(t) \in V_0 \Leftrightarrow X(t+1) \in V_0$.
iii. there is a scaling function $\phi(t) \in V_0$ (also called father wavelet) such that $\phi(t-k)$ is an orthonormal basis of V_0.

$$\phi_{j,k}(t) = 2^{j/2}\phi\left(2^j - k\right) \quad (1)$$

where the level j controls the degree of stretching of the function (the larger the j, the more stretched is the basis function); the smaller the time scale, the higher the frequency of the decomposed series, and k is the parameter that controls the translation of the basis function.

Assuming that the detail spaces, $\{W_j\}$, are orthogonal to each other, we can define a sequence $\{\psi_{j,k}(t)\}_k$ of orthonormal basis functions that spans $L^2(\mathbb{R})$:

$$\psi_{j,k}(t) = 2^{j/2}\psi\left(2^j t - k\right) \quad (2)$$

where wavelets are generated by shifts and stretches of the mother wavelet, $\psi_{j,k}(t)$. Let $X(t)$ be the original time series, we represent the multiresolution representation of $X(t)$ by:

$$\begin{aligned} X(t) &= \sum_k s_{j,k}\phi_{j,k}(t) + \sum_j \sum_k d_{j,k}\psi_{j,k}(t), \quad j = 1, \dots, J \\ &= S_J(t) + D_J(t) + D_{J-1}(t) + \dots + D_1(t) \end{aligned} \quad (3)$$

where

$$S_J(t) = \sum_k s_{j,k}\phi_{j,k}(t) \quad (4)$$

$$D_j(t) = \sum_k d_{j,k}\psi_{j,k}(t). \quad (5)$$

The series $S_J(t)$ provides a smooth of original time series $X(t)$ and represents the approximation that captures the long term properties (i.e. the low-frequency dynamics), and the series $D_j(t)$ for $j = 1, \dots, J$ refers to wavelet details and captures local fluctuations (i.e. the higher-frequency characteristics) over the whole period of $X(t)$ at each scale. The expression (3) represents the decomposition of $X(t)$

[1] For more details, see Mallat (1989) and Percival and Walden (2000).

into orthogonal components at different resolutions and constitutes the so-called wavelet multiresolution analysis (MRA).

In practical applications, we invariably deal with sequences of values indexed by integers rather than functions defined over the entire real axis. Instead of actual wavelets, we use short sequences of values referred to as wavelet filters. The number of values in the sequence is called the width of the wavelet filter. Thus, the wavelet analysis considered from a filtering perspective is then well-suited to time series analysis. For the discrete wavelet transform, the wavelet coefficients can be calculated from the MRA scheme. The recursive MRA scheme,[2] which is implemented by a two-channel filter bank (i.e. a high-pass wavelet filter and its associated low-pass scaling filter) representation of the wavelet transform, is divided into decomposition and reconstruction schemes according to the forward and inverse wavelet transform.

2.2. Discrete wavelet transform

The discrete wavelet transform (DWT) is based on two types of filters called a wavelet filter and scaling filter denoted by h_l, $l = 0, \ldots, L-1$ and g_l, $l = 0, \ldots, L-1$, respectively. The wavelet filters $\{h_l\}_{l=0}^{L-1}$ and the scaling filters $\{g_l\}_{l=0}^{L-1}$ are used to construct the DWT matrix.

The wavelet filter of support $L \in \mathbb{N}$ (L is the length of the filter), is defined so as to satisfy the following properties:

i. zero mean: $\sum_{l=1}^{L} h_l = 0$,
ii. unit energy: $\sum_{l=0}^{L-1} h_l^2 = 1$,
iii. orthogonal to its shifts: $\sum_{l=0}^{L-1} h_l h_{l+2n} = \sum_{l=-\infty}^{\infty} h_l h_{l+2n} = 0, \forall\, n \in \mathbb{N}^*$.

The scaling filter of support L is defined via the quadrature mirror relationship $g_l = (-1)^{l+1} h_{L-1-l}$ similarly satisfies the following conditions:

i. $\sum_{l=0}^{L-1} g_l = \sqrt{2}$,
ii. $\sum_{l=0}^{L-1} g_l^2 = 1$,
iii. $\sum_{l=0}^{L-1} g_l g_{l+2n} = \sum_{l=-\infty}^{\infty} g_l g_{l+2n} = 0,\ \forall\, n \in \mathbb{N}^*$.

The wavelet and scaling coefficients of a time series $X(t)$ at the jth level are defined as:

$$w_{j,t} = \sum_{l=0}^{L-1} h_{j,l} X(t-l) \tag{6}$$

$$v_{j,t} = \sum_{l=0}^{L-1} g_{j,l} X(t-l). \tag{7}$$

Daubechies (1992) defined a useful class of wavelet filters, namely the Daubechies compactly supported wavelet filters of width L and distinguishes between two choices — the extremal phase filters $D(L)$ and the least asymmetric filters $LA(L)$. The Daubechies wavelet has many desirable properties; its most useful property is possessing the smallest support for a given number of vanishing moments.[3]

2.3. Maximum overlap discrete wavelet transform

The maximum overlap discrete wavelet transform (MODWT) is an alternative wavelet transform for the DWT. This transform is developed in order to cope with the DWT shortcomings. The MODWT is a non-orthogonal transform and it has many advantages over the DWT such as non-dyadic length sample size, invariant translation (i.e. shifting the time series by integer unit will shift the MODWT wavelet and scaling coefficients of the same amount), which provides increased resolution at coarser scales and produces more asymptotically efficient wavelet variance estimator than DWT. The MODWT goes by several names in the statistical and engineering literature, such as, the "stationary DWT" (Nason and Silverman, 1995), "translation-invariant DWT" (Coifman and Donoho, 1995) and "time-invariant DWT" (Pesquet et al., 1996).

Instead of using the wavelet and scaling filters from Section 2.2, the MODWT utilizes the rescaled filters $\widetilde{h}_{j,l} = \frac{h_{j,l}}{2^{j/2}}$ and $\widetilde{g}_{j,l} = \frac{g_{j,l}}{2^{j/2}}$, j, 1, …, J, where J is the total number of levels.

The MODWT filters satisfy the following properties:

i. $\sum_{l=0}^{L-1} \widetilde{h}_l = 0$, $\sum_{l=0}^{L-1} \widetilde{g}_l = 1$,
ii. $\sum_{l=0}^{L-1} \widetilde{h}_l^2 = \sum_{l=0}^{L-1} \widetilde{g}_l^2 = \frac{1}{2^l}$,
iii. $\sum_{-\infty}^{+\infty} \widetilde{h}_l \widetilde{h}_{l+2n} = \sum_{-\infty}^{+\infty} \widetilde{g}_l \widetilde{g}_{l+2n}$.

Wavelet and scaling coefficients are obtained as follows:

$$\widetilde{w}_{j,t} = \frac{1}{2^{j/2}} \sum_{l=0}^{L-1} \widetilde{h}_{j,l} X_{t-l} \tag{8}$$

$$\widetilde{v}_{j,t} = \frac{1}{2^{j/2}} \sum_{l=0}^{L-1} \widetilde{g}_{j,l} X_{t-l}. \tag{9}$$

2.4. Maximum overlap discrete Hilbert wavelet transform

The maximum overlap discrete Hilbert wavelet transform (MODHWT) was advocated by Whitcher and Craigmile (2004) (see also Whitcher et al. (2005)) to investigate multiscale coherence and phase properties of time varying non-stationary processes. The MODHWT is implemented using a pair of mother and father wavelet filters based on Hilbert wavelet pairs (HWP)[4] and uses the maximum overlap version of the dual-tree complex wavelet transform.

Let $\left\{\widetilde{h}_l^1\right\}$ and $\left\{\widetilde{g}_l^1\right\}$ a couple of father and mother wavelet filters respectively of $\phi^1(t)$ and $\psi^1(t)$. $\psi^1(t)$ is the Hilbert transform of $\psi(t)$ if

$$\Psi^1(X) = \begin{cases} -i\Psi(X), & X>0 \\ i\Psi(X), & X<0 \end{cases} \tag{10}$$

where $\Psi(X)$ and $\Psi^1(X)$ are the Fourier transform of $\psi(t)$ and $\psi^1(t)$ respectively. This implies that the wavelets are $\frac{\pi}{2}$ out of phase with each other.

The MODHWT utilizes a non-decimated pair of wavelet (and scaling) filters. As a consequence, two sequences of coefficients are obtained, which are the real and imaginary parts of the final wavelet coefficients, such that:

$$\widetilde{\widetilde{h}}_{j,l} = \widetilde{h}_{j,l} + i\widetilde{h}_{j,l}^1 \tag{11}$$

$$\widetilde{\widetilde{g}}_{j,l} = \widetilde{g}_{j,l} + i\widetilde{g}_{j,l}^1 \tag{12}$$

where $\widetilde{h}_{j,l} = \frac{h_{j,l}}{\sqrt{2}}$, $\widetilde{h}_{j,l}^1 = \frac{h_{j,l}^1}{\sqrt{2}}$, $\widetilde{g}_{j,l} = \frac{g_{j,l}}{\sqrt{2}}$ and $\widetilde{g}_{j,l}^1 = \frac{g_{j,l}^1}{\sqrt{2}}$. $\widetilde{h}_{j,l}$, $\widetilde{h}_{j,l}^1$, $\widetilde{g}_{j,l}^1$ and $\widetilde{g}_{j,l}$ are the renormalized Hilbert wavelet and scaling filter pairs.

[2] A robust theoretical framework for critically sampled wavelet transformation is Mallat's multiresolution analysis (for more details, see Mallat (1989)).
[3] For Daubechies wavelets, the number of vanishing moments is half the filter length.
[4] The HWP is a pair of wavelet filters that are designed to be approximate Hilbert transform of one another.

The Hilbert wavelet and scaling coefficients are obtained as follows:

$$\tilde{\tilde{w}}_j(t) = \sum_{l=0}^{L-1} \tilde{\tilde{h}}_{j,l} X(t-j) = \tilde{w}_{j,t} + i\tilde{w}^1_{j,t} \tag{13}$$

$$\tilde{\tilde{v}}_j(t) = \sum_{l=0}^{L-1} \tilde{\tilde{g}}_{j,l} X(t-j) = \tilde{v}_{j,t} + i\tilde{v}^1_{j,t}. \tag{14}$$

2.5. Wavelet coherence

The wavelet coherence was introduced by Whitcher and Craigmile (2004). Let $\left(\tilde{\tilde{w}}^X_{j,t}, \tilde{\tilde{w}}^Y_{j,t}\right)$, $j = 1, \dots, J$, $t \in \mathbb{Z}$ denote the MODHWT coefficients of $(X(t), Y(t))$. The time varying cross spectrum of the bivariate time series $X(t)$ and $Y(t)$ is as follows:

$$\mathcal{S}_{XY}\left(\lambda_j, t\right) = \mathrm{E}\left[\tilde{\tilde{w}}^X_{j,t}\left(\tilde{\tilde{w}}^Y_{j,t}\right)^*\right] = C_{XY}(\lambda j, t) - iQ_{XY}\left(\lambda_j, t\right) \tag{15}$$

where $C_{XY}(\lambda j, t) = \Re\left\{\mathcal{S}_{XY}\left(\lambda_j, t\right)\right\}$ and $Q_{XY}\left(\lambda_j, t\right) = -\Im\left\{\mathcal{S}_{XY}\left(\lambda_j, t\right)\right\}$ denote the time varying cospectrum and quadrature spectrum, respectively.

Let $A_{XY}(\lambda_j, t) = |\mathcal{S}_{XY}(\lambda_j, t)| = \left[C^2_{XY}(\lambda_j, t) + Q^2_{XY}(\lambda_j, t)\right]^{1/2}$ be the cross amplitude spectrum. The time varying phase spectrum is

$$\theta_{XY}\left(\lambda_j, t\right) = \arctan\left[\frac{-Q_{XY}\left(\lambda_j, t\right)}{C_{XY}\left(\lambda j, t\right)}\right]. \tag{16}$$

Following Whitcher et al. (2005) the time varying magnitude squared coherence is given by

$$\mathcal{K}_{XY}(\lambda j, t) = \frac{A^2_{XY}\left(\lambda_j, t\right)}{\mathcal{S}_X\left(\lambda_j, t\right)\mathcal{S}_Y\left(\lambda_j, t\right)} \tag{17}$$

where $\mathcal{S}_X\left(\lambda_j, t\right) = \mathrm{E}\left|\tilde{\tilde{w}}^X_{j,t}\right|^2$ and $\mathcal{S}_Y\left(\lambda_j, t\right) = \mathrm{E}\left|\tilde{\tilde{w}}^Y_{j,t}\right|^2$. $\mathcal{K}_{XY}(\lambda j, t)$ is a normalized and squared version of the time varying cross spectrum.[5]

3. Wavelet-based GARCH–BEKK model

To model the time-varying dynamics and spillovers between stock market indices and WTI crude oil prices, we employ a bivariate GARCH–BEKK model of Baba et al. (1989) and (Engle and Kroner, 1995). The specification of the model is as follows: the conditional mean equation is given by

$$D_t(j) = \begin{bmatrix} D_{s,t}(j) \\ D_{o,t}(j) \end{bmatrix} = \begin{bmatrix} \phi^s_0 \\ \phi^o_0 \end{bmatrix} + \begin{bmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{bmatrix}\begin{bmatrix} D_{s,t-1}(j) \\ D_{o,t-1}(j) \end{bmatrix} + \epsilon_t(j) \tag{18}$$

where $D_t(j)$ is an 2×1 vector of wavelet details for stock s and oil o at time t and for scale j, and $\epsilon_t(j)|_{\Omega_{t-1}} = \left[\epsilon_{s,t}(j),\ \epsilon_{o,t}(j)\right]' \sim \mathcal{N}\{0, H_t(j)\}$. The conditional variance equation is given by:

$$H_t(j) = C'C + A'\epsilon_{t-1}(j)\epsilon'_{t-1}(j)A + B'H_{t-1}(j)B \tag{19}$$

where $H_t(j)$ is the conditional variance matrix at scale j and the elements for A, B and C are given as follows:

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix},\ B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \text{ and } C = \begin{bmatrix} c_{11} & 0 \\ c_{21} & c_{22} \end{bmatrix}.$$

The coefficients a_{12}, a_{21}, b_{12} and b_{21} can reflect the volatility transmission and spillover between oil and stock markets among wavelet scales.

The conditional variance equation at each scale j and for each market can be expanded for the bivariate GARCH(1,1)–BEKK model as follows:

$$\begin{aligned} h_{s,t}(j) = {} & c^2_{11} + a^2_{11}\epsilon^2_{s,t-1}(j) + 2a_{11}a_{12}\epsilon_{s,t-1}(j)\epsilon_{o,t-1}(j) + a^2_{21}\epsilon^2_{o,t-1}(j) \\ & + b^2_{11}h_{s,t-1}(j) + 2b_{11}b_{12}h_{so,t-1}(j) + b^2_{21}h_{o,t-1}(j), \end{aligned} \tag{20}$$

$$\begin{aligned} h_{o,t}(j) = {} & c^2_{12} + c^2_{22} + a^2_{12}\epsilon^2_{s,t-1}(j) + 2a_{12}a_{22}\epsilon_{s,t-1}(j)\epsilon_{o,t-1}(j) + a^2_{22}\epsilon^2_{o,t-1}(j) \\ & + b^2_{12}h_{s,t-1}(j) + 2b_{12}b_{22}h_{so,t-1}(j) + b^2_{22}h_{o,t-1}(j), \end{aligned} \tag{21}$$

$$\begin{aligned} h_{so,t}(j) = h_{os,t}(j) = {} & c_{11}c_{12} + a_{11}a_{21}\epsilon^2_{s,t-1}(j) + a_{22}a_{21}\epsilon^2_{o,t-1}(j) \\ & + [a_{11}a_{22} + a_{21}a_{12}]\epsilon_{s,t-1}(j)\epsilon_{o,t-1}(j) + b_{11}b_{21}h_{s,t-1}(j) \\ & + [b_{21}b_{12} + b_{11}b_{22}]h_{so,t-1}(j) + b_{22}b_{12}h_{o,t-1}(j), \end{aligned} \tag{22}$$

where we denote

$$H_t(j) = \begin{bmatrix} h_{s,t}(j) & h_{so,t}(j) \\ h_{os,t}(j) & h_{o,t}(j) \end{bmatrix}.$$

Eqs. (20), (21) and (22) reveal how shocks and volatility are transmitted across markets and over scales.

We assume that the random errors ϵ_t are normally distributed and we maximize the following likelihood function:

$$\mathcal{L}(\theta) = -T\ln(2\pi) - \frac{1}{2}\sum_{t=1}^{T}\left\{\ln|H_t(j)| + \epsilon'_t H_t(j)^{-1}\epsilon_t\right\} \tag{23}$$

where θ is the vector of parameters to be estimated and T is the number of observations. Numerical maximization techniques were used in order to maximize this non-linear log likelihood function. Engle and Kroner (1995) proposed several iterations which were performed with simplex algorithm to obtain the initial conditions. The BFGS algorithm[6] was then employed to obtain the final estimate of the variance–covariance matrix with corresponding standard errors.

The number of ARCH and GARCH parameters in GARCH(1,1)–BEKK model is $N(5N + 1)/2$, where N is the number of indices (number of used time series: in our case $N = 2$).

4. Empirical application

4.1. Data description

The data set used in this study consists of daily oil price and daily stock market indices. The data for oil market are daily spot prices for West Texas Intermediate (WTI), which are traded on the domestic spot market at the Cashing, Oklahoma Center. These data were obtained from Energy Information Administration (EIA). The stock market indices are: S&P/TSX (Canada), CAC 40 (France), DAX (Germany), FTSE MIB (Italy), NIKKEI 225 (Japan), FTSE 100 (United Kingdom) and S&P 500 (United States). The series for these indices were obtained from

[5] For further details of this approach see Whitcher and Craigmile (2004) and Whitcher et al. (2005).

[6] BFGS (Broyden–Fletcher–Goldfarb–Shanno) is a quasi-Newton optimization method that uses information about the gradient of the function at the current point to calculate the best direction to look in to find a better point. Using this information, the BFGS algorithm can iteratively calculate a better approximation to the inverse Hessian matrix, which will lead to a better approximation of the minimum value.

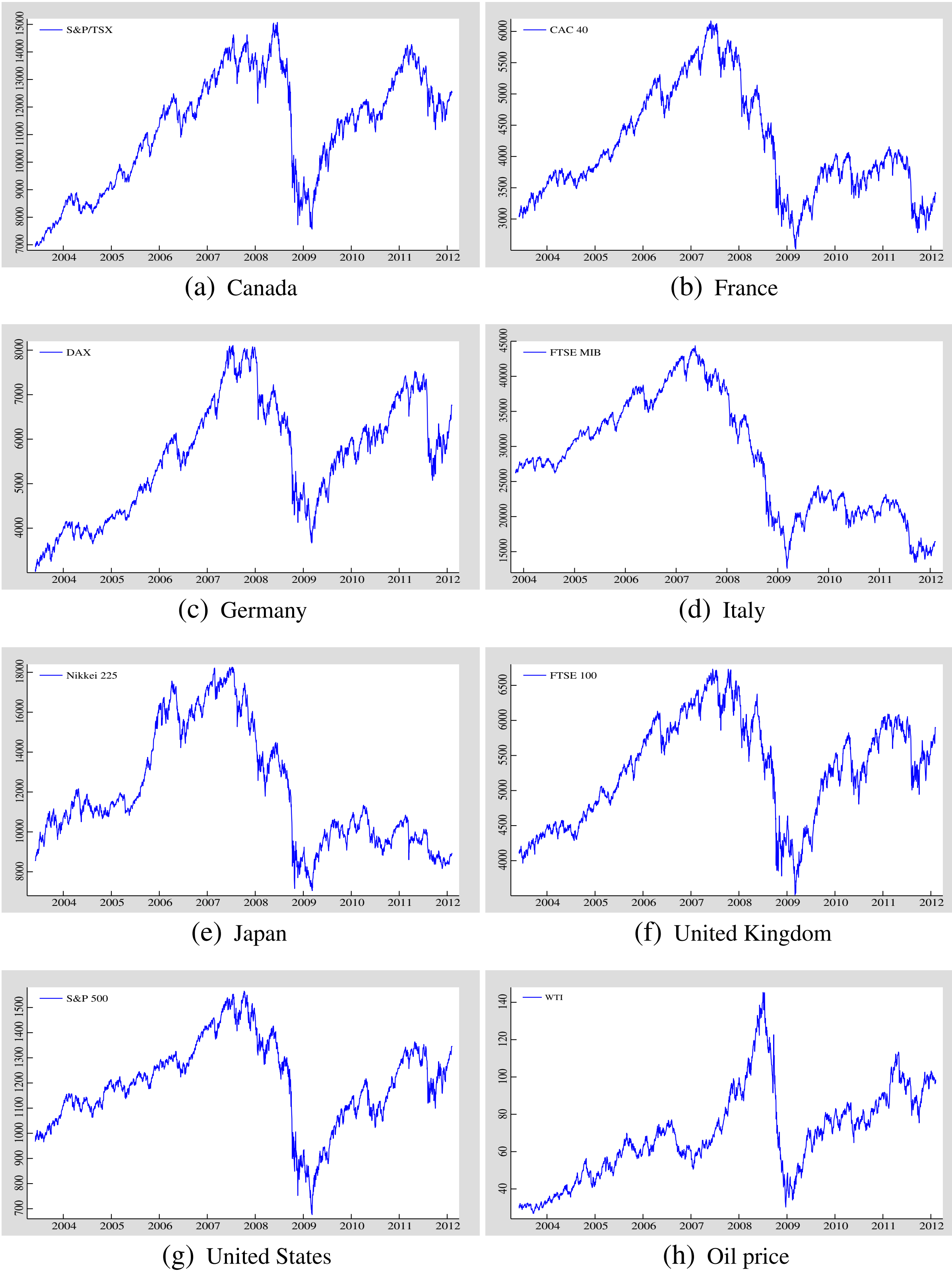

Fig. 1. Time series plots.

Datastream. We have a sample of 2195 daily observations from June 02, 2003 to February 07, 2012, when assets are traded. Fig. 1 plots the oil and stock market prices.

Supplementary Table 1 presents the statistical properties for oil and stock market (G-7 countries) series at each wavelet scale. The results reveal that oil is found to be the best performing asset on risk basis (Std. dev.), it depicts the lowest standard deviation values in all wavelet scales, followed by S&P 500, CAC 40, FTSE 100, DAX, S&P/TSX, Nikkei 225 and FTSE MIB. We also observe that unconditional volatility as represented by standard deviation of wavelet series ($D1$, ..., $D5$)

increases across scales. This is mainly due to the extreme losses that were reported in the aftermath of the crisis, and overall all G-7 countries are severely affected by the currency crisis.

As shown in Supplementary Table 1 all wavelet components ($D1, \ldots, D5$) for oil prices and stock market prices are skewed and leptokurtic. Jarque–Bera (J–B) test statistic, which tests the hypothesis of normal distribution, is consistently rejected at a high significance level. This suggests that wavelet series are not normally distributed.

Our study is based on wavelet series,[7] we decompose the daily oil spot prices and daily stock market indices using the MODWT with Daubechies least asymmetric (LA) wavelet filter of length $L = 8$, (denoted LA(8) wavelet filter).[8] This wavelet filter has been widely used and applied in the financial literature and it has been shown that LA(8) gives the best performance for the wavelet time series decomposition.

Our MODWT decomposition goes up to level $J = 8$, which is given by, $J \le log_2\left[\frac{T}{L-1} + 1\right]$, i.e. T is the length of the given time series and L is the length of the filter (Gencay et al., 2002; Percival and Walden, 2000). Because most information (energy) spreads in lower scales (high frequency fluctuations), and the variances of the wavelet time series at higher scales (lower frequency fluctuations) are too small to make GARCH-type model significant, we use in our empirical study only the first five details ($D_t(1), \ldots, D_t(5)$), which depict more variability.

4.2. Empirical results

In this section we focused on the mean and volatility spillover between the crude oil market (WTI) and seven developed stock markets (G-7 stock markets) based on wavelet series (details) in order to analyze the volatility spillover in different time horizons, i.e., we study the volatility sub-spillover between oil and stock markets. Our application is based on a multivariate GARCH(1,1) model with BEKK parameterizations for each variance equation (Fig. 2).[9] The estimated coefficients of each time scale GARCH(1,1)–BEKK model are reported in Supplementary Tables 2 to 8.

Estimation results based on the two finest wavelet components $D_t(1)$ and $D_t(2)$, which represent short term horizon (high frequency fluctuations), it represents the oil and stock market price variations due to shocks occurring at time scales of 2 to 4 days and of 4 to 8 days (fluctuations of one week), are given respectively by scale 1 and scale 2 (see scale 1 and scale 2 in Supplementary Tables 2–8). Moreover, estimation results based on the third wavelet component $D_t(3)$, which represents the midterm horizon (fluctuations due to shocks occurring from 8 to 16 days or fluctuations occurring in two weeks) are given by scale 3 (see scale 3 at each Supplementary Table: Supplementary Tables 2–8). Estimation results based on the fourth and fifth wavelet components, which represent the long term horizon in our study (fluctuations due to shocks occurring in the periods of 16 to 32 days) (approximatively one month) and of 32 to 64 days (period of one to two months), are given respectively by scale 4 and scale 5 (see scales 4 and 5 in Supplementary Tables 2–8).

4.2.1. Mean equation

We first discuss our findings related to price linkages between WTI crude oil market and each stock market in the G-7 countries. As shown in Supplementary Tables 2–8 and for each wavelet scale, the conditional oil price and the conditional stock prices of each stock market under study are directly affected by its own lagged prices (see significant estimated coefficients ϕ_{11} and ϕ_{22}). Moreover, we remark that the influence of past values (X_{t-1}) depends on scales, at one scale the one period lag of oil or stock market prices affects positively the current period and at the other scale it affects negatively the current period. For instance, the estimated coefficient of WTI oil price in the WTI equation (ϕ_{22}) based on the modeled pair: CAC 40–WTI, is negative and statistically significant at 1% level in scale 1 and it is positive and statistically significant at 1% level in scales 2, 3, 4 and 5. This result is important in distinguishing a positive or negative relationship between current period WTI oil prices and last period WTI oil prices.

The estimation results showed that WTI oil price and stock market prices of each market in the G-7 countries are affected not only by its own lagged term but also by the lagged term of the stock market price/WTI oil price. This leads us to discuss the mean spillovers.

The mean spillovers between WTI oil market and each stock market in the G-7 countries are given by the significance of estimated coefficients ϕ_{12} and ϕ_{21}, i.e. the off-diagonal elements of Φ_1. From (Supplementary Tables 2–8), we can see that the mean linkage between WTI oil market and each stock market in the G-7 countries is unidirectional, bidirectional or there is no linkage. In fact, the price transmission (mean spillovers) varies across wavelet scales. For instance, based on the CAC 40–WTI pairwise, we remark that at scale 4 and scale 5 the estimates reveal bidirectional linkages between WTI oil market and France stock market (see significant estimated coefficients ϕ_{12} and ϕ_{21}). At scale 1, the pairwise estimates reveal unidirectional linkage between WTI oil market and France stock market (see only significant estimated coefficient ϕ_{21}) and the price transmission goes from WTI oil market to France stock market. At scale 2, there exists a unidirectional linkage between WTI oil market and France stock market (see only significant estimated coefficient ϕ_{12}) and the price transmission goes from France stock market to WTI oil market. At scale 3, no statistically significant linkages are found between WTI oil market and France stock market. To see all the mean spillover effects between WTI oil market and each stock market in the G-7 countries over scales refer to Supplementary Table 9.

4.2.2. Variance equation

Our second aim is to capture any spillover effects occurring in the volatilities of the WTI oil market and the G-7 stock markets. The diagonal elements in matrix A capture the own ARCH effects, while the diagonal elements of B capture the own GARCH effects. From Supplementary Tables, 2, 3, 4, 5, 6, 7 and 8 we see that the estimated diagonal parameters a_{11}, a_{22} and b_{11}, b_{22} are all statistically significant at all wavelet scales, indicating that oil volatility and stock market volatility of all countries under study are directly affected by their own shocks, except for oil volatility at scale 4 using the pairwise FTSE 100–WTI which is not affected by its own shocks (see insignificant estimated coefficient of a_{22}). Based on its own shocks, we remark that for all pairwise markets higher levels of oil or stock market price volatility in the past are associated with higher conditional volatility in the current period (see positive and significant coefficients b_{11} and b_{22}) at all wavelet scales. Furthermore, as it can be shown in Supplementary Table 6 at scale 4, the WTI oil and NIKKEI 225 price volatilities are negatively associated with their own news (negative and significant coefficient a_{11} and a_{22}), indicating that a positive shock in the past is associated with a negative one in the current period. We also remark the same results in the pair S&P/TSX–WTI at scale 5.

The off-diagonal elements of matrices A and B capture the cross-market effects over different scales such as volatility spillovers among thirty five pairs. From the estimated coefficients a_{12}, a_{21} and b_{12}, b_{21} we remark that the connection between each stock market of the G-7

[7] The wavelet transform intelligently adapts itself to capture features across a wide range of frequencies and thus has the ability to capture events that are local in time. This makes the wavelet transform an ideal tool for studying non-stationary time series. To measure the local regularity of a signal, it is not so important to use a wavelet with a narrow frequency support, but vanishing moments are crucial. If the wavelet has M vanishing moments, then we show that the wavelet transform can be interpreted as a multiscale differential operator of order M, i.e. The first M moments of the wavelet coefficients are zero: $\int_{-\infty}^{+\infty} t^r \psi(t) dt = 0$, $r = 1, \ldots, M$. This yields a first relation between the differentiability of the time series and its wavelet transform decay at fine scales. Following Gencay et al. (2002) the Daubechies wavelet filter guarantees that the resulting DWT coefficients be stationary and thus protect ourselves from problems caused by a non-stationary time series.

[8] For more details about this wavelet filter see Daubechies (1992) and Gencay et al. (2002).

[9] We cannot fit a GARCH model to S5 series (see Fig. 2) because it is very smooth ($Var(S_5) \simeq 0$).

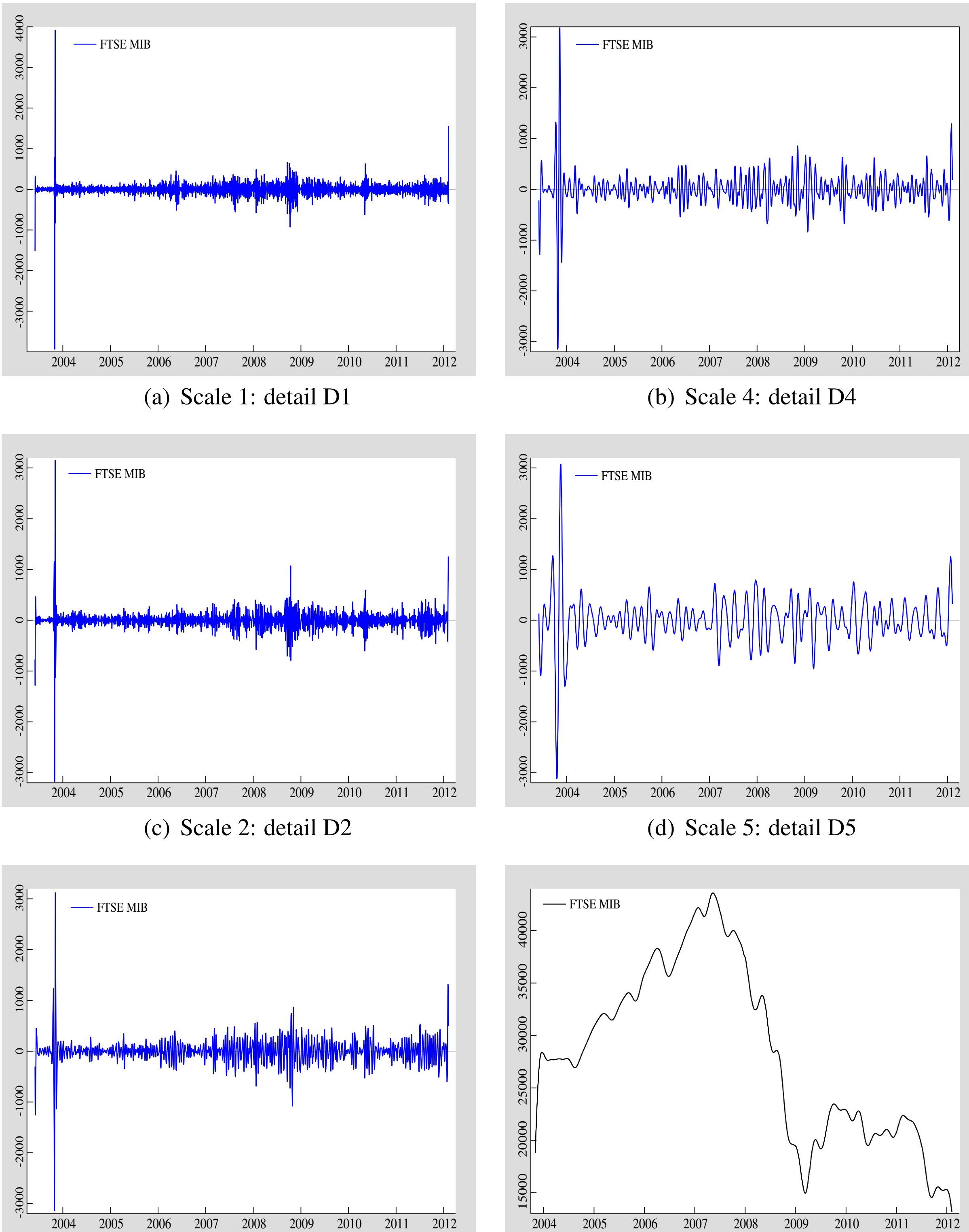

Fig. 2. Wavelet decomposition of the FTSE MIB (Italy) stock market index (as an example). Each detail represents the contribution of fluctuations of a specific time scale to the FTSE MIB price variations, while the smooth S5 represents its trend. The various decomposition levels we obtain correspond to time scales: *D*1 (2 to 4 days), *D*2 (4 to 8 days), *D*3 (8 to 16 days), *D*4 (16 to 32 days) and *D*5 (32 to 64 days).

countries and WTI oil market varies across scales. Sometimes we see unidirectional volatility linkages, other times bidirectional ones and sometimes we see no volatility linkages between markets under study. For instance, the volatility transmission from France stock market to WTI oil market is bidirectional at scale 3 (medium term horizon) and scale 5 (long term horizon) (see the significant estimated coefficients b_{12} and b_{21}), while this volatility transmission is unidirectional at scale 2 (short term horizon), from oil market to France stock market (see significance of only the estimated coefficient b_{21}). Also, we remark no volatility transmission from France stock market to WTI oil market or the inverse at scale 1 (see insignificant estimated coefficients b_{12} and b_{21}). For more details about all volatility transmission between WTI oil market and each stock market of the G-7 countries see the summary in Supplementary Table 9.

4.2.3. Wavelet coherence and phase

Wavelet coherence measures how much two time series co-vary and capture both time and frequency-varying features.[10] If the coherence is close to one, the time series show a strong common behavior. If it is close to zero the time series do not behave in a coherent way.

Since the wavelet coherence coefficient is squared, and we cannot distinguish between negative and positive correlations, we use the wavelet phase differences which indicate delays in the oscillation between the two examined time series. The two time series move together on a particular scale if values of wavelet phase deference range to zero.

The series move in-phase if the wavelet phase $\theta_{XY}(\lambda_j, t) \in [0, \frac{\pi}{2}]$ and the time series X is led by Y time series. Conversely, if the wavelet phase $\theta_{XY}(\lambda_j, t) \in [-\frac{\pi}{2}, 0]$ then X is leading. We have an anti-phase relation (negative correlation between the two time series) if we have a wavelet phase of π (or $-\pi$).

Looking at the wavelet coherence (see Supplementary Table 10: the output from the Hilbert wavelet pairs for stock market vs oil market in terms of mean squared coherence), we notice that the co-movement between stock and oil markets increases over scales. However, we observe that for fluctuations with a duration of less than two weeks (scale 3: 8–16 days), which represents the short and mid-terms, the co-movement is weaker: the wavelet coherence values do not exceed 57%. For instance, the wavelet coherence between CAC 40 and WTI prices at scale 2 is 32%. We can also conclude that oil market presents, in general, a strong co-movement at long term (scale 4 and scale 5) with other stock markets: The wavelet coherence values are more than 70%.

The information on the delay between the oscillations of two time series i.e. lead–lag relationship is provided by phase difference. We calculate the mean phase for each scale (see Fig. 3). As it can be shown in Fig. 3, for the phase difference between S&P/TSX stock and WTI stock phases vary from $[0, \pi/2]$ for all scales. This means that the WTI market leads the S&P/TSX market and the WTI prices are in phase. For the phase difference between FTSE 100–WTI market pairs and DAX–WTI market pairs (from scale 2 to scale 5), it has been found that WTI prices are in-phase or leading FTSE 100 and DAX prices at low frequencies (mid and long term), whereas for higher frequencies (short term: scale 1), it has been found that FTSE 100–WTI market pairs and DAX–WTI market pairs are out of phase. For the Nikkei 225–WTI market pairs, we found that WTI market leads in all scales except in scale 3 the Nikkei 225 market leading the WTI one. We also conclude that at scale 1 (short term) the FTSE MIB, DAX and FTSE 100 stock markets lead the WTI one.

In summary, we find evidence of mean spillovers between WTI oil market and each stock market in the G-7 countries. These linkages appear to be less important than linkages in the volatility spillovers. However, our empirical results suggest that wavelet-based multiresolution analysis joined with multivariate GARCH models has demonstrated its capability to understand the multiscale nature of oil and stock market data. According to wavelet-based GARCH–BEKK approach, we decompose the spillover effect into five sub-spillovers on the five time scales used in the study. We conclude that mean and volatility spillovers depend on wavelet scales.

[10] We use wavelet coherence to measure the extent to which two time series move together over time and across frequencies.

4.3. Hedging performance

In this section, we first consider a portfolio composed of oil and stock market index for which we attempt to minimize the risk without lowering expected returns. We define the optimal weight of holdings of the two assets (oil and stock market index) at each wavelet scale by applying the methods of Kroner and Ng (1998) and Hammoudeh et al. (2010), which is given by:

$$w_{os,t}(j) = \frac{h_{s,t}(j) - h_{os,t}(j)}{h_{o,t}(j) - 2h_{os,t}(j) + h_{s,t}(j)} \tag{24}$$

and

$$w_{os,t}(j) = \begin{cases} 0, & \text{if} \quad w_{os,t}(j) < 0 \\ w_{os,t}(j), & \text{if} \quad 0 \le w_{os,t}(j) \le 1 \\ 1, & \text{if} \quad w_{os,t}(j) > 1 \end{cases} \tag{25}$$

where $w_{os,t}(j)$ is the weight of oil in one-dollar two-assets portfolio at time t and at wavelet scale j, $h_{os,t}(j)$ is the conditional covariance between oil and stock market prices at time t and at wavelet scale j and $h_{o,t}(j)$ and $h_{s,t}(j)$ are the conditional variances at time t and wavelet scale j for oil and stock market prices respectively. The optimal weight of the stock market index is equal to $1 - w_{os,t}(j)$.

The average values of the optimal portfolio weights $w_{os,t}(j)$ for the G-7 countries and computed from the GARCH(1,1)–BEKK model are reported in column 2 in Supplementary Table 11. As shown in optimal portfolio weight values, we remark that at all scales (scale 1 to scale 5), the average portfolio weights are high and vary from 91% to 99%. For instance, the average value of $w_{so,t}(5)$ of a portfolio composed of S&P 500 stock index and WTI computed from the GARCH(1,1)–BEKK is 92% implying that investors (long term traders in this case because this value corresponds to scale 5) should have more oil (WTI) than S&P 500 stock index in their portfolio in order to minimize the risk without lowering expected returns. Overall, our results of higher optimal portfolio weights suggest that investors (short term, midterm and long term traders) should have more oil (WTI) than stock index (stock indices of the G-7 countries) in their portfolios to minimize risk without lowering expected returns.

Second, we focus on computing the risk minimizing hedge ratios using the GARCH(1,1)–BEKK model and based on the Kroner and Sultan's methods. Kroner and Sultan (1993) show that the risk of a portfolio of two assets (oil and stock market index in our study) is minimal if a long position of one dollar in the oil market can be hedged by a short position of $\beta^*_{os,t}(j)$ dollars in the stock market index, that is:

$$\beta^*_{os,t}(j) = \frac{h_{os,t}(j)}{h_{s,t}(j)} \tag{26}$$

where $\beta^*_{os,t}(j)$ is the risk minimizing hedge ratio for oil and stock market prices, $h_{os,t}(j)$ is the conditional covariance between oil and stock market prices at time t and at wavelet scale j and $h_{s,t}(j)$ is the conditional variance of stock market index.

The average values of the hedge ratios computed from the results of the GARCH(1,1)–BEKK model are reported in column 3 and column 5 of Supplementary Table 11. As shown in Supplementary Table 11 and Fig. 4, the wavelet hedge ratios are low in general for all portfolios indicating that hedging effectiveness involving oil and stock markets in the G-7 countries is quite good. Based on the GARCH(1,1)–BEKK model, we also remark that at scale 1 the lowest average wavelet hedge ratio value is of the S&P 500–WTI portfolio, followed by NIKKEI 225–WTI portfolio, FTSE MIB–WTI portfolio, FTSE 100–WTI portfolio, DAX–WTI portfolio, CAC 40–WTI portfolio and S&P/TSX–WTI portfolio.

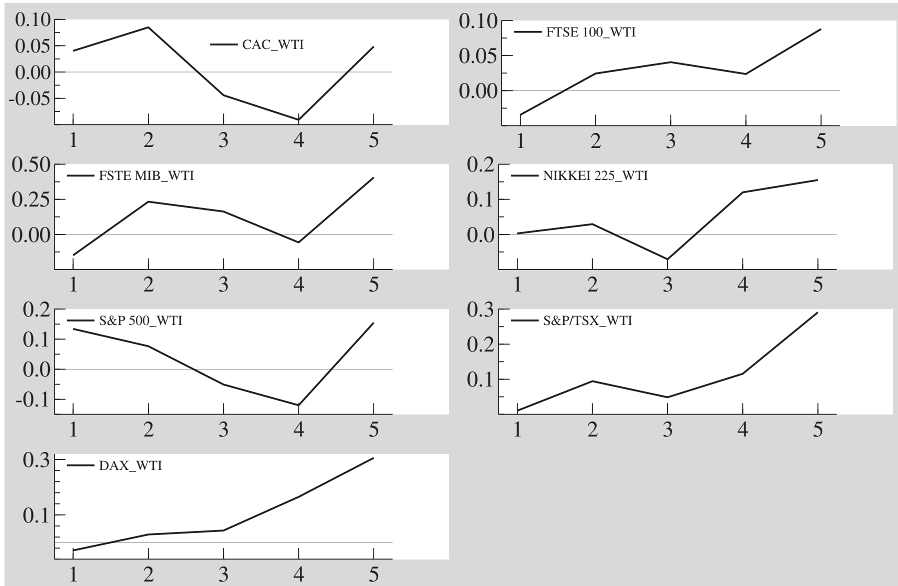

Fig. 3. Wavelet phase difference between the oil market and stock markets.

At scale 2 (corresponds to short term traders), we remark that the NIKKEI 225–WTI portfolio has the lowest average wavelet hedge ratio value, followed by FTSE MIB–WTI portfolio, FTSE 100–WTI portfolio, DAX–WTI portfolio, CAC 40–WTI portfolio, S&P/TSX–WTI portfolio and S&P/TSX–WTI portfolio which has the highest hedge ratio average value. According to these findings, we suggest that short term traders (risk lover traders) can invest in the portfolio which depicts minimum hedge ratio value, that is the S&P 500–WTI portfolio (the hedge ratio is equal to −0.27%) for 2–4 days period and they can invest in NIKKEI 225–WTI portfolio (the hedge ratio is equal to 0.049%) for 4–8 days period. At scale 5 (corresponds to long term horizon), the results show that the lowest wavelet hedge ratio value is of S&P 500–WTI portfolio, followed by CAC 40–WTI portfolio, NIKKEI 225–WTI portfolio, FTSE 100–WTI portfolio, FTSE MIB–WTI portfolio, DAX–WTI portfolio and finally the S&P/TSX–WTI portfolio. Moreover, the results show that at scale 4 (corresponds to long term horizon) the DAX–WTI portfolio depicts the lowest average wavelet hedge ratio value and the S&P 500–WTI portfolio the highest one. As a consequence, we can say that long term traders or fundamentals (risk aversion traders) can invest in S&P 500–WTI portfolio at 32–64 days period, approximately one up to two months and also one trader can invest in DAX–WTI portfolio at a 16–32 days period, approximately two weeks up to one month.

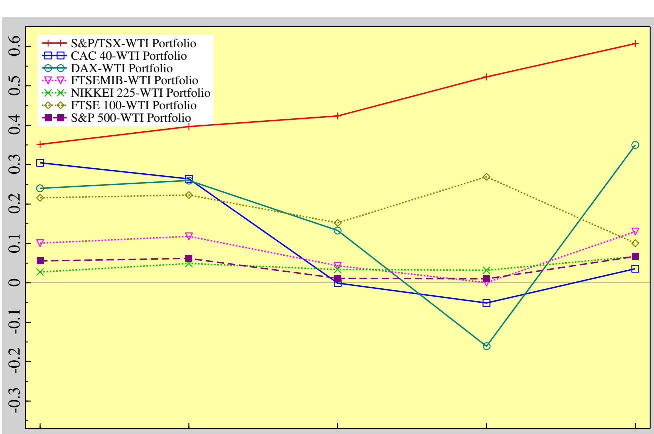

Fig. 4. Optimal hedge ratios $\beta^*_{os,t}(j)$ over scales based on GARCH–BEKK model.

Overall, our results based on wavelet hedge ratio provide important and useful information for risk management and for optimal portfolio allocation decisions, which is the important objective of financial market participants, that is to understand the risk over time and across scales in order to make their strategy decisions.

5. Conclusion

This paper investigates the spillover effects of volatility and shocks between oil prices and the G-7 stock markets using multivariate GARCH models and wavelet analysis. The data set consists of daily oil and stock (G-7 stock market indices) prices, i.e. wavelet details of oil and stock prices. The main objective of this work was to examine the mean and volatility spillover effects at different time horizons. Hence, we combine a GARCH(1,1)–BEKK model and wavelet multiresolution analysis to study this phenomenon. A bivariate GARCH(1,1)–BEKK model was joined with MODWT filter to capture a broad range of possible spillover effects in mean and variances of level prices at various time horizons, i.e. wavelet-based GARCH–BEKK approach.

Generally, empirical results provide strong evidence of time-varying volatility in all markets under study. However, our proposed approach show that oil price and stock market prices are directly affected by their own news and volatilities and indirectly affected by the volatilities of other prices and wavelet scale. The results show also, that mean and volatility spillover effects were decomposed into many sub-spillovers on different time scales according to heterogeneous investors and market participants.

Moreover, in order to take optimal portfolio allocation decisions according to the behaviors of different groups of investors and market traders we compute the hedging ratios at different time horizons. Results show that hedging ratios and optimal weights vary across scales. According to optimal weights we remark that investors and financial market participants should hold less stocks than crude oil. This may be due to the fact that stock prices of the G-7 markets are more volatile than WTI oil prices.

We summarize our results as follows: the oil and stock market volatilities are affected by their own volatilities (risks) at each wavelet scale and the spillover effect was decomposed into many sub-spillovers. Furthermore, hedging ratios vary across scales.

The proposed wavelet-based multivariate GARCH models allows us to analyze the volatility spillover effects at different scales using level time series and to capture the multiscale behavior of WTI oil and stock market (G-7 stock market indices) prices and the hedging ratios.

Acknowledgment

We would like to thank the referees for their constructive remarks and careful reading.

Appendix A. Supplementary data

Supplementary data to this article can be found online at http://dx.doi.org/10.1016/j.eneco.2015.03.023.

References

Arouri, M.E.H., Nguyen, D.K., 2010. Oil prices, stock markets and portfolio investment: evidence from sector analysis in Europe over the last decade. Energy Policy 38, 4528–4539.

Arouri, M.E.H., Lahiani, A., Nguyen, D.K., 2011. Return and volatility transmission between world oil prices and stock markets of the GCC countries. Econ. Model. 28, 1815–1825.

Awartani, B., Maghyereh, A.I., 2013. Dynamic spillovers between oil and stock markets in the Gulf Cooperation Council countries. Energy Econ. 36, 28–42.

Baba, Y., Engle, R.F., Kraft, D.F., Kroner, K.F., 1989. Multivariate simultaneous generalized ARCH. University of California at San. Diego, Mimeo.

Boubaker, H., Boutahar, M., 2011. A wavelet-based approach for modelling exchange rates. Stat. Methods Appl. 20, 201–220.

Coifman, R.R., Donoho, D.L., 1995. Translation-invariant de-noising. Lecture Notes in Statistics: Wavelet and Statisticspp. 125–150.

Daubechies, I., 1992. Ten Lectures on Wavelets. Society for Industrial and Applied Mathematics, Philadelphia, PA, USA.

Du, X., Yu, C.L., Hayes, D.J., 2011. Speculation and volatility spillover in the crude oil and agricultural commodity markets: a Bayesian analysis. Energy Econ. 33, 497–503.

Engle, R.F., Kroner, K.F., 1995. Multivariate simultaneous generalized arch. Econ. Theory 11, 122–150.

Fernandez, V., 2006. The CAPM and value at risk at different time-scales. Int. Rev. Financ. Anal. 15, 203–219.

Fernández-Macho, J., 2012. Wavelet multiple correlation and cross-correlation: a multiscale analysis of Eurozone stock markets. Phys. A Stat. Mech. Appl. 391, 1097–1104.

Filis, G., Degiannakis, S., Floros, C., 2011. Dynamic correlation between stock market and oil prices: the case of oil-importing and oil-exporting countries. Int. Rev. Financ. Anal. 20, 152–164.

Gallegati, M., 2010. A wavelet-based approach to test for financial market contagion. Comput. Stat. Data Anal. 56 (11), 3491–3497.

Gencay, R., Selcuk, F., Whitcher, B., 2002. An Introduction to Wavelets and Other Filtering Methods in Finance and Economics. Academic-Press.

Gencay, R., Selcuk, F., Whitcher, B., 2005. Multiscale systematic risk. J. Int. Money Financ. 24, 55–70.

Genest, C., Masiello, E., Tribouley, K., 2009. Estimating copula densities through wavelets. Insur. Math. Econ. 44, 170–181.

Hammoudeh, S.M., Yuan, Y., McAleer, M., Thompson, M.A., 2010. Precious metals—exchange rate volatility transmissions and hedging strategies. Int. Rev. Econ. Financ. 19, 633–647.

He, K., Xie, C., Chen, S., Lai, K.K., 2009. Estimating var in crude oil market: a novel multi-scale non-linear ensemble approach incorporating wavelet analysis and neural network. Neurocomputing 72, 3428–3438.

He, K., Lai, K.K., Yen, J., 2012. Ensemble forecasting of value at risk via multiresolution analysis based methodology in metals markets. Expert Syst. Appl. 39, 4258–4267.

Jammazi, R., 2012. Oil shock transmission to stock market returns: wavelet-multivariate Markov switching GARCH approach. Energy 37, 430–454.

Kim, S., In, F., 2007. On the relationship between changes in stock prices and bond yields in the G7 countries: wavelet analysis. J. Int. Financ. Mark. Inst. Money 17, 167–179.

Kroner, K.F., Ng, V.K., 1998. Modeling asymmetric comovements of asset returns. Rev. Financ. Stud. 11, 817–844.

Kroner, K.F., Sultan, J., 1993. Time varying distributions and dynamic hedging with foreign currency futures. J. Financ. Quant. Anal. 28, 535–551.

Kumar, S., Managi, S., Matsuda, A., 2012. Stock prices of clean energy firms, oil and carbon markets: a vector autoregressive analysis. Energy Econ. 34, 215–226.

Malik, F., Hammoudeh, S., 2007. Shock and volatility transmission in the oil, us and Gulf equity markets. Int. Rev. Econ. Financ. 16, 357–368.

Mallat, S.G., 1989. A theory for multiresolution signal decomposition: the wavelet representation. IEEE Trans. Pattern Anal. Mach. Intell. 11, 674–693.

Masih, M., Alzahrani, M., Al Titi, O., 2010. Systematic risk and time scales: new evidence from an application of wavelet approach to the emerging Gulf stock markets. Int. Rev. Financ. Anal. 19, 10–18.

Nason, G.P., Silverman, B.W., 1995. The Stationary Wavelet Transform and Some Statistical Applications. Springer-Verlag.

Percival, D.B., Walden, A.T., 2000. Wavelet Methods for Time Series Analysis. Cambridge University Press.

Pesquet, J.C., Krim, H., Carfantan, H., 1996. Time invariant orthonormal wavelet representations. IEEE Trans. Signal Process. 44, 1964–1970.

Rua, A., 2010. Measuring comovement in the time–frequency space. J. Macroecon. 32, 685–691.

Rua, A., Nunes, L.C., 2009. International comovement of stock market returns: a wavelet analysis. J. Empir. Financ. 16, 632–639.

Sadorsky, P., 2012. Correlations and volatility spillovers between oil prices and the stock prices of clean energy and technology companies. Energy Econ. 34, 248–255.

Singh, P., Kumar, B., Pandey, A., 2010. Price and volatility spillovers across North American, European and Asian stock markets. Int. Rev. Financ. Anal. 19, 55–64.

Sun, E.W., Meinl, T., 2012. A new wavelet-based denoising algorithm for high-frequency financial data mining. Eur. J. Oper. Res. 217, 589–599.

Vo, M., 2011. Oil and stock market volatility: a multivariate stochastic volatility perspective. Energy Econ. 33, 956–965.

Whitcher, B.J., Craigmile, P.F., 2004. Multivariate spectral analysis using Hilbert wavelet pairs. Int. J. Wavelets Multiresolution Inf. Process. 2, 567–587.

Whitcher, B.J., Craigmile, P.F., Brown, P., 2005. Time-varying spectral analysis in neurophysiological time series using Hilbert wavelet pairs. Signal Process. 85, 2065–2081.

Zhang, Y.J., Wei, Y.M., 2010. The crude oil market and the gold market: evidence for cointegration, causality and price discovery. Resour. Policy 35, 168–177.