Two numerical test cases of diffusion equations

Porquerolles; June, 22th-24th 2009.

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Equation and data

$$\begin{cases} -\mathrm{div}(\Lambda \nabla u) = f & \text{in } \Omega = (0,1)^2, \\ u = 0 & \text{on } \partial\Omega \end{cases}$$

Diffusion:

$$\Lambda(x,y) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \text{ if } x < 0.5,$$

$$\Lambda(x,y) = \begin{pmatrix} D & 0 \\ 0 & 1 \end{pmatrix} \text{ if } x > 0.5$$

with D **large**.

Exact solution:

$$u(x,y) = x(1-x)y(1-y).$$

Minimum $= 0$; maximum $=$ 6.25E-2.

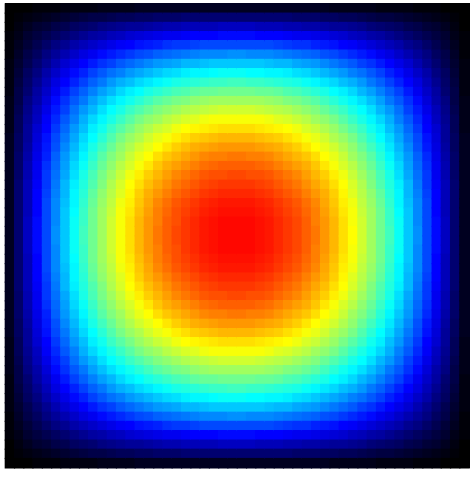
HMM scheme, cartesian grid

HMM = Mimetic Finite Difference = Hybrid Finite Volume = Mixed Finite Volume

Grid: 50×50 regular cartesian grid (2500 cells, h=2.83E-2).

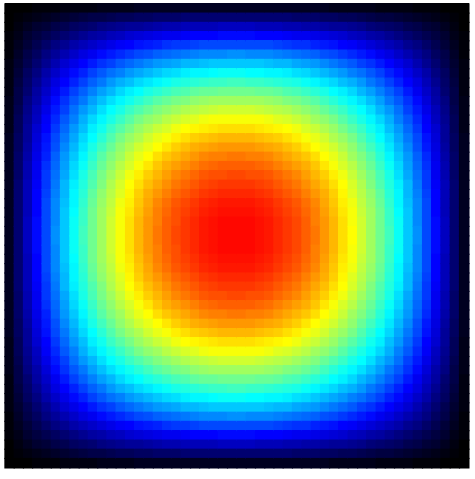
Results:

- D =1E-3



L^2 errors: 5.6E-4 (u), 7.3E-4 (∇u)
min=9.9E-5, max=6.247E-2

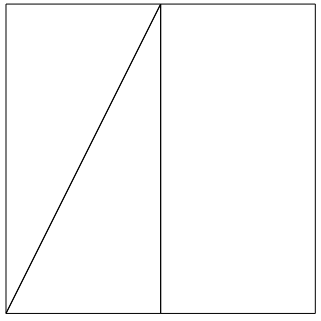
- D =1E-4



5.6E-4 (u), 8.6E-4 (∇u)
min=9.9E-5, max=6.247E-2

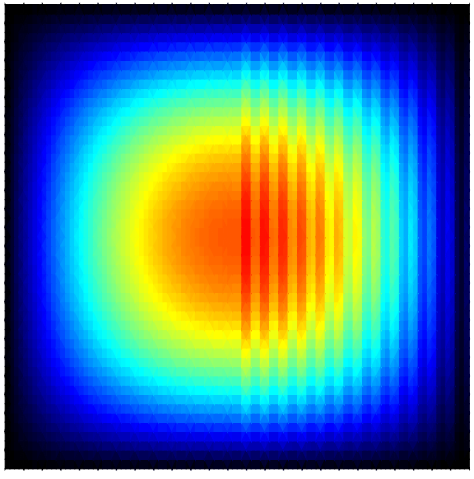
HMM scheme, non-aligned

Grid: 50×50 reproductions by symmetry of this pattern (7500 cells, h=2.24E-2).



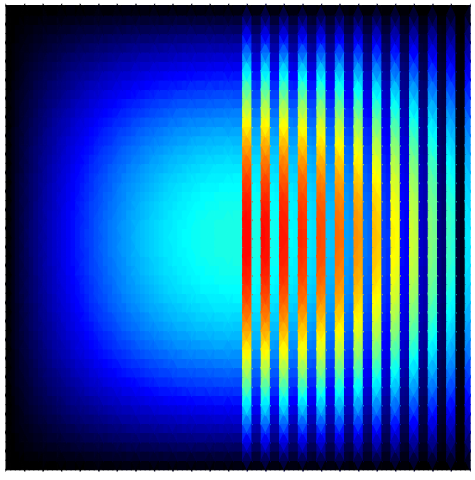
Results:

- D =1E-3



L^2 errors: 6.1E-2 (u), 4E0 (∇u)
min=4.9E-5, max=6.8E-2

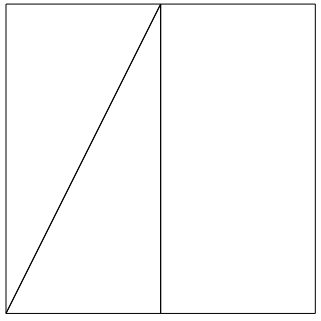
- D =1E-4



6.1E-1 (u), 4E1 (∇u)
min=4.9E-5, max=0.12

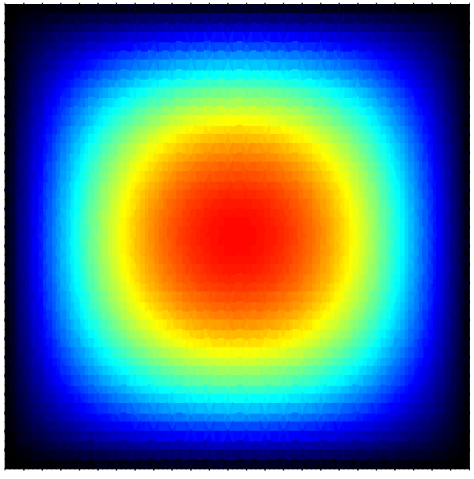
G-scheme, non-aligned

Grid: 50×50 reproductions by symmetry of this pattern (7500 cells, h=2.24E-2).



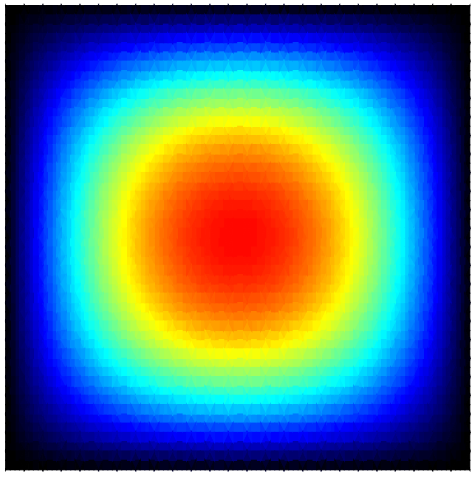
Results:

- D =1E-3



L^2 errors: 1.4e-3 (u), 1.9E-2 (∇u)
min=2.45E-6, max=6.248E-2

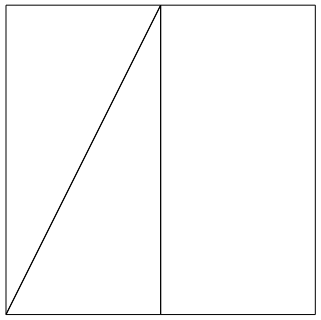
- D =1E-4



1.4E-3 (u), 3E-2 (∇u)
min=2.45E-6, max=6.248E-2

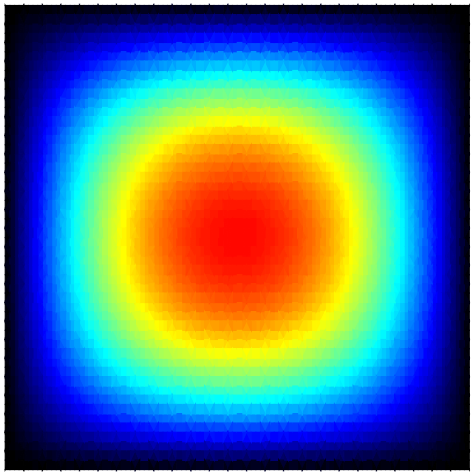
G-scheme, non-aligned

Grid: 50×50 **two different** reproductions of this pattern (7500 cells, h=2.24E-2).



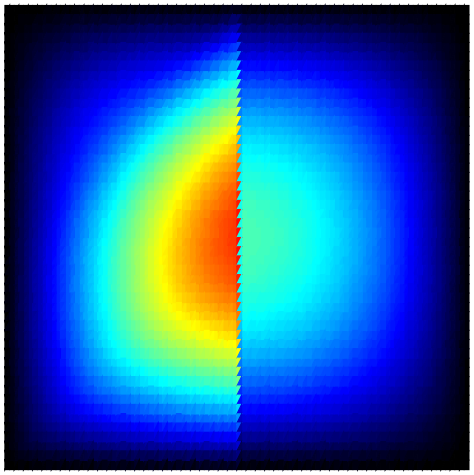
Results: with D =1E-4

Reproduction by symmetry



L^2 errors: 1.4e-3 (u), 3E-2 (∇u)
min=2.45E-6, max=6.248E-2

Reproduction by translation



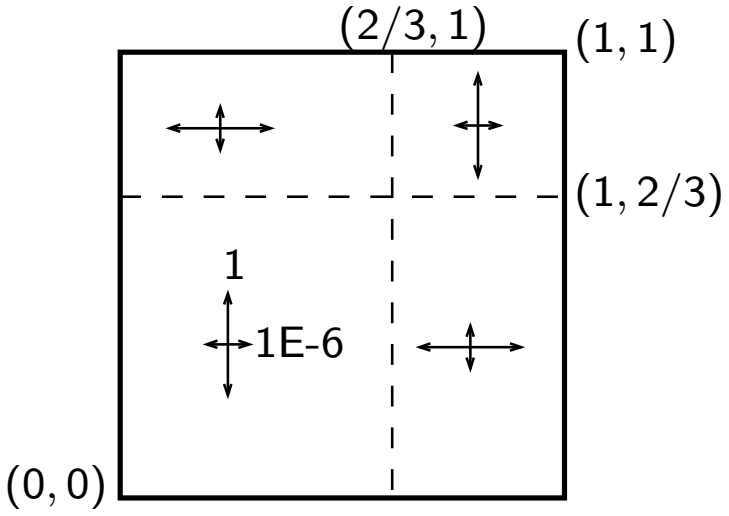
3.5E-1 (u), 2.8E0 (∇u)
min=4.9E-5, max=1.1E-1

Contents

Equation and data

$$\begin{cases} -\mathrm{div}(\Lambda \nabla u) + \mathrm{div}(Vu) + u = f & \text{in } \Omega = (0,1)^2, \\ u = 0 & \text{on } \partial\Omega \end{cases}$$

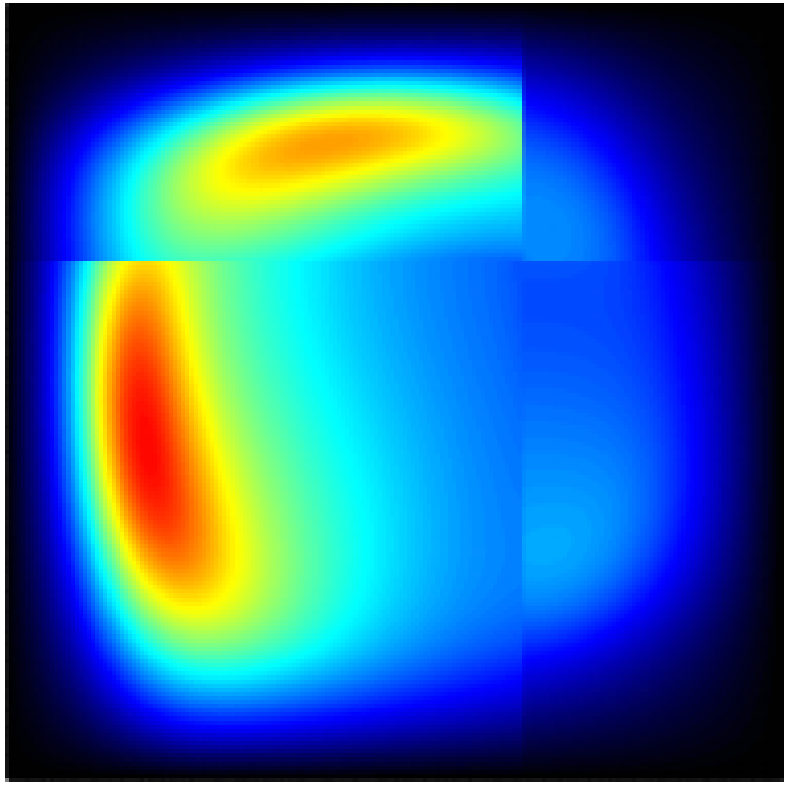
Diffusion: Λ is diagonal with coefficients:



Convection: strong and rotating:
$V(x,y) = (40x(2y-1)(x-1), -40y(2x-1)(y-1))$

Source term: volcano: $f(x,y) = 10^{-2}\exp(-(r-0.35)^2/0.005)$
with $r = \mathrm{dist}((x,y),(0.5,0.5))$.

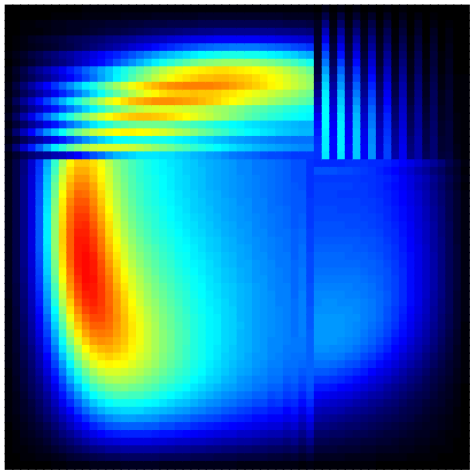
“Exact” solution



min=0, max≈6.7E-4

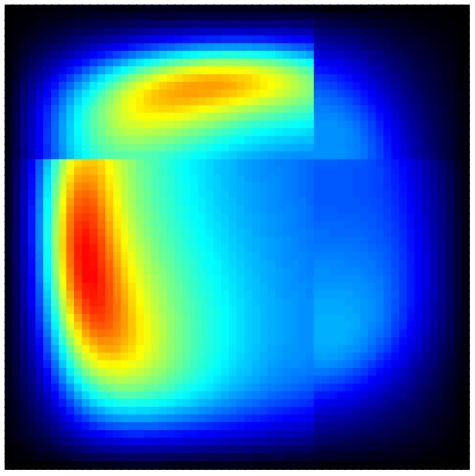
HMM scheme, 60×60 cartesian grid ($h = 2.36\mathrm{E}-2$)

Centered convection



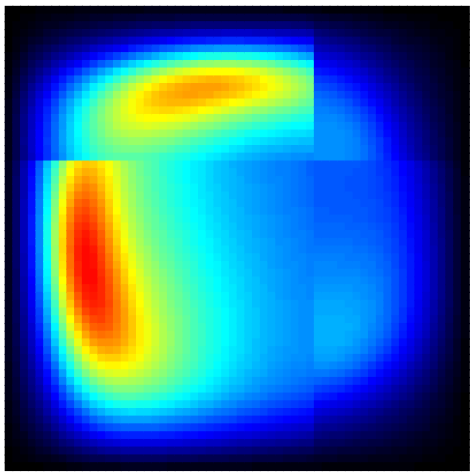
min=1.8E-7, max=6.97E-4

Cell upwind



min=3.7E-7, max=6.41E-4

Edge upwind

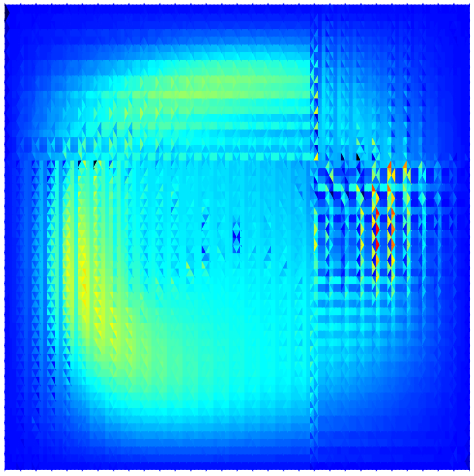


min=3.7E-7, max=6.45E-4

Recall:
true min=0,
true max≈6.7E-4

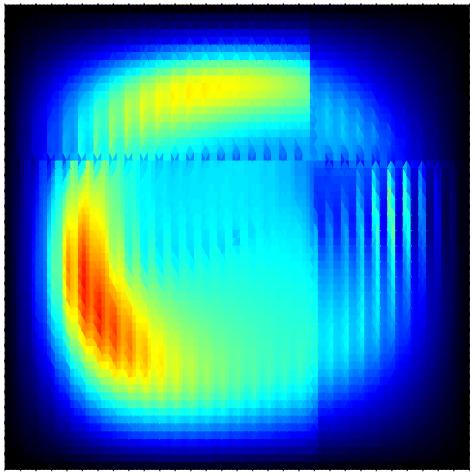
HMM scheme, 60×60 previous patterns ($h = 1.86\mathrm{E}-2$)

Centered convection



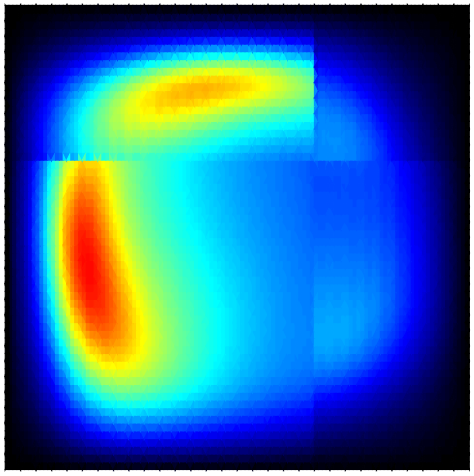
min=-4.7E-4, max=1.4E-3

Cell upwind



min=-1.55E-7, max=7.9E-4

Edge upwind



min=3.3E-8, max=6.62E-4

Recall:
true min=0,
true max≈6.7E-4