Sine-Gordon Model, Interface Models, and Coulomb gas in 2D - an overview

Speaker: Roland Bauerschmidt [1]

Abstract: I will give an overview of selected results and open questions for following related models: the Sine-Gordon model, 2D interface models with Z-symmetry such as the Discrete Gaussian and SOS model, and the neutral 2D Coulomb gas. In particular, I will explain the relation between these models, discuss the difference between the ultraviolet and infrared regimes, and describe some of the main open questions.

[1] University of Cambridge

Percolation and Gaussian fields

Speaker: Vincent Beffara and Vincent Gayet [2]

Abstract: Let f be a centred smooth Gaussian field on the real plane, whose law is invariant under the planar symmetries. It is very natural to see it as a percolation model and ask whether there are large or unbounded connected components of $f > a$ or $f = a$ for various values of the level a. As in the case of Bernoulli percolation, because of symmetry, any square is crossed horizontally by a positive component, that is a connected component of $f > 0$, with probability 1/2. We will show that, under reasonable assumptions on the law of the field, the random region $f > 0$ shares many crucial features with critical percolation, namely that it satisfies Russo-Seymour-Welsh type estimates (large rectangles with fixed shape are crossed with a uniformly positive probability) and that none of its connected components is unbounded. We will then present a related result by Alejandro Rivera and Hugo Vanneuville stating that the level $a = 0$ is critical, in the sense that for any $a < 0$, $f > a$ has almost surely exactly one unbounded component.

[2]Grenoble University

Schur processes and free fermions

Speaker: Jérémie Bouttier[3]

Abstract: The purpose of these lectures is to provide a gentle introduction to Schur processes, which are central models in integrable probability. For pedagogical purposes, we will mostly consider the simplest case, which is the Plancherel measure on integer partitions. It arises in connection with the famous Ulam-Hammersley problem of finding the asymptotic distribution of the length of a longest increasing subsequence of a uniform random permutation.

In the first lecture, we will introduce free fermions and see their connection with the Plancherel measure (which may be seen as a coherent state of a quantum harmonic oscillator). The goal is to prove that the Plancherel measure is closely related with a certain determinantal point process called the discrete Bessel process.

In the second lecture, we will discuss the bulk and edge asymptotics of the Plancherel measure. The limiting determinantal point processes are respectively the discrete sine process and the Airy process. In particular, we shall arrive at a proof of the Baik-Deift-Johansson theorem stating that the fluctuations of the length of a longest increasing subsequence of a uniform random permutation are asymptotically governed by the Tracy-Widom GUE distribution.

In the third lecture, we will discuss the periodic Schur process (via its simplest instance, the cylindric Plancherel measure), which has been recently related to finite temperature ensembles of free fermions. We will see how to generalize the approach of the previous lectures to this setting.

(The material for these lectures comes mostly from papers of Okounkov, Borodin, Olshanski, Johansson, and Reshetikhin. The last lecture is based on a joint work with Dan Betea.)

[3]IPhT Saclay

S- (and t-) embeddings of planar graphs: revealing the conformal structure in the Ising (and dimer) models

Speaker: Dmitry Chelkak[4]

Abstract:

The first part of the talk is devoted to our recent (still unpublished) results on the conformal invariance of the critical planar Ising model *beyond* the standard setup of Z-invariant weights on isoradial graphs. In particular, this includes the convergence of interfaces to SLE(16/3) and dipolar SLE(3) curves on all *periodic* graphs equipped with the critical Ising model. The key role in the proof is played by special embeddings of planar weighted graphs into the complex plane, the so-called *s-embeddings* [Section 5, arXiv:1712.04192], which eventually allow us to prove the convergence of Ising fermionic observables to holomorphic functions. Though the results available at the moment are limited to the periodic case, the techniques seem to be robust enough and we believe that this framework should be relevant in other (deterministic or random) setups when dealing with a (conjecturally) critical Ising model on a planar graph.

In the second part of the talk we plan to discuss a related construction for the bipartite dimer model (which we call *t-embeddings*), developed by Kenyon, Lam, Ramassamy and Russkikh [arXiv:1810.05616] in connection with circle patterns and cluster algebras and, independently, in our joint work with Laslier and Russkikh (in preparation) in the context of discrete complex analysis techniques. (In fact, s-embeddings can be viewed as a particular case of t-embeddings via the 'exact bosonization' correspondence due to Dubédat and de Tiliére, see [Section 7, arXiv:1810.05616].) We discuss the notion of t-holomorphic functions – an analogue of s-holomorphic ones available in the Ising model context – which seems to be the right way of thinking about the convergence of dimer observables to harmonic limits. Finally, we formulate a conjecture on the so-called *perfect t-embeddings* of finite bipartite graphs onto the unit disc, which we believe to be right approximations to the conformal structure arising in the scaling limit of the dimer model (in absence of frozen regions).

[4]DMA ÉNS, 45 rue dUlm, 75005 Paris

On the circle, Kahane's Gaussian Multiplicative Chaos and Circular Random Matrices match

Speaker: Reda Chhaibi[5]

Collaborators:
J. Najnudel [6]

Abstract: In this talk, I would like to advertise an equality between two objects from very different areas of mathematical physics. This bridges the Gaussian Multiplicative Chaos, which plays an important role in certain conformal field theories, and a reference model in random matrices.

On the one hand, in 1985, J.P Kahane introduced a random measure called the Gaussian Multiplicative Chaos (GMC). Morally, this is the measure whose Radon-Nikodym derivative w.r.t to Lebesgue is the exponential of a log correlated Gaussian field. In the cases of interest, this Gaussian field is a Schwartz distribution but not a function. As such, the construction of GMC needs to be done with care. In particular, in 2D, the GFF (Gaussian Free Field) is a random Schwartz distribution because of the logarithmic singularity of the Green kernel in 2D. Here we are interested in the 1D case on the circle.

On the other hand, it is known since Verblunsky (1930s) that a probability measure on the circle is entirely determined by the coefficients appearing in the recurrence of orthogonal polynomials. Furthermore, Killip and Nenciu (2000s) have given a realization of the CBE, an important model in random matrices, thanks to random orthogonal polynomials of the circle.

I will give the precise statement whose loose form is CBE = GMC

[5]Université Paul Sabatier
[6]University of Bristol

Discrete and continuous random surfaces (with unconstrained genus)

Speaker: Nicolas Curien[7]

Collaborators:
J. Najnudel [8]

Abstract: We will survey recent results obtained with Thomas Budzinski and Bram Petri concerning the geometry of random maps with unconstrained genus and the related hyperbolic model of Brooks and Makover. In particular, we conjecture that random maps (with no genus constraint) obey a very strong universality principle and there should be roughly "one law" of random graph associated to those maps. We present some progress towards this conjecture using a version of the peeling process for those objects. Understanding the fine properties of those random maps enables us to study the associated model of Brooks and Makover: we prove that the diameter of the hyperbolic surface obtained by gluing uniformly at random 2N triangles is asymptotically 2 log N with high probability.

[7]Université Paris Sud
[8]University of Bristol

Tightness of Liouville first passage percolation for $\gamma \in (0, 2)$

Speaker: Hugo Falconet[9]

Collaborators:
Jian Ding[10]
Julien Dubédat [11]
Alex Dunlap [12]

Abstract: This talk is related with the construction of the Liouville metric, which can be thought as the Riemannian metric whose metric tensor is given by $e^{2\xi h}(dx^2 + dy^2)$, where ξ is a fixed parameter and h is a Gaussian free field (or a log-correlated Gaussian field). We study Liouville first passage percolation metrics, which are natural approximations of this metric when the field h is mollified. When the parameter ξ corresponds to the subcritical regime of the LQG measure, we show that renormalized metrics $(\lambda_t^{-1} e^{\xi p_t * h} ds)_{t \in (0,1)}$ are tight with respect to the uniform topology. This indicates existence of subsequential limits. We obtain qualitative and quantitative properties satisfied by these limits (Weyl scaling, tail estimates, Hölder bounds, errors bounds for renormalizing constants).

[9]Columbia University
[10]University of Pennsylvania
[11]Columbia University
[12]Stanford University

Speaker: Krzysztof Gawędzki[13]

Collaborators:
Edwin Langmann and Per Moosavi[14]
Karol K. Kozłowski[1]

Abstract: In recent years CFT has been increasingly used to model situations out of equilibrium. I shall describe a simple class of nonequilibrium states in Minkowskian CFT that may be exactly mapped to equilibrium states using conformal symmetries. Such a mapping, combined with the conformal welding technique, leads to an exact expression for the statistics of out-of-equilibrium energy transfers. On the way, one obtains as a byproduct in representation theory a formula for the extension of characters of positive energy Virasoro representations to 1-parameter groups of circle diffeomorphisms.

[13]Laboratoire de Physique, ENS de Lyon
[14]Department of Physics, KTH Royal Institute of Technology, Stockholm

Anomaly non-renormalization in Weyl semimetals

Speaker: Alessandro Giuliani[15]

Collaborators:
Vieri Mastropietro[16]
Marcello Porta[17]

Abstract: Weyl semimetals are three dimensional materials characterized by a highly degenerate Fermi surface, which reduces to two isolated points, around which the dispersion relation is approximately conical. If we choose the Fermi energy to pass through these conical intersections, the low-energy quasi-particle excitations above the ground state have the same qualitative behavior as Dirac fermions in three spatial dimensions. Therefore, in the presence of interactions, the ground state correlations of Weyl semimetals have similar qualitative features as infrared QED_{3+1}. For these reasons, Weyl semimetals have been proposed as a condensed-matter realization of infrared QED_{3+1}. Several peculiar QED phenomena are expected to have their analogues in Weyl semimetals, including the Adler-Bell-Jackiw (ABJ) axial anomaly. As first proposed by Nielsen-Ninomiya in 1983, in the context of *non-interacting* lattice Weyl fermions, the condensed matter analogue of the ABJ anomaly consists in a flow of quasi-particles from one Fermi point to the other, in the presence of constant, *parallel*, electric and magnetic fields: in quadratic response theory, this translates into $\partial_t \langle N_R(t) - N_L(t) \rangle = \frac{1}{2\pi^2} E \cdot B$, the factor $1/(2\pi^2)$ being the ABJ anomaly. Such an anomalous response has been recently observed in real Weyl semimetals by several experimental groups. In QED, the ABJ anomaly is not dressed by radiative corrections, due to a deep cancellation mechanism, known as the 'Adler-Bardeen anomaly non-renormalization theorem'. The (perturbative) proof of this theorem is based on cancellations of loop integrals, which require in a crucial way exact relativistic invariance. Therefore, it is unclear whether the same universal coefficient should be observed in *interacting* Weyl semimetals. Remarkably, we show that, for a class of lattice Weyl semimetals with short range interactions, the analogue of the ABJ anomaly is exactly independent of the interaction. Our proof is based on a combination of: (1) rigorous Renormalization Group methods, which allow us to compute the ground state correlations of the interacting system with explicit control on their regularity in momentum space; (2) lattice Ward Identities, which translate the conservation of the lattice current into exact identities for lattice correlation functions; (3) the computation of the chiral triangle graph in QED_{3+1} with momentum cutoff.

[15]Univ. Roma Tre
[16]Univ. Milano
[17]Univ. Tübingen

Scattering for Liouville quantum gravity

Speaker: Colin Guillarmou [18]

Collaborators:
Antti Kupiainen [19]
Remi Rhodes [20]
Vincent Vargas [21]

Abstract: we describe the spectral structure of the Hamiltonian H appearing in 2D Liouville quantum gravity for the sphere. In particular, we show that there is a "good" scattering theory with generalized eigenfunctions which extend meromorphically in the complex plane (more precisely some ramified cover). This provides meromorphic extensions of some exponentials of the GFF for complex values of the parameters, defined initially for real valued parameters using probabilistic methods. This approach allows to give a fine analysis of the spectrum of H which is the first step toward the conformal bootstrap method for computing n-points correlations functions for the sphere. This is ongoing joint work with Kupiainen, Rhodes, Vargas.

[18] University Paris-Sud
[19] University of Helsinki
[20] Aix-Marseille University
[21] ENS Paris

Basic properties of the Liouville quantum gravity metric for $\gamma \in (0, 2)$

Speaker: Ewain Gwynne[22]

Collaborators:
Jason Miller[23]
Julien Dubédat[24]
Hugo Falconet[25]
Josh Pfeffer[26]
Xin Sun[27]

Abstract: In recent joint work with Jason Miller, we proved that for each $\gamma \in (0, 2)$, there is a unique metric associated with γ-Liouville quantum gravity (LQG). It can be constructed as the limit of Liouville first passage percolation, a family of random metrics obtained by exponentiating a mollified version of the Gaussian free field, which was recently shown to be tight by Ding, Dubédat, Dunlap, and Falconet (2019).

I will review the construction and axiomatic characterization of the metric, then discuss some of its most important properties, including conformal covariance, the KPZ formula, and confluence of geodesics. I will also present some open problems.

This talk complements the earlier talk by Jason Miller, but will be self-contained.

Based on four joint papers with Jason Miller, one joint paper with Julien Dubédat, Hugo Falconet, Josh Pfeffer, and Xin Sun, and one joint paper with Josh Pfeffer.

[22]Cambridge
[23]Cambridge
[24]Columbia
[25]Columbia
[26]MIT
[27]Columbia

Liouville quantum gravity with central charge in $(1, 25)$: a probabilistic approach

Speaker: Nina Holden[28]

Collaborators:
Ewain Gwynne[29]
Joshua Pfeffer[30]
Guillaume Remy[31]

Abstract: There is a substantial literature about Liouville quantum gravity (LQG) with coupling constant $\gamma \in (0, 2]$. In this setting, the central charge of the corresponding matter field satisfies $\mathbf{c} = 25 - 6(2/\gamma + \gamma/2)^2 \in (-\infty, 1]$. LQG with $\mathbf{c} > 1$ has been considered in the physics literature, but the probabilistic behavior in this regime is rather mysterious in part because the corresponding value of γ is complex, so analytic continuations of various formulas give non-physical complex answers.

We introduce and study a discretization of LQG which makes sense for all $\mathbf{c} \in (-\infty, 25)$. Our discretization is a random planar map, defined as the adjacency graph of a tiling of the plane by dyadic squares which all have approximately the same "LQG size" with respect to a Gaussian free field. We prove that several formulas for dimension-related quantities are still valid for $\mathbf{c} \in (1, 25)$, with the caveat that the dimension is infinite when the formulas give a complex answer. In particular, we prove an extension of the KPZ formula for $\mathbf{c} \in (1, 25)$, which gives a finite quantum dimension if and only if the Euclidean dimension is at most $(25 - \mathbf{c})/12$. We also show that the graph distance between typical points with respect to our discrete model grows polynomially while the cardinality of a graph distance ball of radius r grows faster than any power of r (which suggests that the Hausdorff dimension of LQG for $\mathbf{c} \in (1, 25)$ is infinite).

At the end of the talk I will present a number of open problems.

[28]ETH Zürich
[29]Cambridge
[30]Massachusetts Institute of Technology
[31]Columbia

Conformal field theory on the Riemann sphere and its boundary version for SLE

Speaker: Nam-Gyu Kang [32]

Collaborator:
Nikolai G. Makarov [33]

Abstract: Applying the Schottky double construction to conformal field theory on the Riemann sphere, I will implement its boundary version in a simply connected domain. The statistical fields under consideration are generated by background charge modification of the Gaussian free field under the OPE multiplications. I will show that the correlation functions of such fields under the insertion of (effective) one-leg operator form a collection of martingale-observables for chordal/radial SLE with force points (and spins).

[32] KIAS
[33] Caltech

Geometry and large N limits of Quantum Hall states

Speaker: Semyon Klevtsov [34]

Abstract: Laughlin states dene probability measures for N particle congurations on Riemann surfaces, describing the fractional Quantum Hall e?ect. The physical parameters can be inferred when QH states are dened on Riemann surfaces, with an arbitrary genus, metric, moduli etc, in the asymptotic limit of large N. I will give an introduction to this approach to QHE and talk about recent work.

[34]university of Cologne

Prospects in Constructive Field Theory

Speaker: Antti Kupiainen[35]

Abstract:

[35] Université Lyon 1

The renormalization of the log-gas via its Sine-Gordon representation (in dimension one)

Speaker: Hubert Lacoin [36]

Collaborators:
R.Rhodes and V.Vargas

Abstract: Consider a model gas of electrically charged particles in D a bounded subset of $\mathbb{R}^d$. The interaction given by $\lambda_i\lambda_j V(|x_i - x_j|)$ where $\lambda_i, \lambda_j \in \{-1, 1\}$ stands for the particle charge and $x_i, x_j \in D$ for the particle position and V represents the electrical potential (which is a function of the distance). Given α and β two positive parameters, our model is described by the following partition function which induces a probability distribution on the set of charged particles

$$\mathcal{Z}^V_{\alpha,\beta}(\mathcal{O}) = \sum_{n=0}^{\infty} \frac{\alpha^n}{n!} \sum_{(\lambda_i)_{i=1}^n \in \{-1,1\}^n} \int_{D^n} \exp\Big(-\beta \sum_{1\leq i<j\leq n} \lambda_i \lambda_i V(|z_i - z_j|)\Big) \prod_{i=1}^n dz_i$$

We are interested specifically in the case when V diverges logarithmically in zero $V(r) = -\log r + O(1)$. In that case, we observe that the above partition function is finite if and only if $\beta \geq d$. The aim of this talk is to introduce renormalization techniques using connection with Gaussian fields which allows for a definition of the Log-gas when $\beta \in [d, 2d)$.

[36]IMPA Rio

Brownian disks and Brownian motion indexed by the Brownian tree

Jean-François Le Gall[37]

Collaborators:
Céline Abraham[38]
Armand Riera[39]

Abstract: The Brownian sphere or Brownian map is the Gromov-Hausdorff limit of large random planar maps equipped with the (suitably rescaled) graph distance. Brownian disks have been introduced and studied by Bettinelli and Miermont as scaling limits of large quadrangulations with a boundary, when both the boundary size and the volume tend to infinity in a suitable way, and they also appear in the recent work of Gwynne, Miller and Sheffield. We will provide a new construction of Brownian disks based on the excursion theory for Brownian motion indexed by the Brownian tree that has been developed in a joint work with C. Abraham. This construction yields a natural definition of the uniform measure on the boundary, and makes it possible to derive new relations between the Brownian disk and the Brownian sphere. We will also discuss the remarkable growth-fragmentation process that arises when considering the boundary sizes of connected components of the set of points of a Brownian disk whose distance from the boundary is greater than r (joint work with A. Riera). This growth-fragmentation process process already appeared in the earlier work of Bertoin, Curien and Kortchemski studying the cycles obtained by slicing random Boltzmann triangulations at varying heights.

[37]Université Paris-Sud
[38]Université Paris-Sud
[39]Université Paris-Sud

AGT relation and combinatorics of conformal blocks

Speaker: Alexey Litvinov[40]

Abstract:

This is an introductory talk to Alday-Gaiotto-Tachikawa correspondence (2009), a remarkable intersection between two different branches of mathematical physics: two-dimensional conformal field theories and $\mathcal{N} = 2$ supersymmetric gauge theories, which already influenced both areas of research bringing a lot of new methods and ideas. One of the advantages of this relation is an explicit combinatorial formula for the so called conformal blocks, a building elements for correlation functions.

I'll start with basic concepts of 2D CFT: conformal Ward identities, operator product expansion, Virasoro algebra and its representation theory, conformal families, conformal blocks decomposition of correlation functions. Then I'll review free-field methods in 2D CFT, invented by Dotsenko and Fateev. Then I'll postulate the AGT combinatorial formula for $4-$point conformal block and comment on its geometrical origin. The rest of my talk will be devoted to its proof. Depending on time and interest I could also discuss an integrable structure hidden behind AGT relation and Maulik-Okounkov $\mathcal{R}$ matrix.

[40]Landau Institute for theoretical physics

Uniform Lipschitz functions on the triangular lattice have logarithmic variations

Speaker: Ioan Manolescu[41]

Collaborators:
Alexander Glazman[42]

Abstract: Uniform integer-valued Lipschitz functions on a finite domain of the triangular lattice are shown to have variations of logarithmic order in the radius of the domain. This is in agreement with the conjectured convergence of uniform integer-valued Lipschitz functions to the Gaussien Free Field.

The level lines of such functions form a loop $O(2)$ model on the edges of the hexagonal lattice with edge-weight one. An infinite-volume Gibbs measure for the loop $O(2)$ model is constructed as a thermodynamic limit and is shown to be unique. It contains only finite loops and has properties indicative of scale-invariance: macroscopic loops appearing at every scale. The existence of the infinite-volume measure carries over to height functions pinned at 0; the uniqueness of the Gibbs measure does not.

The proof is based on a representation of the loop $O(2)$ model via a pair of spin configurations that are shown to satisfy the FKG inequality. We prove RSW-type estimates for a certain connectivity notion in the aforementioned spin model.

[41]University of Fribourg
[42]Tel-Aviv University

Delocalization of two-dimensional random surfaces

Speaker: Ron Peled [43]

Abstract: We give an overview of the problem of proving delocalization for two-dimensional random surfaces, discussing both real- and integer-valued surfaces dened over domains in the square lattice.

[43]Tel Aviv university

Speaker: Ellen Powell[44]

Collaborators:
Nina Holden [45]

Abstract: I will discuss the conformal welding of critical ($\gamma = 2$) Liouville quantum gravity surfaces according to quantum boundary length. If one takes two such surfaces with infinite boundary length, then we show that the conformal welding exists, and results in a further critical LQG surface decorated by an independent SLE_4. Combined with the proof of uniqueness for such a welding, recently established by McEnteggart, Miller, and Qian, this shows that the welding operation for critical LQG is well-defined. This result is an analogue of Sheffield's quantum gravity zipper in the subcritical regime $\gamma < 2$.

[44]ETH Zürich
[45]ETH Zürich

Conformal welding, Loewner equation, and trees

Speaker: Steffen Rohde [46]

Abstract: Conformal welding plays a role in conformal geometry, complex dynamics, Teichmueller theory and more recently also in image processing, string theory, and random geometry. After giving a brief overview, I will describe the conformal welding of trees and joint work with Peter Lin on the convergence of random Shabat trees to the conformal continuum random tree.

[46] University of Washington

Fuchsian equations and bootstrap solutions in Toda field theories

Speaker: Raoul Santachiara [47]

Collaborator:

Abstract: The Toda theories are the most direct generalizations of the Liouville theories. Nevertheless, they are considerably more complicated, as new structures and phenomena arise. For instance, contrary to the Liouville theory, the Ward identities are not anymore sufficient correlation functions of descendants in terms of primary fields.

I will discuss these issues, focusing in particular on the computation of the Toda conformal blocks, on the Fuchsian equations they satisfy and on the construction of crossing invariant correlation function

[47]Univ. Paris-Sud

Extremal distance and conformal radius of a CLE_4 loop.

Speaker: Avelio Sepúlveda[48]

Collaborators:
Juhan Aru[49]
Titus Lupu[50]

Abstract: In this talk, we will discuss the geometry of the loop of a CLE_4 surrounding the origin. In particular, we show how to compute the joint law between the extremal distance from this loop to the boundary together with the conformal radius seen from the origin of the domain surrounded by the loop.

[48]Université Lyon 1
[49]EPFL
[50]CNRS, Sorbonne Université

Stochastic Ricci Flow on Compact Surfaces

Speaker: Hao Shen[51]

Collaborators:
Julien Dubédat[52]

Abstract: We introduce the Stochastic Ricci flow (SRF) in two spatial dimensions. On the torus it is formally given by

$$\partial_t g = -2R_g - 2\lambda g + 2\sigma\xi_g g$$

where space-time noise ξ_g is "white" with respect to g, and $\lambda, \sigma \in \mathbf{R}$; or in terms of the conformal factor ϕ with $g = e^{2\phi} g_0$ and g_0 a flat (reference) metric and Δ_g the Beltrami-Laplacian

$$\partial_t \phi = \Delta_g \phi - \lambda + \sigma\xi_g \ .$$

The flow is symmetric with respect to a measure induced by Liouville Conformal Field Theory. Using the theory of Dirichlet forms, we construct a weak solution to the associated equation of the area measure on a flat torus, in the full "L^1 regime" $\sigma < \sigma_{L^1} = 2\sqrt{\pi}$. Modifications are needed for the SRF on general compact surfaces due to conformal anomaly. Under the flow, the total area of the surface follows a squared Bessel process. We also discuss some open questions.

[51]University of Wisconsin - Madison
[52]Columbia University

Liouville quantum gravity and mating of trees

Speaker: Xin Sun[53]

Abstract: Liouville quantum gravity (LQG) describes the scaling limit of random planar maps under conformal embeddings. LQG coupled with independent conformal invariant processes describe the scaling limit of statistical mechanical models on random planar maps. Mating-of-trees theory is an approach to study the annealed scaling limit of certain statical mechanical models on random planar maps. In the discrete, it is a collection of bijections that encode model-decorated planar maps by 2D lattice walks. In the continuum, it gives an encoding of Schramm Loewner evolution coupled with LQG in terms of 2D Brownian motion. In the first lecture, I will give an overview of this theory and describe two simple mating-of-trees bijections, one for spanning-tree-decorated planar maps, one for site-percolation-decorated triangulations. In the second lecture, I will (try my best to) provide a precise statement of the fundamental mating-of-trees theorem due to Duplantier-Miller-Sheffield (2014), and explain some ideas entering its proof. In the third lecture I will discuss some applications and open problems.

[53]Columbia university

Conformal Field Theories in any dimension and Bootstrap

Speaker: Alessandro Vichi[54]

Abstract:

The purpose of these lectures is to provide a pedagogical introduction to conformal field theories, their application to critical phenomena in statistical physics and modern techniques to extract quantitative informations in this context. For decades it has been a dream to study these intricate strongly coupled theories nonperturbatively using symmetries and other consistency conditions. This idea, called the conformal bootstrap, saw some successes in two dimensions but it is only in the last ten years that it has been fully realized in three, four, and other dimensions of interest. After reviewing the properties of a field theory invariant under conformal symmetry, I will show how to set up the bootstrap equations and discuss numerical techniques based on convex optimization for finding or constraining their solutions. I will explain how these techniques can outperform field theory approaches based on perturbative expansion or MonteCarlo simulations. I will conclude by presenting a few concrete applications, such as the three dimensional Ising model.

[54]École Polytechnique Fédérale de Lausanne

Interplay between the Loewner and Dirichlet energies: Conformal welding and Flow-lines

Speaker: Yilin Wang[55]

Collaborators:
Fredrik Viklund [56]

Abstract: The Loewner's energy defined for Jordan curves is the action functional of SLE (and also its large deviation rate function when the parameter goes to 0). It was shown in a previous work that a Jordan curve has finite energy if and only if it is a Weil-Petersson quasicircle.

In this talk we present identities that relate the Loewner energy to the Dirichlet energy of ambient fields. They are deterministic analogs of both the welding and flow-line couplings of SLEs with the Gaussian free field on the level of action functionals. We deduce also an identity on complex valued fields that combines both welding and flow-line identities. We apply these results to show that the operation of arclength isometric welding of two finite energy domains is sub-additive in the energy and that the energy of equipotentials in a simply connected domain is monotone.

[55]ETH Zürich
[56]KTH