Compression for Coinductive Infinitary Rewriting
(A Preliminary Account)

Rémy Cerda[1] and Alexis Saurin[1,2]

[1] Université Paris Cité, CNRS, IRIF, F-75013 Paris, France
[2] INRIA π^3, Paris, France
Remy.Cerda@math.cnrs.fr, Alexis.Saurin@irif.fr

In "traditional" infinitary rewriting based on ordinal-indexed rewriting sequences and strong Cauchy convergence, a key property of rewriting systems is *compression*, that is, the fact that rewriting sequences of arbitrary ordinal length can be compressed to sequences of length ω. Famous examples of compressible systems are left-linear first-order systems and infinitary λ-calculi.

In this work, we investigate compression in the equivalent setting of coinductive infinitary rewriting, introduced by Endrullis et al. [End+18], which we recall in Section 1 in a slightly augmented form: the original work only covered first-order rewriting, we extend it to rewriting of (possibly non-wellfounded) derivations in an arbitrary sytem of derivation rules.

Then in Section 2 we define the coinductive counterpart of compressed rewriting sequences, and we present a general coinductive procedure turning arbitrary infinitary rewriting derivations into compressed ones, without relying on a topological formalism. The coinductive presentation of the two aforementioned examples, that is left-linear first-order systems and the full infinitary λ-calculus, are endowed with compression lemmas as instances of our general method.

This is a preliminary work, as our main motivation is to tackle the rewriting induced on non-wellfounded proofs by eliminating cuts. For future work, we will focus on the system $\mu\mathsf{MALL}^\infty$ for multiplicative-additive linear logic with fixed points, the cut-elimination theorem of which crucially relies on a compression lemma [Sau23]. In particular, we hope to be able to use a coinductive compression step as a component of a fully coinductive cut-elimination proof.

1 Coinductive infinitary rewriting

In this first section, we recall the coinductive presentation of infinitary first-order rewriting from [End+18]. Then we provide an extension of this presentation to infinitary λ-calculus, and to a generic notion of rewriting system for non-wellfounded derivations.

1.1 First order infinitary rewriting

Fix a countable set $\mathcal{V}$ of variables. A first-order signature is a countable set Σ equipped with an arity function $\mathrm{ar} : \Sigma \to \mathbf{N}$; we fix such a signature. The set T_Σ of first-order terms on this signature can be defined inductively by the derivation rules:

$$\frac{x \in \mathcal{V}}{x \in \mathrm{T}_\Sigma} \qquad \frac{t_1 \in \mathrm{T}_\Sigma \quad \dots \quad t_{\mathrm{ar}(\mathsf{c})} \in \mathrm{T}_\Sigma}{\mathsf{c}(t_1, \dots, t_{\mathrm{ar}(\mathsf{c})}) \in \mathrm{T}_\Sigma} \quad (\text{for each } \mathsf{c} \in \Sigma).$$

The set T^∞_Σ of infinitary first-order terms on the signature Σ can then be defined by completing T_Σ with respect to the metric defined by $d(s,t) \coloneqq 2^{-(\text{the smallest depth at which } s \text{ and } t \text{ differ})}$, or equivalently by treating the above rules coinductively [Bar93] (which we will denote by using double inference bars).

In this setting, an (infinitary) rewrite rule is a couple (p, q) where $p \in \mathrm{T}_\Sigma$ and $q \in \mathrm{T}_\Sigma^\infty$. An infinitary term rewriting system (ITRS) is a countable set of rewrite rules; we fix an ITRS $\mathcal{R}$. Two terms $s, t \in \mathrm{T}_\Sigma^\infty$ are related by a rewrite step, denoted $s \longrightarrow t$, whenever there are a rule $(p, q) \in \mathcal{R}$, a (single-hole) context $u[*]$ and a substitution $\sigma : \mathcal{V} \to \mathrm{T}_\Sigma^\infty$ such that $s = u[\sigma \cdot p]$ and $t = u[\sigma \cdot q]$ (where $\sigma \cdot p$ denotes the substitution of each $x \in \mathcal{V}$ by $\sigma(x)$ in p).

A rewriting sequence of ordinal length γ is given by terms $(s_\delta)_{\delta \le \gamma}$ together with rewrite steps $(s_\delta \longrightarrow s_{\delta+1})_{\delta < \gamma}$. Such a rewriting sequence is said to be strongly convergent if for all limit ordinal $\delta \le \gamma$, $\lim_{\epsilon \to \delta} d(s_\epsilon, s_{\epsilon+1}) = 0$ and in addition, for all limit ordinal $\delta < \gamma$, the steps $s_\epsilon \longrightarrow s_{\epsilon+1}$ occur at depths tending to infinity when $\epsilon \to \delta$ [Ken+95].

Definition 1. We say that $s \longrightarrow^\infty t$ when there is an ordinal γ such that $s \xrightarrow[\gamma]{}^\infty t$ is derivable in the following system of rules (wheres simple bars denote inductive inferences and double bars denote coinductive inferences, *i.e.* non-wellfounded derivations are allowed provided each infinite branch crosses infinitely often a double bar):

$$\frac{s \underset{\gamma,n}{\rightsquigarrow} s' \qquad s' \xrightarrow[\gamma]{}^\infty t}{s \xrightarrow[\gamma]{}^\infty t}\ (\mathsf{split}) \qquad \frac{x \in \mathcal{V}}{x \underset{\gamma}{\rightharpoonup}^\infty x}\ (\mathsf{var}) \qquad \frac{\forall 1 \le i \le n,\ s_i \xrightarrow[\gamma]{}^\infty t_i}{\mathsf{c}(s_1, \dots, s_n) \underset{\gamma}{\rightharpoonup}^\infty \mathsf{c}(t_1, \dots, t_n)}\ (\mathsf{lift_c})$$

where $s \underset{\gamma,n}{\rightsquigarrow} s'$ denotes any sequence $s \longrightarrow^* s_1' \underset{\delta_1}{\rightharpoonup}^\infty t_1 \longrightarrow^* s_2' \underset{\delta_2}{\rightharpoonup}^\infty \dots \underset{\delta_n}{\rightharpoonup}^\infty t_n \longrightarrow^* s'$ such that $\forall 1 \le i \le n,\ \delta_i < \gamma$. (Notice that we are in fact defining *two* relations, namely $\xrightarrow[\gamma]{}^\infty$ and $\underset{\gamma}{\rightharpoonup}^\infty$, the latter one indicating that the former one occurs under a constructor.)

This coinductive presentation was introduced (with slightly different notations) by Endrullis et al. [End+18], who prove that it is equivalent to the "traditional", topology-based one. Indeed:

Theorem 2 ([End+18])**.** For $s, t \in \mathrm{T}_\Sigma^\infty$, there is a strongly converging rewriting sequence from s to t iff. $s \longrightarrow^\infty t$.

1.2 Infinitary λ-calculus

An identical path can be followed to present an infinitary λ-calculus. (Finite) λ-terms are the elements of the set Λ defined by:

$$\frac{x \in \mathcal{V}}{x \in \Lambda} \qquad \frac{x \in \mathcal{V} \qquad M \in \Lambda}{\lambda x.M \in \Lambda} \qquad \frac{M \in \Lambda \qquad N \in \Lambda}{MN \in \Lambda}$$

and the set Λ^∞ of infinitary λ-terms can be defined either by metric completion [Ken+97] or by treating these rules coinductively [EP13]. In both cases we work modulo α-equivalence (which raises some subtleties, see [Kur+13] for a formal treatment.)

As usual, the reduction $\longrightarrow$ of β-reduction is defined on Λ^∞ by $(\lambda x.M)N \longrightarrow M[N/x]$ for all $M, N \in \Lambda^\infty$ (where $M[N/x]$ denotes the term obtained by substituting N to each free x in M), as well as lifting to contexts. As above, we define:

Definition 3. For $M, N \in \Lambda^\infty$, we say that $M \longrightarrow^\infty N$ when there is an ordinal γ such that $M \xrightarrow[\gamma]{}^\infty N$ is derivable with the rules (split), (var),

$$\frac{M \xrightarrow[\gamma]{}^\infty M'}{\lambda x.M \underset{\gamma}{\rightharpoonup}^\infty \lambda x.M'}\ (\mathsf{lift}_\lambda) \qquad \text{and} \qquad \frac{M \xrightarrow[\gamma]{}^\infty M' \qquad N \xrightarrow[\gamma]{}^\infty N'}{MN \underset{\gamma}{\rightharpoonup}^\infty M'N'}\ (\mathsf{lift}_@).$$

Theorem 4. For $M, N \in \Lambda^\infty$, there is a strongly converging β-reduction sequence from M to N iff. $M \longrightarrow^\infty N$.

This can in fact be extended to all *abc*-infinitary λ-calculi from [Ken+97]. The corresponding work (but the missing piece we intend to add) is presented in [Cer24; Cer25].

1.3 Rewriting non-wellfounded derivations

Consider the set $\mathcal{D}$ of all derivations obtained from a set of rules of one of the following shapes:

$$\frac{p_1 \quad \dots \quad p_k}{c}\ (\mathsf{r}) \qquad \text{or} \qquad \frac{p_1 \quad \dots \quad p_k}{c}\ (\mathsf{r})$$

and some rewriting relation $\longrightarrow$ on $\mathcal{D}$. Just as we did in Definitions 1 and 3, we can define a relation $\underset{\gamma}{\longrightarrow}^{\infty}$ on $\mathcal{D}$ by the rules (split) as well as:

$$\frac{}{d \underset{\gamma}{\longrightarrow}^{\infty} d}\ (\mathsf{refl}) \qquad \text{and} \qquad \frac{d_1 \underset{\gamma}{\longrightarrow}^{\infty} d_1' \quad \dots \quad d_k \underset{\gamma}{\longrightarrow}^{\infty} d_k'}{d \underset{\gamma}{\longrightarrow}^{\infty} d'}\ (\mathsf{lift}_{\mathsf{r}})$$

where d (resp. d') is a derivation concluded by a rule (r) as above, $d_1, \dots, d_k$ (resp. $d_1', \dots, d_k'$) are the sub-derivations rooted at the premises of this rule, and $(\mathsf{lift}_{\mathsf{r}})$ is coinductive whenever (r) is. We say that $d \longrightarrow^{\infty} d'$ whenever there is γ such that $d \underset{\gamma}{\longrightarrow}^{\infty} d'$.

Our presentations of first-order rewriting and β-reduction are instances of this construction. Furthermore, as we did in Theorems 2 and 4, one can show that $d \longrightarrow^{\infty} d'$ iff. there is a strongly convergent rewriting sequence from d to d'.

What remains to be investigated is whether this construction is compatible with validity criteria, that is, global criteria used on top of non-wellfounded derivation systems to sort out "incorrect" derivations, typically to avoid inconsistencies in non-wellfounded proof systems. Our hope is that reasonable validity criteria on the rewritten derivations can be transported to restrict the derivations defining $\longrightarrow^{\infty}$ in such a way that $\longrightarrow^{\infty}$ rewrites valid derivations into valid derivations.

2 Compression lemmas

Two standard instances of the Compression lemma, which we would like to transport to the coinductive setting we just presented, are the following:

Theorem 5 ([Ken+95])**.** Let $\mathcal{R}$ be a left-linear ITRS, that is, no variable occurs twice in the left component of a rule of $\mathcal{R}$. Then for all $s, t \in \mathrm{T}^{\infty}_{\Sigma}$, there is a strongly convergent rewriting sequence from s to t iff. there is such a sequence of length at most ω.

Theorem 6 ([Ken+97])**.** For all $M, N \in \Lambda^{\infty}$, there is a strongly convergent β-reduction sequence from M to N iff. there is such a sequence of length at most ω.

2.1 Compressed rewriting sequences, coinductively

Since the "length" of a derivation $d \longrightarrow^{\infty} d'$ is not defined, we first need to introduce a coinductive counterpart of strongly converging rewriting sequences of length ω, extending again a definition from [End+18].

Definition 7. With the notation of Section 1.3, a relation $\longrightarrow^{\omega}$ is defined on $\mathcal{D}$ by:

$$\frac{d \longrightarrow^{*} e \quad e \longrightarrow^{\omega} d'}{d \longrightarrow^{\omega} d'}\ (\mathsf{split}^{\omega}) \qquad \frac{}{d \longrightarrow^{\omega} d}\ (\mathsf{refl}^{\omega}) \qquad \frac{d_1 \longrightarrow^{\omega} d_1' \quad \dots \quad d_k \longrightarrow^{\omega} d_k'}{d \longrightarrow^{\omega} d'}\ (\mathsf{lift}^{\omega}_{\mathsf{r}})$$

Lemma 8. For $d, d' \in \mathcal{D}$, there is a strongly convergent rewriting sequence of length at most ω from d to d' iff. $d \longrightarrow^{\omega} d'$.

In particular, this construction defines relations $\longrightarrow^{\omega}$ on $\mathrm{T}^{\infty}_{\Sigma}$ and Λ^{∞}, capturing exactly strongly convergent sequences of rewriting steps through $\mathcal{R}$ and β-reductions, respectively.

A relation $\longrightarrow^{\infty}$ defined as in Section 1.3 has the (coinductive) compression property if any derivation of $d \longrightarrow^{\infty} d'$ can be turned into a derivation of $d \longrightarrow^{\omega} d'$, that is, if $\longrightarrow^{\infty} = \longrightarrow^{\omega}$. In particular for the relation $\longrightarrow^{\infty}$ on $\mathrm{T}^{\infty}_{\Sigma}$, the compression property can be obtained by translating $s \longrightarrow^{\infty} s'$ as a strongly convergent rewriting sequence (Theorem 2), compressing it (Theorem 5), and translating the compressed sequence again (Lemma 8). Similarly, the compression property in Λ^{∞} is a consequence of Theorems 4 and 6 and Lemma 8. However, we would like to build a direct, explicit proof, without resorting to ordinal-based infinitary rewriting.

2.2 A general proof structure

With the general notations from Section 1.3, consider the following properties:

$\mathfrak{P}_{\gamma,n}$: For all $d, d' \in \mathcal{D}$, if $d \underset{\gamma,n}{\rightsquigarrow} d'$ then there are $d'' \in \mathcal{D}$ and an ordinal $\delta < \gamma$ such that $d \longrightarrow^{*} d'' \xrightarrow[\delta]{}^{\infty *} d'$.

$\mathfrak{Q}$: For any ordinal δ, if $\forall m \in \mathbf{N}$, $\mathfrak{P}_{\delta,m}$ holds, then for any reduction $d'_n \xrightarrow[\delta]{}^{\infty} e'_n \longrightarrow d'$ there is a $d''_n \in \mathcal{D}$ such that $d'_n \longrightarrow^{*} d''_n \xrightarrow[\delta]{}^{\infty} d'$.

Theorem 9. If $\mathfrak{Q}$ holds then $\longrightarrow^{\infty}$ has the compression property.

Lemma 10. If $d \xrightarrow[\gamma]{}^{\infty} e$, then for any ordinal $\epsilon \geq \gamma$ there is also a derivation of $d \xrightarrow[\epsilon]{}^{\infty} e$.

Lemma 11. If $d \xrightarrow[\gamma]{}^{\infty} e$ and $e \xrightarrow[\delta]{}^{\infty} f$, then $d \xrightarrow[\epsilon]{}^{\infty} f$ for $\epsilon \coloneqq \max(\gamma + 1, \delta)$.

Lemma 12. If $\mathfrak{Q}$ holds then $\forall \gamma$, $\forall n \in \mathbf{N}$, $\mathfrak{P}_{\gamma,n}$.

Proof. We proceed by well-founded induction over γ, and we suppose that $\forall \delta < \gamma$, $\forall m \in \mathbf{N}$, $\mathfrak{P}_{\delta,m}$. Then we proceed by induction on $n \in \mathbf{N}$.

If $n = 0$ the result is immediate since $d \underset{\gamma,0}{\rightsquigarrow} d'$ means that $d \longrightarrow^{*} d'$. Otherwise, suppose that $d \underset{\gamma,n}{\rightsquigarrow} d'$. This can be decomposed as: $d \underset{\gamma,n-1}{\rightsquigarrow} d'_n \xrightarrow[\delta_n]{}^{\infty} e'_n \longrightarrow^{*} d'$, with $\delta_n < \gamma$. Using $\mathfrak{Q}$ and the induction hypothesis (*i.e.* the fact that $\forall m \in \mathbf{N}$, $\mathfrak{P}_{\delta_n,m}$ holds), there is a $d''_n \in \mathcal{D}$ such that $d \underset{\gamma,n-1}{\rightsquigarrow} d'_n \longrightarrow^{*} d''_n \xrightarrow[\delta_n]{}^{\infty} d'$. Notice that $d \underset{\gamma,n-1}{\rightsquigarrow} d'_n \longrightarrow^{*} d''_n$ can be reformulated as $d \underset{\gamma,n-1}{\rightsquigarrow} d''_n$, whence we can apply the induction hypothesis on $n-1$ and obtain a term d'' and an ordinal $\delta' < \gamma$ such that $d \longrightarrow^{*} d'' \xrightarrow[\delta']{}^{\infty *} d''_n \xrightarrow[\delta_n]{}^{\infty} d'$, which can be simplified as $d \longrightarrow^{*} d'' \xrightarrow[\delta]{}^{\infty *} d'$ with $\delta \coloneqq \max(\delta', \delta_n) < \gamma$ thanks to Lemma 10. ☐

Proof of Theorem 9. Suppose $\mathfrak{Q}$. We start with a derivation of $d \xrightarrow[\gamma]{}^{\infty} e$. It can only be obtained through the rule (split), hence $d \underset{\gamma,n}{\rightsquigarrow} d' \xrightarrow[\gamma]{}^{\infty} e$. Then:

1. We apply Lemma 12 to $d \underset{\gamma,n}{\rightsquigarrow} d'$, and obtain $d \longrightarrow^{*} d'' \xrightarrow[\delta]{}^{\infty *} d' \xrightarrow[\gamma]{}^{\infty} e$ for some $d'' \in \mathcal{D}$ and some ordinal $\delta < \gamma$.

2. We apply the transitivity Lemma 11, and obtain $d \longrightarrow^{*} d'' \xrightarrow[\gamma]{}^{\infty} e$.

3. We proceed coinductively in $d'' \xrightarrow[\gamma]{}^{\infty} e$, building $d \longrightarrow^{*} d'' \longrightarrow^{\omega} e$. We conclude with the rule (split^{ω}). ☐

Our proof departs from the one presented in the Coq formalisation of [End+18] in three directions: first, as already stressed, it is parametric in the kind of rewriting we consider (whereas their proof only covers first-order rewriting); second, our definition of $\longrightarrow^{\infty}$ features ordinal annotations to constrain the use of coinduction, whereas their definition relies on mixing least and a greatest fixed points, which results in different treatments of the inductive part of the proof; third, our proof provides an explicit coinductive procedure for compressing derivations of infinitary rewritings. This suggests that compression may be computable, in a sense and under conditions that are yet to be made precise (which we leave for further work).

2.3 Back to our main examples

In the property $\mathfrak{Q}$, we isolated the precise step where the specific properties of the considered rewriting system come into play. Let us come back to the two previously described examples, that is left-linear ITRS and infinitary λ-calculus, and instantiate Theorem 9.

Lemma 13. If $\mathcal{R}$ is a left-linear ITRS, then the relation $\longrightarrow$ it defines on $\mathrm{T}^{\infty}_{\Sigma}$ satisfies the property $\mathfrak{Q}$.

Proof sketch. Let δ be an ordinal such that $\forall m \in \mathbf{N}$, $\mathfrak{P}_{\delta,m}$ holds, and consider a derivation of $s'_n \xrightarrow[\delta]{}^{\infty} t'_n \longrightarrow s'$. The last step can be described as $p[\sigma] \longrightarrow q[\sigma]$ for a substitution σ. The key observation is that since p is finite, we can analyse $s'_n \xrightarrow[\delta]{}^{\infty} t'_n$ inductively (using the hypothesis on δ when we meet $\underset{\delta,m}{\rightsquigarrow}$) and produce, on one hand a finite reduction $s'_n \longrightarrow^{*} p[\tau]$ for some substitution τ, on the other hand derivations $\tau(x) \xrightarrow[\delta]{}^{\infty} \sigma(x)$ for each $x \in \mathcal{V}$. This allows to conclude: $s'_n \longrightarrow^{*} p[\tau] \longrightarrow q[\tau] \xrightarrow[\delta]{}^{\infty} q[\sigma]$. The left-linearity assumption is used when we define τ: if a variable x appeared several times in p, then we might define $\tau(x)$ in several conflicting ways. ☐

Corollary 14. If $\mathcal{R}$ is a left-linear ITRS, then the relation $\longrightarrow^{\infty}$ it defines on $\mathrm{T}^{\infty}_{\Sigma}$ has the compression property.

The same holds in the infinitary λ-calculus:

Lemma 15. The relation $\longrightarrow$ defined on Λ^{∞} satisfies the property $\mathfrak{Q}$.

Proof sketch. Let δ be an ordinal such that $\forall m \in \mathbf{N}$, $\mathfrak{P}_{\delta,m}$ holds, and consider a derivation of $M'_n \xrightarrow[\delta]{}^{\infty} N'_n \longrightarrow M'$. If the last β-reduction step occurs at top-level, *i.e.* $N'_n = (\lambda x.P')Q'$ and $M' = P'[Q'/x]$, then by analysing $M'_n \xrightarrow[\delta]{}^{\infty} N'_n$ and using the hypothesis $\mathfrak{P}_{\delta,m}$ we are able to identify terms P, Q such that $M'_n \longrightarrow^{*} (\lambda x.P)Q \longrightarrow P[Q/x]$, as well as $P \xrightarrow[\delta]{}^{\infty} P'$ and $Q \xrightarrow[\delta]{}^{\infty} Q'$. For the last two hypotheses we are able to deduce that $P[Q/X] \xrightarrow[\delta]{}^{\infty} P'[Q'/x] = N'_n$. In general the last redex occurs in context, *i.e.* $N'_n = C[(\lambda x.P')Q']$, and we have to scan this context inductively, collecting finite reductions $P_0 \longrightarrow^{*} P_1 \longrightarrow^{*} \dots \longrightarrow^{*} P_K$ as in Lemma 13. ☐

Corollary 16. The relation $\longrightarrow$ defined on Λ^{∞} has the compression property.

In particular, notice that this justifies the coinductive definition of infinitary β-reduction as it is usually written, that is, using $\longrightarrow^{\omega}$ instead of the rather impractical $\longrightarrow^{\infty}$ [EP13; Cza20; Cer24].

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