Uniqueness of least-energy solutions to the fractional Lane-Emden equation in the ball

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We prove uniqueness of least-energy solutions to the fractional Lane-Emden equation, under homogeneous Dirichlet exterior conditions, when the underlying domain is a ball $B \subset \mathbb{R}^N$. The equation reads as follows:

$$\begin{cases} (-\Delta)^s u & = & u^p & \text{in } B, \\ u & = & 0 & \text{in } \mathbb{R}^N \setminus B. \end{cases}$$

Here $s \in (0, 1)$, and $p \in \left(1, \frac{N+2s}{N-2s}\right)$, which makes the nonlinearity superlinear and subcritical. The proof makes use of Morse theory and is inspired by some results obtained by C. S. Lin in the '90s. A new Hopf's Lemma-type result is an essential ingredient in order to prove nondegeneracy of least-energy solutions. This is a joint work with Azahara DelaTorre.