

A Fourier extremal problem

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This talk addresses a long-standing problem in Fourier analysis: identifying the function with the smallest possible “size” (integral of its absolute value) among all functions whose value at the origin is one and whose frequencies are limited to a specific range. While the existence of this “extremal function” and its numerical value have been known for decades, its precise mathematical structure has remained a mystery.

We present a complete description of this function. We show that it can be broken down into two simpler components that satisfy a second-order differential equation and a functional equation. These mathematical relationships allow us to prove several properties that were previously only guessed at through computer simulations. Specifically, we show that the function’s behavior is extremely smooth and we also give a precise description of its zeros.

Finally, we describe a new method to calculate the constants associated with this function to any level of accuracy.