An obstacle to a characterisation of rectifiability of sets with high codimensions

March 5, 2026

Recent developments have led to the understanding that, essentially, $n-1$-rectifiability of the boundary of a domain in $\mathbb{R}^n$ is necessary and sufficient for the harmonic measure to be absolutely continuous with respect to the Hausdorff measure on that boundary. This can be viewed as a characterisation of the geometry of the boundary E of a domain. One might ask: can we characterise the geometry (or d-rectifiability, for $d < n-1$) of a set E of a smaller dimension in some similar way?

An attempt to do this was made recently in a series of papers by G. David, M. Engelstein, J. Feneuil, L. Li, S. Mayboroda, and collaborators. They introduced a class of degenerate elliptic operators defined on the domain $\mathbb{R}^n \setminus E$, the simplest of which is

$$L_\alpha = -\mathrm{div} D_\alpha^{-n+d+1} \nabla,$$

where D_α is a regularised distance function. We will discuss the key obstacle to completing the aforementioned characterisation in terms of the analogue of harmonic measure associated with these operators. It turns out to be a question concerning solutions of the equation $L_\alpha D_\alpha = 0$, which can be boiled down to a comparison between certain function spaces.