THE MAXIMAL REGULARITY OPERATOR ON TENT SPACES

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Abstract. Recently, Auscher and Axelsson gave a new approach to non-smooth boundary value problems with \( L^2 \) data, that relies on some appropriate weighted maximal regularity estimates. As part of the development of the corresponding \( L^p \) theory, we prove here the relevant weighted maximal estimates in tent spaces \( T^{p,2} \) for \( p \) in a certain open range. We also study the case \( p = \infty \).

1. Introduction. Let \(-L\) be a densely defined closed linear operator acting on \( L^2(\mathbb{R}^n) \) and generating a bounded analytic semigroup \((e^{-tL})_{t \geq 0}\). We consider the maximal regularity operator defined by

\[
\mathcal{M}_L f(t, x) = \int_0^t L e^{-(t-s)L} f(s, .)(x) ds,
\]

for functions \( f \in C_c(\mathbb{R}_+ \times \mathbb{R}^n) \). The boundedness of this operator on \( L^2(\mathbb{R}_+ \times \mathbb{R}^n) \) was established by de Simon in [16]. The \( L^p(\mathbb{R}_+ \times \mathbb{R}^n) \) case, for \( 1 < p < \infty \), turned out, however, to be much more difficult. In [10], Kalton and Lancien proved that \( \mathcal{M}_L \) could fail to be bounded on \( L^p \) as soon as \( p \neq 2 \). The necessary and sufficient assumption for \( L^p \) boundedness was then found by Weis [17] to be a vector-valued strengthening of analyticity, called \( R \)-analyticity. As many differential operators \( L \) turn out to generate \( R \)-analytic semigroups, the \( L^p \) boundedness of \( \mathcal{M}_L \) has subsequently been successfully used in a variety of PDE situations (see [14] for a survey).

Recently, maximal regularity was used in a different manner as an important tool in [2], where a new approach to boundary value problems with \( L^2 \) data for divergence form elliptic systems on Lipschitz domains, is developed. More precisely, in [2], the authors establish and use the boundedness of \( \mathcal{M}_L \) on weighted spaces \( L^2(\mathbb{R}_+ \times \mathbb{R}^n; t^\beta dt dx) \), for certain values of \( \beta \in \mathbb{R} \), under the additional assumption that \( L \) has bounded holomorphic functional calculus on \( L^2(\mathbb{R}^n) \). This additional

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assumption was removed in [3, Theorem 1.3]. Here is the version when specializing
the Hilbert space to be $L^2(\mathbb{R}^n)$.

**Theorem 1.1.** With $L$ as above, $M_L$ extends to a bounded operator on $L^2(\mathbb{R}_+ \times \mathbb{R}^n; t^\beta dt dx)$ for all $\beta \in (-\infty, 1)$.

The use of these weighted spaces is common in the study of boundary value
problems, where they are seen as variants of the tent space $T^{2,2}$ which occurs for
$\beta = -1$, introduced by Coifman, Meyer and Stein in [6]. For $p \neq 2$, the corre-
sponding spaces are weighted versions of the tent spaces $T^{p,2}$, which are defined,
for parameters $\beta \in \mathbb{R}$ and $m \in \mathbb{N}$, as the completion of $C_c(\mathbb{R}_+ \times \mathbb{R}^n)$ with respect
to

$$
\|g\|_{T^{p,2}\alpha(t^\beta dt dy)} = \left( \int_{\mathbb{R}^n} \left( \int_0^\infty \int_{\mathbb{R}^n} \frac{1_{B(x, t^{\frac{1}{2}})}}{t^n} |y|^{2 \frac{\alpha}{\tau}} dy dt \right)^{\frac{2}{p}} dx \right)^{\frac{1}{p}},
$$

the classical case corresponding to $\beta = -1$, $m = 1$, and being denoted simply
by $T^{p,2}$. The parameter $m$ is used to allow various homogeneities, and thus to
make these spaces relevant in the study of differential operators $L$ of order $m$. To
develop an analogue of [2] for $L^p$ data, we need, among many other estimates yet
to be proved, boundedness results for the maximal operator $M_L$ on these tent
spaces. This is the purpose of this note. Another motivation is well-posedness of
non-autonomous Cauchy problems for operators with varying domains, which will
be presented elsewhere. In the latter case, $M_L$ can be seen as a model of the
evolution operators involved. However, as $M_L$ is an important operator on its own,
we thought interesting to present this special case alone.

In Section 3 we state and prove the adequate boundedness results. The proof
is based on recent results and methods developed in [9], building on ideas from [5]
and [8]. In Section 2 we recall the relevant material from [9].

2. Tools. When dealing with tent spaces, the key estimate needed is a change of
aperture formula, i.e., a comparison between the $T^{p,2}$ norm and the norm

$$
\|g\|_{T^{p,2}\alpha} := \left( \int_{\mathbb{R}^n} \left( \int_0^\infty \int_{\mathbb{R}^n} \frac{1_{B(x, \alpha t)}}{t^n} |y|^{2 \frac{\alpha}{\tau}} dy dt \right)^{\frac{2}{\alpha}} dx \right)^{\frac{1}{\alpha}},
$$

for some parameter $\alpha > 0$. Such a result was first established in [6], building on
similar estimates in [7], and analogues have since been developed in various contexts.
Here we use the following version given in [9, Theorem 4.3].

**Theorem 2.1.** Let $1 < p < \infty$ and $\alpha \geq 1$. There exists a constant $C > 0$ such
that, for all $f \in T^{p,2}$,

$$
\|f\|_{T^{p,2}} \leq \|f\|_{T^{p,2}\alpha} \leq C(1 + \log \alpha)\alpha^{n/\tau} \|f\|_{T^{p,2}},
$$

where $\tau = \min(p, 2)$ and $C$ depends only on $n$ and $p$.

Theorem 2.1 is actually a special case of the Banach space valued result obtained
in [9]. Note, however, that it improves the power of $\alpha$ appearing in the inequality
from the $n$ given in [6] to $\frac{n}{\tau}$. This is crucial in what follows, and has been shown
to be optimal in [9].
Applying this to \((t, y) \mapsto t^{\frac{(n+1)}{2}} f(t^{m}, y)\) instead of \(f\), we also have the weighted result, where
\[
g \in T_{\kappa}^{\alpha, \beta}(t^{\beta} dt dy) = \left( \int_{\mathbb{R}^n} \left( \int_{0}^{\infty} \int_{\mathbb{R}^n} \frac{1}{t^{m}} B(x, \alpha t^{\frac{1}{m}}) (y) \|g(t, y)\|^2 t^\beta dy dt \right)^{\frac{p}{2}} dx \right)^{\frac{1}{\beta}}.
\]

**Corollary 2.2.** Let \(1 < p < \infty, m \in \mathbb{N}, \alpha \geq 1, \text{ and } \beta \in \mathbb{R}\). There exists a constant \(C > 0\) such that, for all \(f \in T_{p, m}^{\beta}(t^{\beta} dt dy)\),
\[
\|f\|_{T_{p, m}^{\beta}(t^{\beta} dt dy)} \leq \|f\|_{T_{\kappa}^{\alpha, \beta}(t^{\beta} dt dy)} \leq C (1 + \log \alpha)^{\alpha \eta / \tau} \|f\|_{T_{p, m}^{\beta}(t^{\beta} dt dy)},
\]
where \(\tau = \min(p, 2)\) and \(C\) depends only on \(n\) and \(p\).

To take advantage of this result, one needs to deal with families of operators, that behave nicely with respect to tent norms. As pointed out in [4], this does not mean considering \(R\)-bounded families (which means \(R\)-analytic semigroups when one considers \((t Le^{-tL})_{t \geq 0}\) as in the \(L^p(\mathbb{R}^+ \times \mathbb{R}^n)\) case, but tent bounded ones, i.e. families of operators with the following \(L^2\) off-diagonal decay, also known as Gaffney-Davies estimates.

**Definition 3.3.** A family of bounded linear operators \((T_t)_{t \geq 0} \subset B(L^2(\mathbb{R}^n))\) is said to satisfy off-diagonal estimates of order \(M\), with homogeneity \(m\), if, for all Borel sets \(E, F \subset \mathbb{R}^n\), all \(t > 0\), and all \(f \in L^2(\mathbb{R}^n)\):
\[
\|1_{E} T_{t} 1_{F} f\|_2 \lesssim \left( 1 + \frac{\text{dist} (E, F)^m}{t} \right)^{-M} \|1_{F} f\|_2.
\]

In what follows \(\|\cdot\|_2\) denotes the norm in \(L^2(\mathbb{R}^n)\).

As proven, for instance, in [4], many differential operators of order \(m\), such as (for \(m = 2\)) divergence form elliptic operators with bounded measurable complex coefficients, are such that \((t Le^{-tL})_{t \geq 0}\) satisfies off-diagonal estimates of any order, with homogeneity \(m\). This condition can, in fact, be seen as a replacement for the classical gaussian kernel estimates satisfied in the case of more regular coefficients.

### 3. Results.

**Theorem 3.1.** Let \(m \in \mathbb{N}, \beta \in (-\infty, \beta), p \in \left( -\frac{2n}{n+m(1-\beta)}, \infty \right) \cap (1, \infty), \text{ and } \tau = \min(p, 2).\) If \((t Le^{-tL})_{t \geq 0}\) satisfies off-diagonal estimates of order \(M > \frac{n}{m \tau}\), with homogeneity \(m\), then \(\mathcal{M}_L\) extends to a bounded operator on \(T_{p, m}^{\beta}(t^{\beta} dt dy)\).

**Proof.** The proof is very much inspired by similar estimates in [5] and [9]. Let \(f \in C_c(\mathbb{R}^+ \times \mathbb{R}^n)\). Given \((t, x) \in \mathbb{R}^+ \times \mathbb{R}^n, j \in \mathbb{Z}^n, \text{ we consider}
\[
C_j(x, t) = \begin{cases} B(x, t) & \text{if } j = 0, \\ B(x, 2^j t) \setminus B(x, 2^{j-1} t) & \text{otherwise.} \end{cases}
\]

We write \(\|\mathcal{M}_L f\|_{T_{p, m}} \leq \frac{\infty}{\sum_{k=1}^{\infty} \sum_{j=0}^{\infty} I_{k, j} + \sum_{j=0}^{\infty} J_{j}}\) where
\[
I_{k, j} = \left( \int_{\mathbb{R}^n} \left( \int_{0}^{\infty} \int_{\mathbb{R}^n} \frac{1}{t^{m}} B(x, \alpha t^{\frac{1}{m}}) (y) \int_{2^{-k} t}^{2^{-k+1} t} L e^{-L(s)} (1 C_j(x, 4t^{\frac{1}{m}}) f(s, \cdot) (y) ds)^2 t^\beta dy dt \right)^{\frac{p}{2}} dx \right)^{\frac{1}{\beta}},
\]
\[
J_{j} = \left( \int_{\mathbb{R}^n} \left( \int_{0}^{\infty} \int_{\mathbb{R}^n} \frac{1}{t^{m}} B(x, \alpha t^{\frac{1}{m}}) (y) \int_{\frac{t}{2}}^{t} L e^{-L(s)} (1 C_j(x, 4t^{\frac{1}{m}}) f(s, \cdot) (y) ds)^2 t^\beta dy dt \right)^{\frac{p}{2}} dx \right)^{\frac{1}{\beta}}.
\]
Fixing $j \geq 0$, $k \geq 1$ we first estimate $I_{k,j}$ as follows. For fixed $x \in \mathbb{R}^n$,

$$
\int_0^\infty \int_{B(x, t \frac{1}{2})} \left| \int_{2^{-k-1}t}^{2^{-k}t} \frac{Le^{-(t-s)L}(1_{C_j(x, at \frac{1}{2})} f(s, \cdot))(y)}{t - s} ds \right|^2 t^{\beta - \frac{n}{m}} dy dt
$$

\leq \int_0^\infty \int_{B(x, t \frac{1}{2})} \left( \int_{2^{-k-1}t}^{2^{-k}t} \left| Le^{-(t-s)L}(1_{C_j(x, at \frac{1}{2})} f(s, \cdot))(y) \right| \frac{ds}{t - s} \right)^2 t^{\beta - \frac{n}{m}} dy dt

\leq \int_0^\infty \int_{2^{-k-1}t}^{2^{-k}t} 2^{-k} \left( \int_{B(x, t \frac{1}{2})} \left| Le^{-(t-s)L}(1_{C_j(x, at \frac{1}{2})} f(s, \cdot))(y) \right|^2 dy \right) t^{\beta - \frac{n}{m}} ds dt

\leq 2^{-k} \int_0^\infty \left( \int_{2^{-k-1}t}^{2^{-k}t} t^{\beta - \frac{n}{m} - 1} dt \right) \left\| 1_{B(x, 2j + 1 \frac{k}{m} + 1 \frac{j}{m^2}} f(s, \cdot) \right\|_2^2 ds

\leq 2^{-k} \int_0^\infty \left( \int_{2^{-k-1}t}^{2^{-k}t} t^{\beta - \frac{n}{m} - 1} ds \right) \left\| 1_{B(x, 2j + 1 \frac{k}{m} + 1 \frac{j}{m^2}} f(s, \cdot) \right\|_2^2 ds.

In the second inequality, we use Cauchy-Schwarz inequality for the integral with respect to $t$, the fact that $t - s \sim t$ for $s \in \cup_{k \geq 1} [2^{-k-1}t, 2^{-k}t] \subset [0, \frac{1}{2}]$ and Fubini’s theorem to exchange the integral in $t$ and the integral in $y$. The next inequality follows from the off-diagonal estimate verified by $(t-s)Le^{-(t-s)L}$ and again the fact that $t - s \sim t$. By Corollary 2.2 this gives

$$
I_{k,j} \lesssim (j + k) 2^{-k} \left( \frac{1}{m} + 1 - \beta \right) t^{\beta - \frac{n}{m} - 1} \left\| f \right\|_{TP, 2, m(t^\beta dt dy)},
$$

where $\tau = \min(p, 2)$. It follows that $\sum_{k=1}^{\infty} \sum_{j=0}^{\infty} I_{k,j} \lesssim \left\| f \right\|_{TP, 2, m(t^\beta dt dy)}$ since $M > \frac{n}{m\tau}$ and $\frac{n}{m} + 1 - \beta > \frac{2n}{m\tau}$ (Note that for $p \geq 2$, this requires $\beta < 1$).

We now turn to $J_0$ and remark that $J_0 \leq \left( \int_{\mathbb{R}^n} J_0(x) \frac{2}{x} dx \right)^{\frac{1}{2}}$, where

$$
J_0(x) = \int_0^\infty \int_{\mathbb{R}^n} \left| \int_t^s \frac{Le^{-(t-s)L}(g(s, \cdot))(y)}{t - s} ds \right|^2 t^{\beta - \frac{n}{m}} dy dt
$$

with $g(s, y) = 1_{B(x, 4s \frac{1}{m})}(y)f(s, y)$. The inside integral can be rewritten as

$$
\mathcal{M}_L g(t, \cdot) - e^{-\frac{t}{2}L} \mathcal{M}_L g(t \frac{1}{2}, \cdot).
$$

As $\mathcal{M}_L$ is bounded on $L^2(\mathbb{R}_+ \times \mathbb{R}^n; t^{\beta - \frac{n}{m}} dy dt)$ by Theorem 1.1 and $(e^{-tL})_{t \geq 0}$ is uniformly bounded on $L^2(\mathbb{R}^n)$, we get

$$
J_0(x) \lesssim \int_0^\infty \left\| 1_{B(x, 4s \frac{1}{m})} f(s, \cdot) \right\|_2^2 s^{\beta - \frac{n}{m}} ds.
$$
We finally turn to \( J_j \), for \( j \geq 1 \). For fixed \( x \in \mathbb{R}^n \),
\[
\int_0^\infty \int_{\mathbb{R}^n} 1_{B(x,t^{1/2})}(y) \left| \int_{\frac{t}{2}}^t L e^{-(t-s)L} (1_{C_j(x,4s^{1/2})} f(s,\cdot))(y) \right| ds \, t^{\beta - \frac{m}{2}} \, dy \, dt 
\]
\[
\leq \int_0^\infty \int_{\mathbb{R}^n} 1_{B(x,t^{1/2})}(y) \left| \int_{\frac{t}{2}}^t (t-s) L e^{-(t-s)L} (1_{C_j(x,4s^{1/2})} f(s,\cdot))(y) \right| ds \left( \frac{t}{t-s} \right)^2 \, t^{\beta - \frac{m}{2}} \, dy \, dt 
\]
\[
\lesssim \int_0^\infty \int_{\mathbb{R}^n} 1_{B(x,t^{1/2})}(y) \left( \int_{\frac{t}{2}}^t (t-s) L e^{-(t-s)L} (1_{C_j(x,4s^{1/2})} f(s,\cdot))(y) \right)^2 \, ds \left( \frac{t}{t-s} \right)^2 \, t^{\beta - \frac{m}{2}} \, dy \, dt 
\]
\[
\lesssim \int_0^t (t-s)^{-2} \left( 1 + \frac{2^{jm} t}{t-s} \right)^{-2M} \left( \int_s^\infty (t-s)^{-2} \left( 1 + \frac{2^{jm} t}{t-s} \right)^{-2} dt \right) \left\| 1_{B(x,2^{j+2}s^{1/2})} f(s,\cdot) \right\|_{2^{j}\beta - \frac{m}{2}+1} \, ds 
\]
\[
\lesssim 2^{-jm(2M-2)} \int_0^\infty \left( \int_s^\infty (t-s)^{-2} \left( 1 + \frac{2^{jm} t}{t-s} \right)^{-2} dt \right) \left\| 1_{B(x,2^{j+2}s^{1/2})} f(s,\cdot) \right\|_{2^{j}\beta - \frac{m}{2}} \, ds 
\]
where we have used Cauchy-Schwarz inequality in the second inequality, the off-
diagonal estimates and the fact that \( s \leq t \) in the third, Fubini’s theorem and
the fact that \( s \geq \frac{t}{2} \) in the fourth, and the change of variable \( \sigma = \frac{1}{t-s} \) in the last. An
application of Corollary 2.2, then gives
\[
J_j \lesssim 2^{-jm} \int_{|\mathbb{T}|=2} \left\| f \right\|_{T^{\infty,2}_{\mathbb{T}}} = 2^{-jm} \int_{|\mathbb{T}|=2} \left\| f \right\|_{T^{\infty,2}_{\mathbb{T}}},
\]
and the proof is concluded by summing the estimates. \( \square \)

An end-point result holds for \( p = \infty \). In this context the appropriate tent space
consists of functions such that \( |g(t,y)|^{2 \, dydt} \) is a Carleson measure, and is defined
as the completion of the space \( \mathcal{C} \subset \mathbb{R}_+ \times \mathbb{R}^n \) with respect to
\[
\left\| g \right\|_{T^{\infty,2}_{\mathbb{T}}} = \sup_{(x,r) \in \mathbb{R}_+ \times \mathbb{R}^n} r^{-n} \int_{B(x,r)} \int_0^r \left| g(t,y) \right|^2 \, dydt 
\]
We also consider the weighted version defined by
\[
\left\| g \right\|_{T^{\infty,2}_{\mathbb{T}}} = \sup_{(x,r) \in \mathbb{R}_+ \times \mathbb{R}^n} r^{-\frac{m}{2}} \int_{B(x,r)} \int_0^r \left| g(t,y) \right|^2 t^{\beta} \, dydt.
\]

**Theorem 3.2.** Let \( m \in \mathbb{N} \), and \( \beta \in (-\infty,1) \). If \((tLe^{-L})_{t \geq 0}\) satisfies off-
diagonal estimates of order \( M > \frac{n}{2m} \), with homogeneity \( m \), then \( \mathcal{M}_L \) extends to a bounded
operator on \( T^{\infty,2}_{\mathbb{T}} \).

**Proof.** Pick a ball \( B(z,r^{1/2}) \). Let
\[
I^2 = \int_{B(z,r^{1/2})} \int_0^r \left( (\mathcal{M}_L f)(t,x) \right)^2 t^{\beta} \, dx \, dt.
\]
We want to show that \( I^2 \lesssim r^{\frac{m}{2}} \left\| f \right\|_{T^{\infty,2}_{\mathbb{T}}}. \) We set
\[
I_j^2 = \int_{B(z,r^{1/2})} \int_0^r \left( (\mathcal{M}_L f_j)(t,x) \right)^2 t^{\beta} \, dx \, dt
\]
where \( f_j(s, x) = f(s, x)1_{C_{j,(\varepsilon, \delta, t)}^c}(x)1_{(0, r)}(s) \) for \( j \geq 0 \). Thus by Minkowsky inequality, \( I \leq \sum I_j \). For \( I_0 \) we use again Theorem 1.1 which implies that \( M_L \) is bounded on \( L^2(\mathbb{R}_+ \times \mathbb{R}^n, t^\beta \, dx \, dt) \). Thus

\[
I_0^2 \lesssim \int_{\mathcal{B}_1} \int_0^r |f(t, x)|^2 \, t^\beta \, dx \, dt \lesssim r^{\frac{n}{m}} \|f\|_{T_{t, \infty}^{2, \alpha, m}(t^\beta \, dt \, dy)}^2.
\]

Next, for \( j \neq 0 \), we proceed as in the proof of Theorem 3.1 to obtain

\[
I_j^2 \lesssim \sum_{k=1}^{\infty} \int_0^r \int_{2^{-k-1}t}^{2^{-k}t} 2^{-kM} \left( 1 + \frac{2^jm \, r}{t-s} \right)^{-2M} \|f_j(s, \cdot)\|_{L^2}^2 \, t^\beta \, ds \, dt
\]

\[
+ \int_0^r \int_0^t (t-s)^{-2} \left( 1 + \frac{2^jm \, r}{t-s} \right)^{-2M} \|f_j(s, \cdot)\|_{L^2}^2 \, t^\beta \, ds \, dt.
\]

Exchanging the order of integration, and using the fact that \( t \sim t-s \) in the first part and that \( t \sim s \) in the second, we have the following.

\[
I_j^2 \lesssim \sum_{k=1}^{\infty} 2^{-kM} \int_0^2 \int_{2^{k+1} \beta}^{2^k \beta} \|f_j(s, \cdot)\|_{L^2}^2 \, t^\beta \, ds \, dt
\]

\[
+ \int_0^r \int_0^t (t-s)^{-2} \left( 1 + \frac{2^jm \, r}{t-s} \right)^{-2M} \|f_j(s, \cdot)\|_{L^2}^2 \, t^\beta \, ds \, dt
\]

\[
\lesssim \sum_{k=1}^{\infty} 2^{-kM} \int_0^2 \int_0^{(2^k \beta) \|f_j(s, \cdot)\|_{L^2}^2 \, ds \, ds
\]

\[
+ \int_0^r \int_0^{(1 + 2^jm \sigma)^{-2M} \|f_j(s, \cdot)\|_{L^2}^2 \, ds \, ds
\]

\[
\lesssim 2^{-2jM} \int_0^r \|f_j(s, \cdot)\|_{L^2}^2 \, ds,
\]

where we used \( \beta < 1 \). We thus have

\[
I_j^2 \lesssim 2^{-2jM} (2^j \frac{\pi}{m})^{n} \|f\|_{T_{t, \infty}^{2, \alpha, m}(t^\beta \, dt \, dy)}^2,
\]

and the condition \( M > \frac{n}{2m} \) allows us to sum these estimates. \( \square \)

**Remark 3.3.** Assuming off-diagonal estimates, instead of kernel estimates, allows to deal with differential operators \( L \) with rough coefficients. The harmonic analytic objects associated with \( L \) then fall outside the Calderón-Zygmund class, and it is common (see for instance [1]) for their boundedness range to be a proper subset of \((1, \infty)\). Here, our range \((\frac{2n}{n+1-\beta}, \infty]\) includes \([2, \infty]\) as \( \beta < 1 \), which is consistent with [2]. In the case of classical tent spaces, i.e., \( m = 1 \) and \( \beta = -1 \), it is the range \([2, \infty]\), where \( 2s \) denotes the Sobolev exponent \( \frac{2n}{n+2} \). We do not know, however, if this range is optimal.

**Remark 3.4.** Theorem 3.2 is a maximal regularity result for parabolic Carleson measure norms. This is quite natural from the point of view of non-linear parabolic PDE (where maximal regularity is often used), and such norm have, actually, already been used in the context of Navier-Stokes equations in [11], and, subsequently, for some geometric non-linear PDE in [12]. Theorem 3.1 is also reminiscent of Krylov’s Littlewood-Paley estimates [13], and of their recent far-reaching generalization in [15]. In fact, the methods and results from [9], on which this paper relies,
use the same circle of ideas (R-boundedness, Kalton-Weis $\gamma$ multiplier theorem...) as [15]. The combination of these ideas into a “conical square function” approach to stochastic maximal regularity will be the subject of a forthcoming paper.

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