

## Zhongwei Shen, Professor

Department of Mathematics
773 Patterson Office Tower
Lexington KY 40506-0027
(859) 257-3231 [TEL] • (859) 257-4078 [FAX]
shenz@ms.uky.edu
http://www.ms.uky.edu/~shenz

March 6, 2007

Professor Bernard PICHAUD Université Paul Cézanne- Aix-Marseille III Faculté des Sciences et Techniques Avenue Escadrille Normandie Niémen 13397 Marseille Cedex 20 France

## Dear Professor Bernard PICHAUD:

This is in response to your request for an evaluation of the research and scholarship of Professor Sylvie MONNIAUX in consideration of HDR Sciences.

Prof. MONNIAUX is a very active researcher working in the field of partial differential equations. In the last ten years or so, she has produced a substantial number of high quality original results on optimal regularities of solutions to abstract parabolic equations, the Stokes systems, and to the Navier-Stokes equations. I have never met Prof. MONNIAUX in person. But I have read several of her papers in the past with great interests. In particular I am very impressed by her recent work related to the strong solutions of the Navier-Stokes equations in Lipschitz domains. Below I will comment on this body of work in some details.

A Lipschitz domain is a bounded domain whose boundary is locally given by the graph of a Lipschitz continuous function. The class of Lipschitz domains is a dilation-invariant class which allows the domains to have faces, edges, and corners on their boundaries. The boundary regularities of solutions to partial differential equations on Lipschitz domains, which are of great importance both in terms of theoretical interests and engineering applications, have been studied extensively in the last thirty years, using sophisticated techniques from harmonic analysis. Many sharp tools and results have been established for linear equations. In the light of these development, it is natural to examine some of the classical approaches to nonlinear equations in the non-smooth setting. This is the focus of Prof. MONNIAUX's research program in last few years.

In a series of papers [43, 44, 45, 36, 37], Prof. MONNIAUX has made significant progress on the existence, uniqueness, and optimal regularities of strong solutions to the Navier-Stokes equations on non-smooth domains. In particular, She and Marius Mitrea [37] investigated the functional analytical approach of Fujita and Kato in Lipschitz domains. The results in [37] improve earlier ones by Deuring-von Wahl and Brown-Shen. The proof, which uses an elaborated interpolation schemes, is very impressive.

The use of the Fujita and Kato approach relies heavily on the better understanding of the Stokes operator. However, many questions remains open for the operator in non-smooth domains, even in lower dimensions. For example, it is not known that the resolvent estimates in  $L^p$  hold for  $p \neq 2$ . In [36,38] MONNIAUX and Mitrea studied the Stokes operator with a set of Neumann type boundary conditions and are able to establish the resolvent estimates for  $3/2 \leq p \leq 3$ . This result is highly original.

In summary I think Prof. MONNIAUX is a strong analyst, and certainly one of leading experts on the Navier-Stokes equations in non-smooth domains. If she were is US, her research record in the last ten years would be strong enough to warrant a tenure appointment at any major research university.

Sincerely,

Zhongwei Shen