PH.D. PROPOSAL IN SPECTRAL THEORY AND ASYMPTOTIC ANALYSIS

In the framework of the ANR project *Geometrically Dependent Hamiltonian* (GeoDHa), we are hiring a Ph.D. student. The Ph.D. student will be working at the *Institut de Mathématiques de Marseille* in the Applied Analysis team, located on the Saint-Charles campus, and will work under the supervision of F. Monteghetti and T. Ourmières-Bonafos.

Key-words: Partial Differential Equations, Spectral Theory, Asymptotic Analysis, Differential geometry, Finite elements.

Scientific context: We propose to investigate the impact of geometry on the energy levels of quantum mechanical models that incorporate the effects of special relativity. Such models arise in particle physics and in the study of graphene-like materials.

From a mathematical standpoint, for a given domain $\Omega \subset \mathbb{R}^d$, this involves finding eigenpairs $(\lambda, u) \in \mathbb{R} \times L^2(\Omega, \mathbb{C}^N)$ that satisfy

$$\begin{cases} \mathscr{D}u = \lambda u & \text{in } \Omega, \\ \mathscr{B}u = 0 & \text{on } \partial\Omega. \end{cases}$$

where $\mathscr{D}$ is a first-order differential operator known as the Dirac operator and $\mathscr{B}$ represents a boundary operator that imposes boundary conditions ensuring the self-adjointness of the operator.

The overall objective of the thesis is to understand how such eigenpairs (λ, u) depend on the geometry of Ω. While answering this question in full generality is challenging, there are simpler cases where a precise description of the spectrum is possible, providing clear insights into the impact of the geometry.

One example of such a situation is to consider a family of domains in $\mathbb{R}^d$, denoted by $(\Omega_t)_{t>0}$, which degenerate to a submanifold of $\mathbb{R}^d$ with co-dimension 1 as $t \to 0$. One may for example consider for $t > 0$ the family

$$\Omega_t = \{(x, y) \in \mathbb{R}^{d-1} \times \mathbb{R} : x \in \omega,\ 0 < y < tf(x)\},$$

where ω is a smooth simply connected domain of $\mathbb{R}^{d-1}$ and $f : \overline{\omega} \to \mathbb{R}_+$ is a smooth function that meets the conditions $f|_{\partial\omega} \equiv 0$, strictly positive on ω with a unique non-degenerate global maximum. In this setting, it is possible to obtain asymptotic expansions for the eigenmodes (λ_t, u_t) where the geometry appears explicitly in the leading terms of the asymptotic expansion.

Several problems in this spirit, approached from different perspectives, could be explored by the Ph.D. student, either from a theoretical or numerical standpoint, depending on her/his interests.

Pre-requisite: A master degree in mathematics and applications, with an emphasis on analysis. Knowledge of basic operator theory, spectral theory, differential geometry and/or of finite elements method would be a plus *but is not mandatory.*

Application: Send a CV to Thomas Ourmières-Bonafos (thomas.ourmieres-bonafos@univ-amu.fr) with Florian Monteghetti in copy (florian.monteghetti@univ-amu.fr). A recommendation letter for a Ph.D. program from one of your master's teacher have to be sent to the same e-mail address.

Deadline for application: 3 February 2025.

Expected beginning of the position: September or October 2025. We may also discuss with the selected candidate the possibility of doing her/his master's internship in Marseille on a similar subject.

Contact: If you need more informations do not hesitate to contact Thomas Ourmières-Bonafos (thomas.ourmieres-bonafos@univ-amu.fr).

Below is a list of references to provide an initial overview of the subject (this list is far from being exhaustive).

References

[1] Borisov, D., Freitas, P.: *Asymptotics of Dirichlet eigenvalues and eigenfunctions of the Laplacian on thin domains in* $\mathbb{R}^d$, Journal of functional analysis Vol. 258, **3**, 2010, 893–912.

[2] Lotoreichik, V., Ourmières-Bonafos, T.: *Spectral asymptotics of the Dirac operator in a thin shell*, Preprint ArXiv:2307.09033.

[3] Ourmières-Bonafos, T.: *D*irichlet Eigenvalues of Asymptotically Flat Triangles, Asymptotic Analysis, vol. 92, no. 3-4, pp. 279–312, 2015.

[4] B. Thaller, *The Dirac equation*, Springer-Verlag, Berlin Heidelberg, 1992.