β-NMF and sparsity promoting regularizations for complex mixture unmixing: Application to 2D HSQC NMR.

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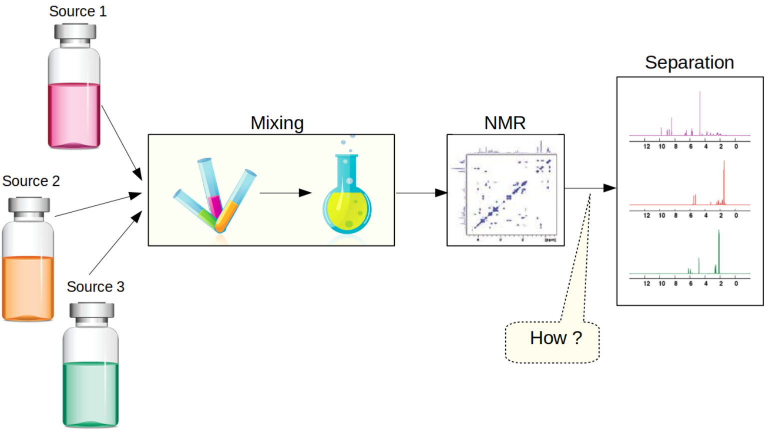
Outline

Introduction

- HSQC NMR experience
  - **NMR** : Nuclear Magnetic Resonance, a spectroscopy technique used to identify molecules in a given chemical mixture.
  - **2D HSQC** : Heteronuclear Single Quantum Coherence, a NMR experience used to determinate the correlations between a carbon and its attached proteins.
  - **BSS** : Blind Source Separation, an efficient mathematical method used to analyze data which are modeled as the linear combination of elementary sources or components.

Introduction



Explanatory scheme

Introduction

■ Problem statement

$$\mathbf{X} = \mathbf{A}\mathbf{S} + \mathbf{N}$$

- $\mathbf{X} = (x_{m,\ell}) \in \mathbb{R}^{M \times L}$: given mixtures
- $\mathbf{S} = (s_{n,\ell}) \in \mathbb{R}^{N \times L}$: unknown sources
- $\mathbf{A} = (a_{m,n}) \in \mathbb{R}^{M \times N}$: unknown mixing matrix
- $\mathbf{N} = (n_{m,\ell}) \in \mathbb{R}^{M \times L}$: acquisition noise

■ Difficulties

✗ Indeterminacies of solutions

$$(\exists \Lambda \in \mathbb{R}^{N \times N}) \quad \text{such that} \quad \mathbf{A}' = \mathbf{A}\Lambda \ \text{ et } \ \mathbf{S}' = \Lambda^{-1}\mathbf{S}$$

where Λ is a diagonal or a permutation matrix.

✗ 2D NMR spectra present a high level of sparsity with a spectral overlap and poor resolution.

Variational formulation

- Regularized approach

$$\underset{\mathbf{A},\,\mathbf{S}}{\text{minimize}}\;\; \Theta(\mathbf{A},\mathbf{S}) := \underbrace{\Phi(\mathbf{A},\mathbf{S})}_{\text{Data fidelity}} + \underbrace{\Psi(\mathbf{A},\mathbf{S})}_{\text{Regularization term}}$$

- Standard choices [Cherni et al., 2019]

$$\underset{\mathbf{A},\mathbf{S}}{\text{minimize}}\;\; \frac{1}{2}\|\mathbf{X}-\mathbf{A}\mathbf{S}\|_F^2 + \lambda_{\mathbf{A}}\Psi_{\mathbf{A}}(\mathbf{A}) + \lambda_{\mathbf{S}}\Psi_{\mathbf{S}}(\mathbf{S})$$

$\lambda_{\mathbf{A}}$ and $\lambda_{\mathbf{S}}$ are regularization parameters.
$\Psi_{\mathbf{A}}$ and $\Psi_{\mathbf{S}}$ are regularization functions.

$$\iota_+(\mathbf{u}) = \begin{cases} 0 & \text{if } u_i \geq 0\ \forall i \\ +\infty & \text{otherwise.} \end{cases}$$

$$\ell_1(\mathbf{u}) = \left(\sum_{i=1}^{L} |u_i|\right)$$

$$\mathrm{Ent}(\mathbf{u}) = \sum_{i=1}^{L} \mathrm{ent}(u_i)$$

$$\mathrm{ent}(u) = \begin{cases} u\log(u) & \text{if } u > 0 \\ 0 & \text{if } u = 0 \\ +\infty & \text{otherwise} \end{cases}$$

Proposed approach

- Generalization

$$\underset{\mathbf{A},\,\mathbf{S}}{\text{minimize}} \;\; \beta\text{-}\mathbf{div}(\mathbf{A},\mathbf{S}) + \lambda_{\mathbf{A}}\Psi_{\mathbf{A}}(\mathbf{A}) + \lambda_{\mathbf{S}}\Psi_{\mathbf{S}}(\mathbf{S})$$

where

$$(\forall\,(\mathbf{u},\mathbf{v}) \in (\mathbb{R}_+^L)^2) \quad \beta\text{-}\mathbf{div}(\mathbf{u},\mathbf{v}) = \sum_{i=1}^{L} \beta\text{-div}(u_i|v_i)$$

and for all $(u,v) \in \mathbb{R}_+^2$

$$\beta\text{-div}(u|v) = \begin{cases} \frac{1}{\beta(\beta-1)}\left(u^\beta + (\beta-1)v^\beta - \beta u v^{\beta-1}\right) & \text{if } \beta \in \mathbb{R} \setminus \{0,1\} \\ \frac{u}{v} - \log\left(\frac{u}{v}\right) - 1 & \text{if } \beta = 0 \\ u\log\left(\frac{u}{v}\right) - u + v & \text{if } \beta = 1 \end{cases}$$

- Frobenius norm is a special case of β-**div** where $\beta = 2$.

MM based multiplicative algorithm

- Generic alternating algorithm [Hunter and Lange, 2000]

For $k = 0, 1, \ldots$

$$\left\lfloor \begin{array}{ll} \mathbf{A}_{k+1} = \underset{\mathbf{A}}{\operatorname{argmin}} \ \beta\text{-}\mathbf{div}(\mathbf{X}, \mathbf{A}\mathbf{S}_k) + \lambda_{\mathbf{A}} \Psi_{\mathbf{A}}(\mathbf{A}) & (\mathbf{I}) \\ \mathbf{S}_{k+1} = \underset{\mathbf{S}}{\operatorname{argmin}} \ \beta\text{-}\mathbf{div}(\mathbf{X}, \mathbf{A}_{k+1}\mathbf{S}) + \lambda_{\mathbf{S}} \Psi_{\mathbf{S}}(\mathbf{S}) & (\mathbf{II}) \end{array} \right.$$

and for $\beta > 2$

(**I**) $\Psi_{\mathbf{A}} = \iota_+$

$$\mathbf{A}_{k+1} = \left(\frac{\left(\mathbf{X} \odot (\mathbf{A}_k\mathbf{S})^{\odot(\beta-2)}\right)\mathbf{S}^T}{(\mathbf{A}_k\mathbf{S})^{\odot(\beta-1)}\mathbf{S}^T} \right)_+^{\odot\frac{1}{\beta-1}} \odot \mathbf{A}_k$$

(**II**)-a) $\Psi_{\mathbf{S}} = \iota_+$

$$\mathbf{S}_{k+1} = \left(\frac{\mathbf{A}^T\left(\mathbf{X} \odot (\mathbf{A}\mathbf{S}_k)^{\odot(\beta-2)}\right)}{\mathbf{A}^T(\mathbf{A}\mathbf{S}_k)^{\odot(\beta-1)}} \right)_+^{\odot\frac{1}{\beta-1}} \odot \mathbf{S}_k$$

(II)-b) $\Psi_{\mathbf{S}} = \ell_1 + \iota_+$

$$\mathbf{S}_{k+1} = \left(\frac{\mathbf{A}^T (\mathbf{X} \odot (\mathbf{A}\mathbf{S}_k)^{\odot(\beta-2)}) - \lambda_{\mathbf{S}}}{\mathbf{A}^T (\mathbf{A}\mathbf{S}_k)^{\odot(\beta-1)}} \right)_+^{\odot \frac{1}{\beta-1}} \odot \mathbf{S}_k$$

(II)-c) $\Psi_{\mathbf{S}} = \mathrm{Ent} + \iota_+$

$$\mathbf{S}_{k+1} = \left(\frac{\gamma}{\alpha} \mathbf{W} \left(\frac{\alpha}{\gamma} \exp(-\frac{\delta}{\gamma}) \right) \right)_+^{\odot \frac{1}{\beta-1}} \odot \mathbf{S}_k$$

where $\odot$ denotes the Hadamard product, $\mathbf{W}$ is the W-Lambert function [Corless et al., 1996] and

$$\begin{cases} \alpha = \mathbf{A}^T (\mathbf{A}\mathbf{S}_k)^{\odot(\beta-1)} \odot \mathbf{S}_k, \\ \gamma = \frac{\lambda_{\mathbf{S}}}{\beta-1} \mathbf{S}_k, \\ \delta = \lambda_{\mathbf{S}} (\mathbf{S}_k + \mathbf{S}_k \odot \log(\mathbf{S}_k)) - \mathbf{A}^T (\mathbf{X} \odot (\mathbf{A}\mathbf{S}_k)^{\odot(\beta-2)}) \odot \mathbf{S}_k. \end{cases}$$

2D HSQC NMR data

- Data

Real case

$\mathbf{X} \in \mathbb{R}^{5\times1024\times2048}$: 5 mixtures

$\mathbf{S} \in \mathbb{R}^{4\times1024\times2048}$: 4 sources

$\mathbf{A} \in \mathbb{R}^{5\times4}$: a mixture matrix

Simulated case

$\mathbf{X}$ is simulated following model

$$\mathbf{X} = \mathbf{AS} + \mathbf{N}$$

$\mathbf{N} \sim \mathcal{N}(0, \sigma^2)$, $\sigma = 2 \times 10^4$

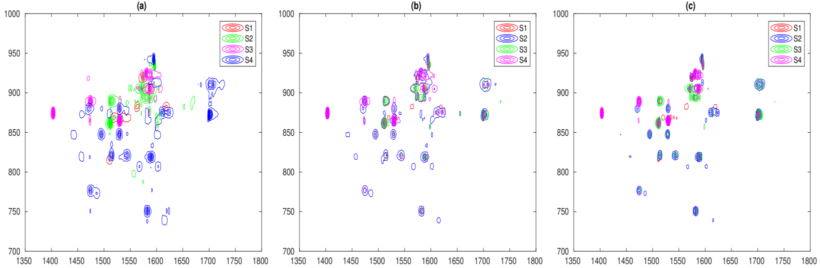
- Performance criteria [Vincent et al., 2006, Moreau et al., 1994]
  - SIR : Source to Interference Ratio
  - SDR : Source to Distortion Ratio
  - SAR : Source to Artifacts Ratio
  - Amari Index

Results on simulated data

Data fidelity term	λ_S	Ψ_S	SDR	SIR	SAR	Amari-index
Squared Frobenius	ι_+		18.073	28.854	18.514	0.0121
	0.1σ	$\ell_1 + \iota_+$	**30.299**	31.475	**39.462**	0.0272
		Ent $+\iota_+$	18.287	36.859	18.354	0.0090
	σ	$\ell_1 + \iota_+$	21.140	21.788	29.872	0.0492
		Ent $+\iota_+$	17.334	**36.909**	17.421	0.0198
	10σ	$\ell_1 + \iota_+$	17.041	25.581	22.104	0.0189
		Ent $+\iota_+$	16.021	30.625	18.216	0.0861
β-divergence	ι_+		**36.711**	40.854	**41.571**	**0.0054**
	0.1σ	$\ell_1 + \iota_+$	36.531	40.853	41.255	**0.0054**
		Ent $+\iota_+$	**36.711**	40.854	41.570	**0.0054**
	σ	$\ell_1 + \iota_+$	32.041	40.868	34.135	**0.0054**
		Ent $+\iota_+$	36.710	40.852	41.570	**0.0054**
	10σ	$\ell_1 + \iota_+$	22.906	**41.140**	23.102	**0.0054**
		Ent $+\iota_+$	36.688	40.851	41.513	**0.0054**

Average criteria obtained with various λ_S for $\beta = 3$.

Results on real data



Pure sources (a), estimated sources using Frobenius norm (b), estimated sources using β-**div** (c).

Data fidelity term	Ψ_S	SDR	SIR	SAR	Amari-index
Squared Frobenius	$\ell_1 + \iota_+$	04.984	13.956	07.951	0.1804
	Ent $+\iota_+$	05.755	**14.434**	08.446	0.1793
β-divergence	$\ell_1 + \iota_+$	**07.240**	11.487	10.574	**0.1610**
	Ent $+\iota_+$	07.220	11.396	**10.632**	0.1657

Average criteria obtained with $\lambda_S = 10\sigma$ for $\beta = 3$.

Conclusion & perspectives

✓ The β-divergence combined with ℓ_1 norm or Ent function ensures the BSS of the 2D HSQC NMR.
✓ In the real case, better SDR and SAR values are obtained using β-divergence. However, a slight deterioration on the SIR values is noticed.

- Optimize the choice of $\lambda_{\mathbf{S}}$.
- Verify the linearity of the model in the context of 2D NMR.

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