Gravitational-wave polarimetry with quaternions and application to precessing binaries

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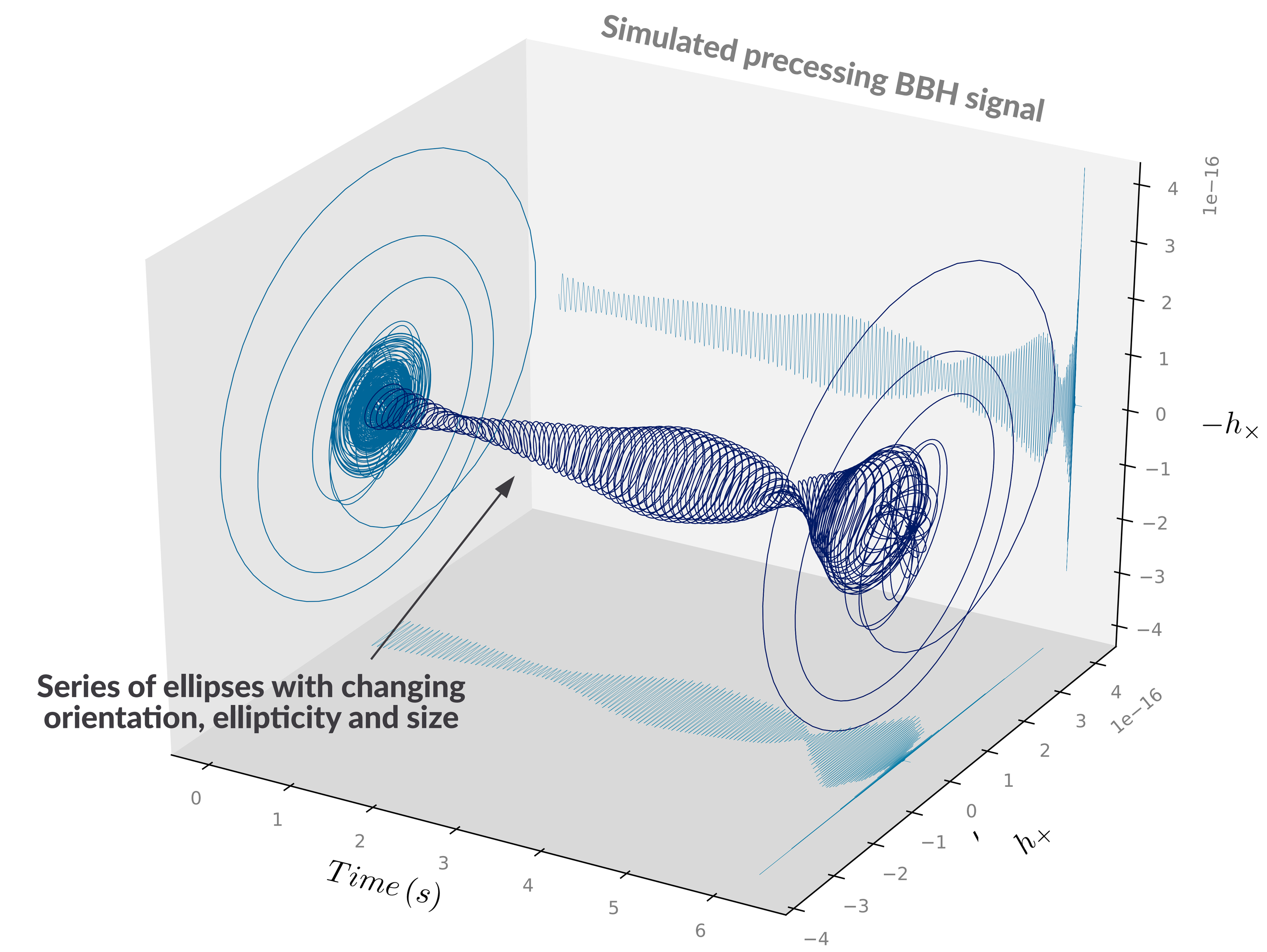
In this poster we introduce a new model free polarimetric analysis method based on quaternion Fourier transform and we also present a method to reconstruct both gravitational-wave polarizations from the full network data. Interestingly, this formalism allows to formulate generic priors on the polarization state that can guide the problem inversion. As an illustration we perform the polarimetric spectral analysis on precessing BBH signal and show that the precessional motion of the binary orbital plane can be tracked from the polarization time evolution.

1. Polarization encodes the source physics : the example of precessing binary mergers

- Polarizations h_+ and $h_\times$ form a 2D or bivariate signal
- Relationship between polarizations carries astrophysical information
- Example with precessing BBH merger :
  - Precession of the orbital plane induces 'modulation' of the signal trajectory in the h_+ vs $h_\times$ phase space (see figure)



2. Polarimetric analysis of bivariate signals

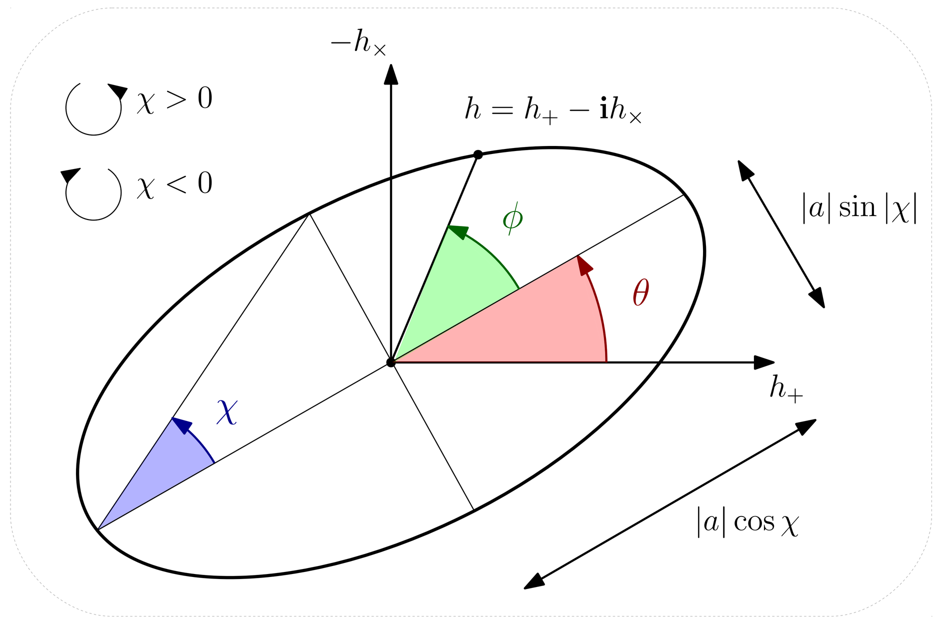
- Map the polarization to complex strain $h = h_+ - \mathbf{i}h_\times$
- Quaternionic analysis give access to geometrical parameters
- Time-frequency : quaternionic short term Fourier transform

$$\underbrace{\mathrm{S}_h(f,\tau)}_{\text{Quaternion}} = \int_{\mathbb{R}} \underbrace{h(t)}_{\text{Complex}}\underbrace{g(t-\tau)}_{\text{Real window}}\underbrace{e^{-\mathbf{j}f\tau}}_{\text{Quaternion}}dt$$

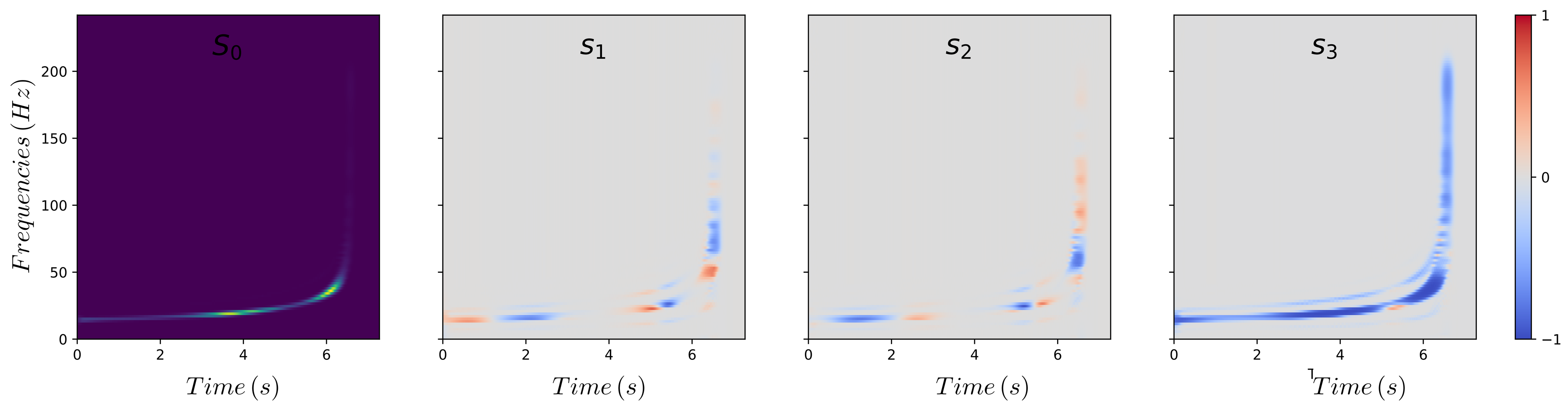
Derive polarization observables (no model required) : Stokes parameters

$$\text{Energy}\ \left\{ S_0(f,\tau) = |a(f,\tau)|^2 \right.$$

$$\text{Polarization}\ \left\{ \begin{aligned} s_1(f,\tau) &= \cos(2\chi(f,\tau))\cos(2\theta(f,\tau)) \\ s_2(f,\tau) &= \cos(2\chi(f,\tau))\sin(2\theta(f,\tau)) \\ s_3(f,\tau) &= \sin(2\chi(f,\tau)) \end{aligned} \right.$$



Geometrical parametrisation of instantaneous polarization



Polarized spectrogram for noise-free precessing BBH signal

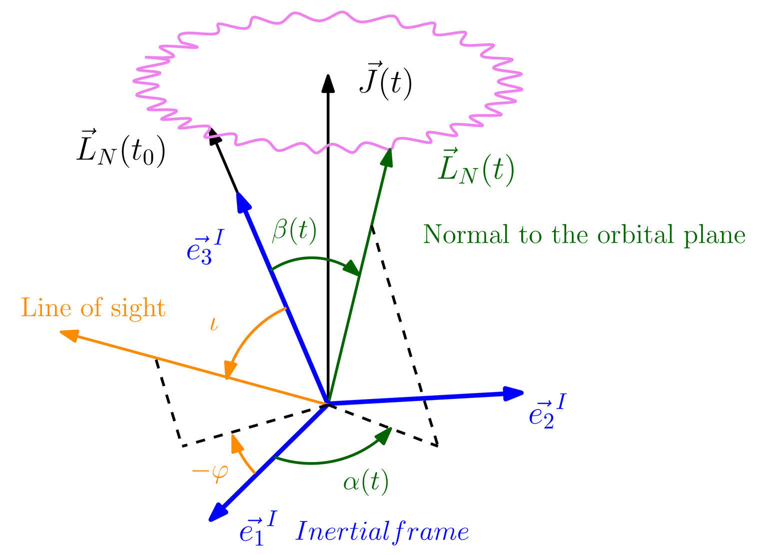
3. Interpretation and polarization characterization

- Stokes obtained from the time frequency ridge
- For precessing BBH : Stokes related to precession parameters such as β (Precession opening angle)

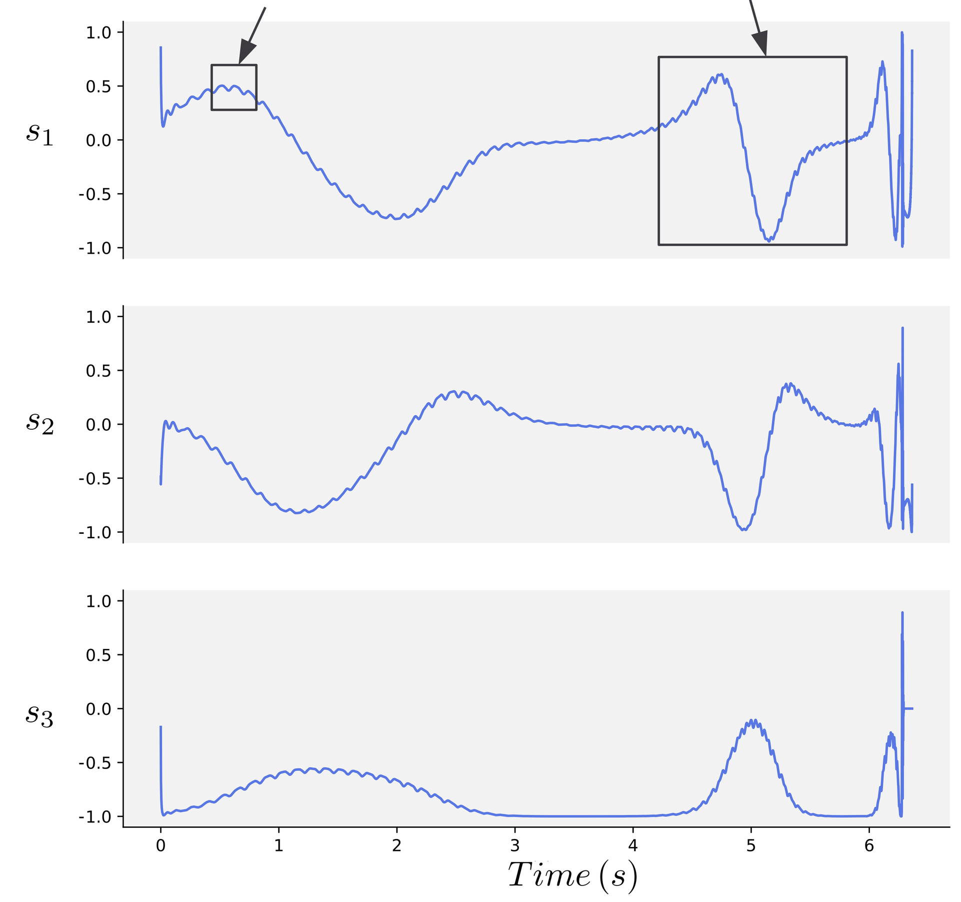
  Possibility to infer on the presence of precession

- For the edge on (largest effect) and small precession case :

$$s_1 = 1 + \mathcal{O}(\beta^2)$$
$$s_2 = 4\sin(\alpha)\,\beta + \mathcal{O}(\beta^2)$$
$$s_3 = -\cos(\alpha)\,\beta + \mathcal{O}(\beta^2)$$



Precession parameters : $\iota, \varphi, \alpha, \beta$



Stokes obtained from noise-free precessing BBH signal

4. Reconstruction of h_+ and $h_\times$ with polarization constraint

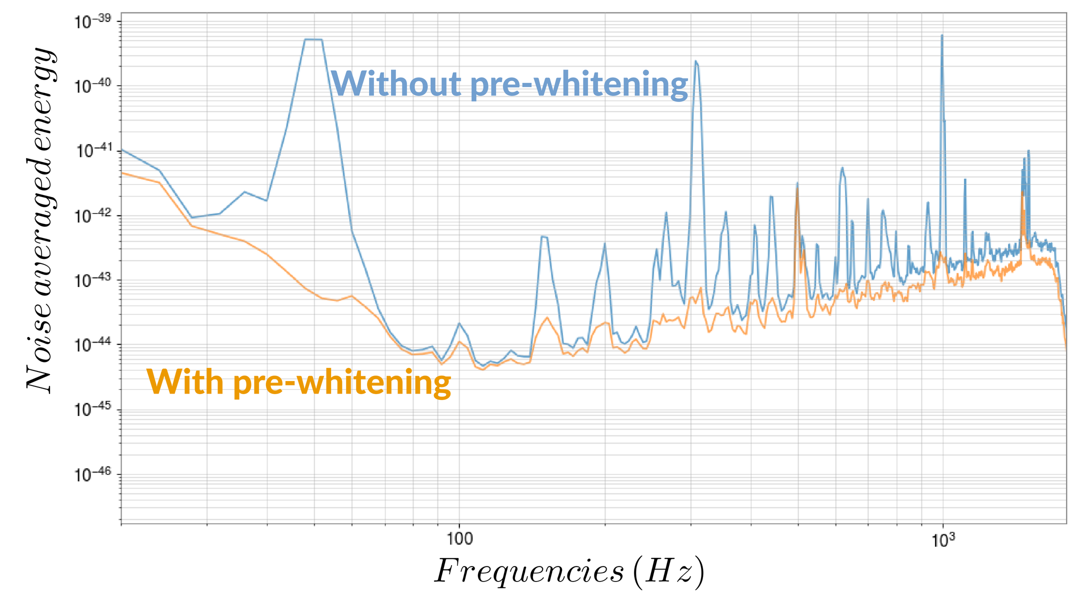
$$\underbrace{x(f,\tau)}_{\text{Network data}} = \underbrace{Fh(f,\tau)}_{\text{Beam pattern}} + \underbrace{n(f,\tau)}_{\text{Noise}} \in \mathbb{C}^{N_{\text{det}}}$$

- Quaternionic representations offer a natural formalism to formulate and resolve the inverse problem with a polarization prior
- For simplicity : here we assume known sky source localization
- Two solutions : work with pre-whitening or without

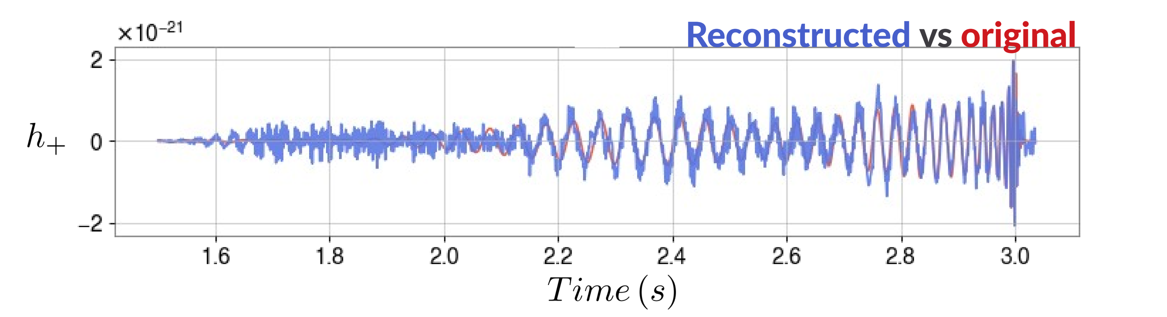
$$x(f,\tau) \begin{cases} \xrightarrow{\text{Without pre-whitening}} F^\dagger x(f,\tau) \\ \xrightarrow{\text{With pre-whitening}} \tilde{x}(f,\tau) \to \tilde{F}^\dagger(f,\tau)\tilde{x}(f,\tau) \end{cases} \to y(f,\tau) = h(f,\tau) + n(f,\tau) \in \mathbb{C}^2$$

$$\hat{h} = \arg\min_h \underbrace{\sum_{(f,\tau)\in\Gamma} |\tilde{y}(f,\tau) - \tilde{h}(f,\tau)|^2}_{\text{Mean squared error}} + \underbrace{\Omega(h)}_{\text{Prior}}$$

(Γ: Time-frequency cluster)



Example of prior :
- Sparsity promoting prior $|h|$ (LASSO)
- Polarization targetting prior



Reconstruction with Lasso to select Γ and then least square (high SNR, HLV)

Take-home message and perspectives

- Quaternionic time-frequency transform allows the 'unmodelled' characterization of the polarization state
- Offer a natural formalism for the inversion of $h_{+,\times}$ with polarization targetting priors
- Allows the detection of precession for high SNR and highly precessing binaries

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