Partial Shape Matching in the Space of Varifolds

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Objective: Build an asymmetric dissimilarity term that allows to compare a shape to a subset of another one.

Challenges:

- Shape comparison with topological changes.
- Deal with local minima.
- Build diffeomorphic deformations without bijective correspondances assumption.

Proposed approach:

- Build the dissimilarity term in the shape space of Varifolds.
- Adapt the function to the Large Deformations Diffeomorphic Metric Mapping (LDDMM) [1].

From the classic distance to the partial dissimilarity term in the space of Varifolds

Source **S** and target **T** are finite reunion of m-dimensional submanifolds of $\mathbb{R}^d$. Here $\mathrm{d} = 3$ and $\mathrm{m} = 2$ (surfaces) or $\mathrm{m} = 1$ (curves). We consider **W** a RKHS* of RK $\boldsymbol{k_w} = \boldsymbol{k_e} \times \boldsymbol{k_t}$, continuously embedded in $C_0(\mathbb{R}^d \times \mathbb{S}^{d-1}, \mathbb{R})$. Then $C_0(\mathbb{R}^d \times \mathbb{S}^{d-1}, \mathbb{R})'$ is continuously embedded in W'. We define $\omega_S \in C_0(\mathbb{R}^d \times \mathbb{S}^{d-1}, \mathbb{R})$:

$$\omega_S(y,\eta) = \int_S k_W\big((y,\eta),(x,\tau_x S)\big)dx = \int_S k_e(y,x).k_t(\eta,\tau_x S)dx\,.$$

We define the varifold $\mu_S \in W'$:

$$\mu_S(\omega) = \int_S \omega(x,\tau_x S)dx\,.$$

$$\langle \mu_S | \mu_T \rangle_{W'} = \int_S \int_T k_e(y,x).k_t(\tau_y T,\tau_x S)dydx.$$

$$k_e(x_i,x_j) = e^{-\frac{\|x_i-x_j\|_2^2}{\sigma^2}}$$

$$k_t(\tau_i,\tau_j) = e^{\langle \tau_i | \tau_j \rangle_{\mathbb{R}^d}}$$

Hence the distance as in [2]:

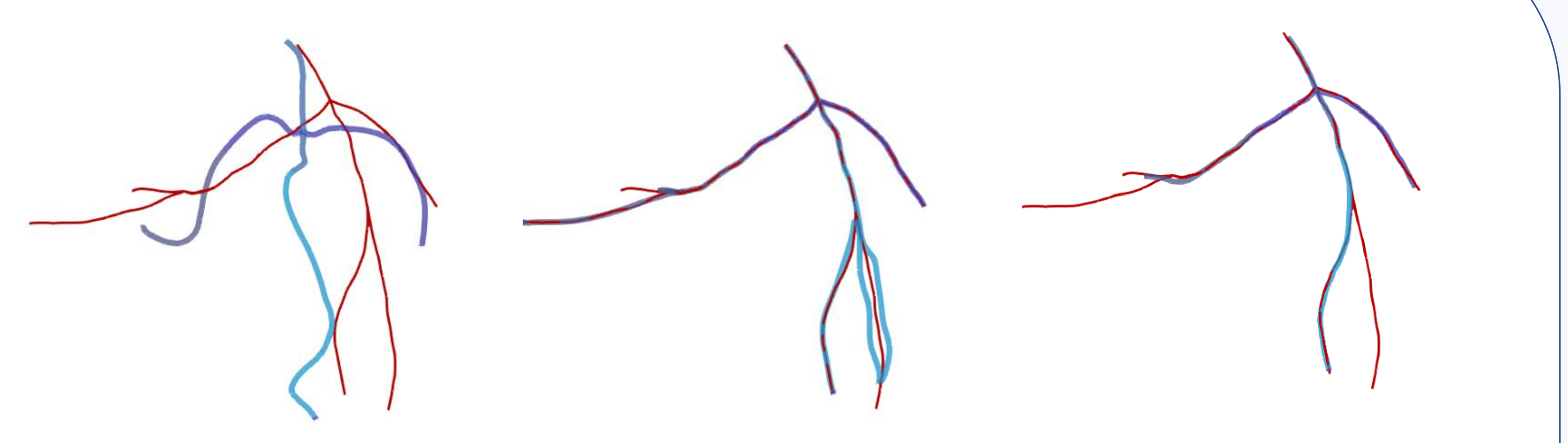
$$d(S,T) = \|\mu_S - \mu_T\|^2{}_{W'} = \|\mu_S\|^2{}_{W'} - 2.\langle \mu_S | \mu_T \rangle_{W'} + \|\mu_T\|^2{}_{W'}. \qquad (1)$$

The partial dissimilarity term is inspired from half the expression (1) : $\left(\|\mu_S\|^2{}_{W'} - \langle \mu_S | \mu_T \rangle_{W'}\right)^2 = \langle \mu_S | \mu_S - \mu_T \rangle_{W'}{}^2$

$$\boldsymbol{\Delta(S,T) = \int_S g\big(\omega_S(x,\tau_x S) - \widetilde{\omega_T}(x,\tau_x S)\big)\,dx},$$

with $g(x) = \max(0,x)^2$ and $\widetilde{\omega_T}(x,\tau_x S) = \int_T \min\left(1,\frac{\omega_S(x,\tau_x S)}{\omega_T(y,\tau_y T)}\right) k_e(y,x).k_t(\tau_y T,\tau_x S)dy\,.$

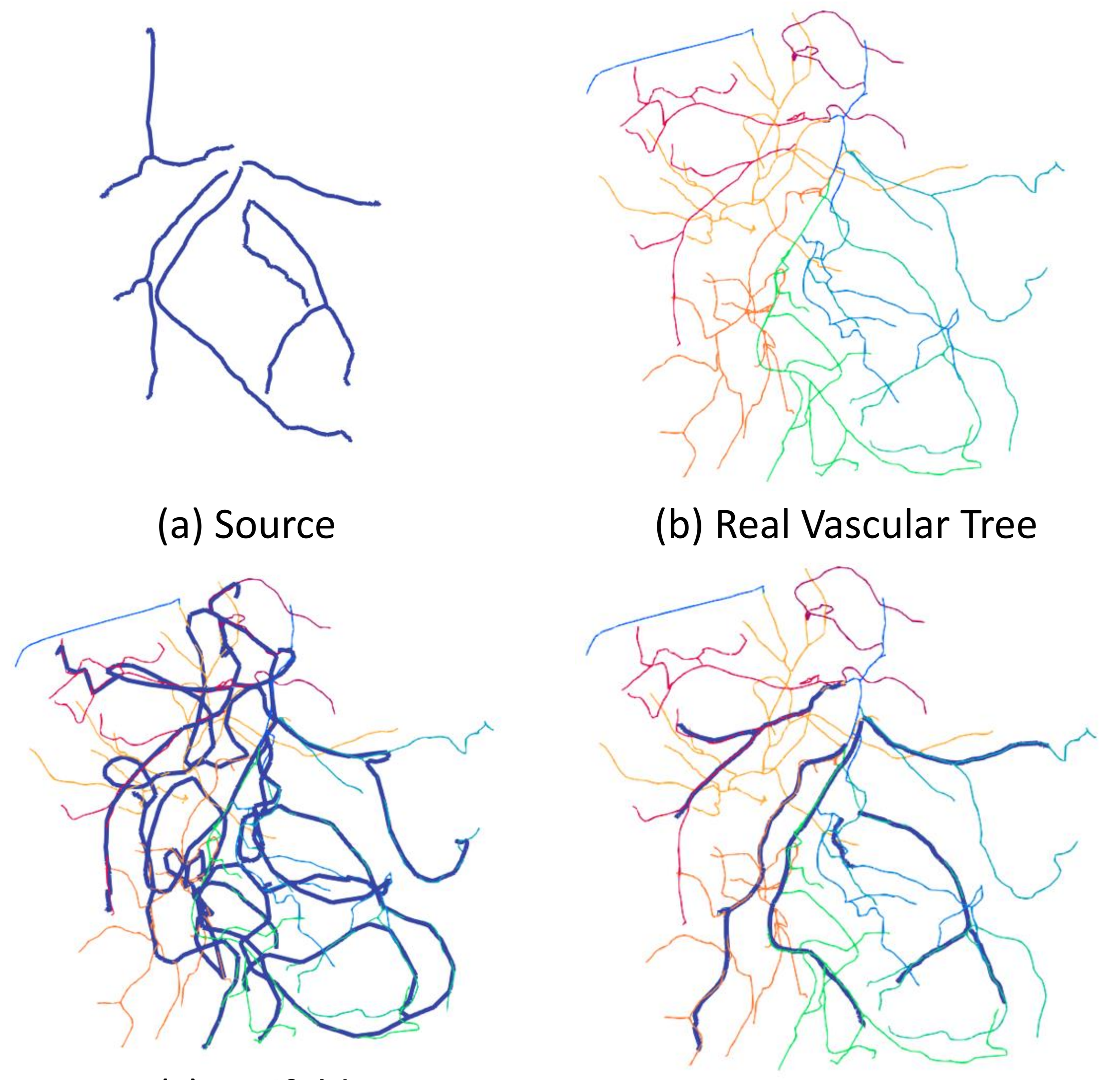
Application to a synthetic case



(a) Source (blue) and target (red) (b) Varifold registration (c) Proposed partial matching

(a) Registration of a tree of 3D curves (blue) onto a target (red). The varifold registration (b) leads to abnormal distortions of the source to match the whole target. The partial matching (c) solution corresponds to the inclusion of the source in the target.

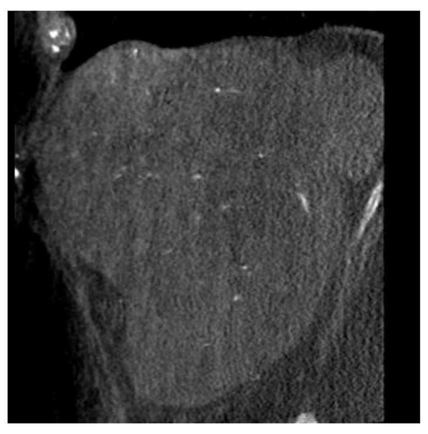
Registration of a template onto a real Vascular Tree



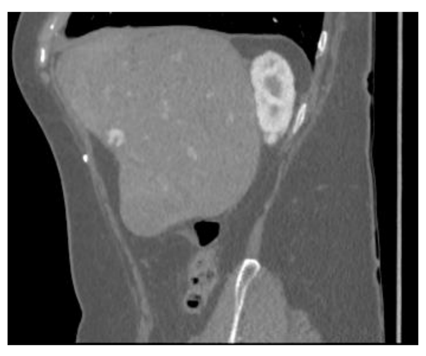
(a) Source (b) Real Vascular Tree (c) Varifold (d) Partial matching

Cone Beam Computed Tomography (CBCT) of a pelvic anatomy

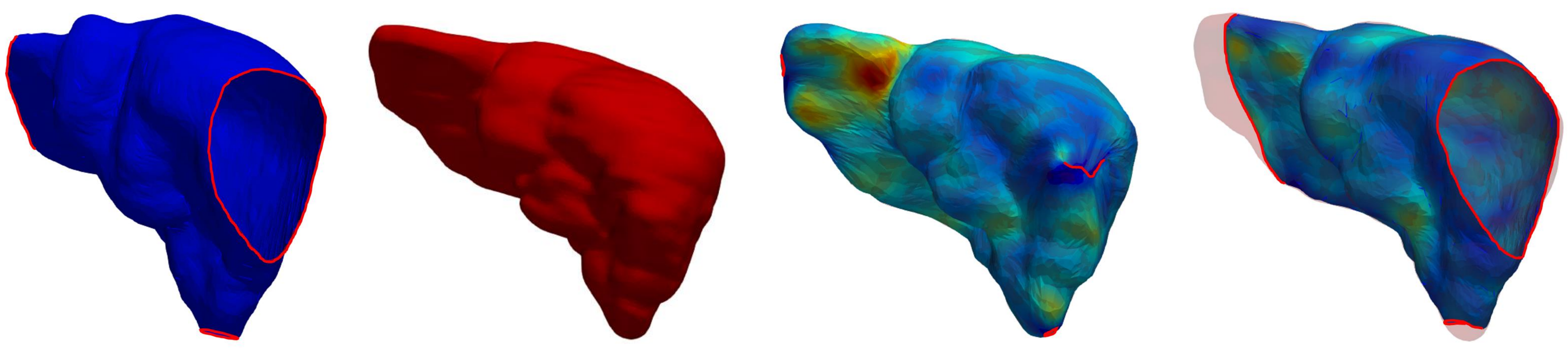
Application to multi modality liver surface registration



Sagittal view of a liver in CBCT



Sagittal view of a liver in CT



(a) (b) (c) (d)

The liver is truncated in CBCT (a) because of the reduced field of view, while the entire liver is seen in Computed Tomography (CT, b).

A coherent non-rigid deformation of the liver surface between modalities provides tools for image fusion or comparison. The varifold distance (b) leads to anatomically inconsistent solution when the partial matching (c) provides realistic result. (c,d) are colored by the determinant of the Jacobian of the deformation.

Discussion

The proposed partial matching provides a new dissimilarity term for the LDDMM framework allowing the registration of one shape onto a subset of another one. It shows interesting results on both real and synthetic examples of 3D trees and surfaces. The same term can be written to include the target into a subset of the source.

Next steps

- Extend to a bidirectional approach, such as finding the largest subpart of each shapes that are isomorph.
- Adapt the partial dissimilarity term to different shape spaces such as Normal Cycles.

* Reproducing Kernel Hilbert Space

[1] L. Younes.: Shapes and Diffeomorphisms, Applied Mathematical Sciences. Springer Berlin Heidelberg (2010)

[2] Charon, N., Trouve, A.: The varifold representation of nonoriented shapes for diffeomorphic registration. SIAM Journal on Imaging Sciences (2013)

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