Transformée de Riesz multi-échelles et Applications à l'image[1]

Valérie Perrier

Laboratoire Jean Kuntzmann
Université de Grenoble-Alpes

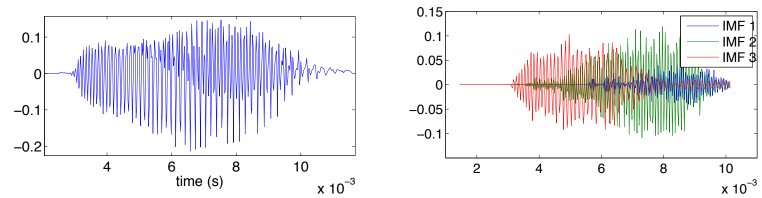
Collaborateurs : Marianne Clausel, Sylvain Meignen, Kévin Polisano (LJK), Laurent Desbat (TIMC-Imag) and Thomas Oberlin (IRIT, Toulouse)

1. *Journée "Temps-Fréquence et Non-Stationnarité", Marseille, 19 juin 2015*

Starting point : Multicomponent signals (1)

$$s(t) = \sum_{\ell=1}^{L} a_\ell(t) \cos(\varphi_\ell(t)) \ , \qquad t \in \mathbb{R}$$

- Decomposition problem : extraction of the different components (IMF_ℓ).
- Demodulation problem for a mode : estimation of the instantaneous amplitudes $a_\ell(t)$, phases $\varphi_\ell(t)$, and frequencies $\varphi'_\ell(t)$.

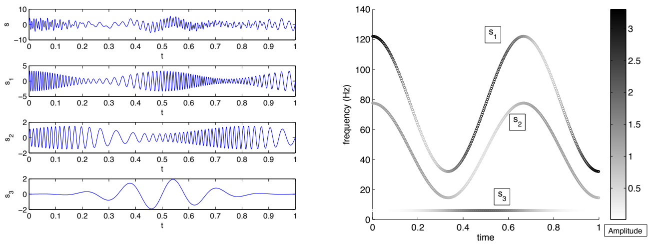


Bat echolocation call signal

Starting point : Multicomponent signals (2)

$$s(t) = \sum_{\ell=1}^{L} a_\ell(t) \cos(\varphi_\ell(t)) \ , \qquad t \in \mathbb{R}$$

- Decomposition problem : extraction of the different components.
- Demodulation problem for a mode : estimation of the instantaneous amplitudes $a_\ell(t)$, phases $\varphi_\ell(t)$, and frequencies $\varphi'_\ell(t)$.



Academic multicomponent signal

Decomposition/demodulation of signals in AM-FM modes[2]

Multicomponent signal :

$$s(t) = \sum_{\ell=1}^{L} a_\ell(t) \cos(\varphi_\ell(t))$$

- $a_\ell(t)\cos(\varphi_\ell(t))$: Intrinsec Mode Function (IMF_ℓ), (*decomposition pb*).
- a_ℓ : amplitude, φ'_ℓ : instantaneous frequency (*demodulation pb*).

The problem of finding the a_ℓ, φ_ℓ is ill-posed in general. Under suitable assumptions (separation of modes in Fourier domain, slowly variations of a'_ℓ, φ'_ℓ..), several methods have been developed in the 90th by the wavelet community, based on reallocation techniques in a time-frequency representation :

- Reassignment method [Auger-Flandrin 1995],
- Squeezing method [Daubechies-Maes 1996],
- Wavelet ridges [Carmona-Hwang-Torrésani 1997, 1999].

Another point of view :

- Empirical Mode Decomposition (EMD) and Hilbert-Huang Transform (HHT) [Huang *et al* 1998]

Review : SPM 2013, [Auger et al, SPM 2013]

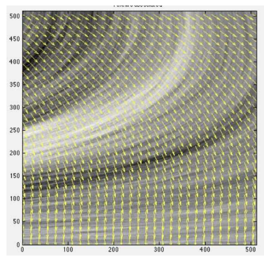
2. *ANR Astres 2013-2016 (coord. P. Flandrin)*

Motivation : Image decomposition/demodulation AM-FM

$$f(x) = a(x)\cos(\varphi(x)) + f_1(x)\ , \qquad x \in \mathbb{R}^2$$

- Decomposition problem : extraction of the different components.
- Demodulation problem for a mode : estimation of the local amplitude $a(x)$, phase $\varphi(x)$, frequency $\nabla\varphi(x)$.

Motivation : anisotropic texture analysis



Texture with prescribed orientation

[Polisano et al 2014]

Locally parallel textures

[Maurel-Aujol-Peyre 2011]

Outline

1. Definitions
   - Hilbert transform and Analytic signal
   - Riesz transform and Monogenic signal
2. Computation of the Riesz Transform
   - via the Fourier domain
   - via the Radon domain
   - In the direct space via a multiscale decomposition
3. Applications
   - Local orientations from local Radon data
   - Decomposition/demodulation of Multicomponent Images
4. Conclusion

Outline

1. Definitions
   - Hilbert transform and Analytic signal
   - Riesz transform and Monogenic signal
2. Computation of the Riesz Transform
   - via the Fourier domain
   - via the Radon domain
   - In the direct space via a multiscale decomposition
3. Applications
   - Local orientations from local Radon data
   - Decomposition/demodulation of Multicomponent Images
4. Conclusion

Hilbert transform and Analytic signal

- **1D : Hilbert Transform**. Let $f : \mathbb{R} \to \mathbb{R}$
  - $\mathcal{H}f$ in time domain :

$$\mathcal{H}f(t) = \left(\frac{1}{\pi}\mathrm{vp}\left(\frac{1}{t}\right) * f\right)(t) = \lim_{\varepsilon\to 0}\left(\frac{1}{\pi}\int_{|t-s|>\varepsilon}\frac{f(s)}{t-s}\,ds\right) \quad \text{for a.e. } t \in \mathbb{R}.$$

  - $\mathcal{H}f$ in Fourier domain : $\widehat{\mathcal{H}f}(\xi) = -\mathrm{i}\,\frac{\xi}{|\xi|}\widehat{f}(\xi) = -\mathrm{i}\ \mathrm{sgn}(\xi)\ \widehat{f}(\xi)$
- **Analytic signal** (complex) : $F(t) = f(t) + \mathrm{i}\ \mathcal{H}f(t) \quad (\hat{F} = 0$ on $\mathbb{R}_-)$
  - AM-FM analysis : $F(t) = A(t)\ e^{i\varphi(t)}$
    - Instantaneous amplitude : $A(t) = |F(t)|$
    - Instantaneous frequency : $\omega(t) = \varphi'(t)$
- **Example** : $f(t) = A\cos(\omega t)$. Then $\mathcal{H}(f) = A\sin(\omega t)$ and $F(t) = Ae^{i\omega t}$
  $\longrightarrow A(t) = A,\ \varphi(t) = \omega t$

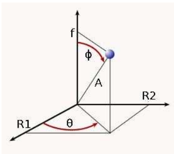
Riesz transform and Monogenic Signal [Felsberg-Sommer 2001]

- **2D (or n-D) : Riesz Transform** $\vec{R}f = \begin{pmatrix} R_1 f \\ R_2 f \end{pmatrix}$
  - $\vec{R}f$ in space domain :

$$R_i f(x) = \lim_{\varepsilon\to 0^+}\left(\frac{1}{\pi}\int_{\|x-y\|>\varepsilon}\frac{(x_i-y_i)}{\|x-y\|^3}\,f(y)\,\mathrm{d}y\right)$$

  - $\vec{R}f$ in Fourier domain : $\widehat{R_i f}(\xi) = -\mathrm{i}\,\frac{\xi_i}{\|\xi\|}\widehat{f}(\xi)$, for $i = 1, 2$.
- **Monogenic** (quaternionique) signal : $\mathcal{M}f = \begin{pmatrix} f \\ \vec{R}f \end{pmatrix} = f + \mathrm{i}\ \mathrm{R_1 f} + \mathrm{j}\ \mathrm{R_2 f}$
  - AM-FM analysis : $\mathcal{M}f = A(x)\ \mathrm{e}^{\varphi(x)\mathrm{n}_\theta(x)}$
    - Local amplitude : $A(x) = |\mathcal{M}f(x)|$
    - Local frequency : $\omega(x) = \nabla\varphi(x)$
    - Local orientation : $\theta(x)$ ($n_\theta = \cos\theta\ \mathrm{i} + \sin\theta\ \mathrm{j}$)
    (link with the orientation of $\nabla\varphi(x)$?)

Monogenic signal



$$\mathcal{M}f = \begin{pmatrix} f \\ \vec{R}f \end{pmatrix} = A(x)\ \mathrm{e}^{\varphi(x)n_\theta(x)} = A(x)\begin{pmatrix} \cos(\varphi(x)) \\ \sin(\varphi(x))\cos(\theta(x)) \\ \sin(\varphi(x))\sin(\theta(x)) \end{pmatrix}$$

- **Example** : $f(x) = A_0\cos(k\cdot x)$. Let $k = (k_1, k_2)$, $\theta_0 = \mathrm{Arctan}(\frac{k_2}{k_1})$.

Then

$$\mathcal{R}f(x) = A_0\begin{pmatrix} \sin(k\cdot x)\cos\theta_0 \\ \sin(k\cdot x)\sin\theta_0 \end{pmatrix} = A_0\frac{k}{|k|}\sin(k\cdot x)$$

and

$$\mathcal{M}f(x) = \begin{pmatrix} f(x) \\ \mathcal{R}f(x) \end{pmatrix} = A_0\begin{pmatrix} \cos(k\cdot x) \\ \sin(k\cdot x)\cos\theta_0 \\ \sin(k\cdot x)\sin\theta_0 \end{pmatrix} = A_0\ \mathrm{e}^{(k\cdot x)(\cos\theta_0\ \mathrm{i} + \sin\theta_0\ \mathrm{j})}$$

Finally

$$A(x) = A_0,\ \varphi(x) = k\cdot x,\ \theta(x) = \theta_0 = \mathrm{Arctan}(k_2/k_1)\ .$$

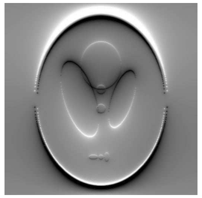
Outline

Computation of the Riesz Transform (1)

- **Via the Fourier domain** : $\widehat{R_i f}(\xi) = -\mathrm{i}\ \frac{\xi_i}{|\xi|}\widehat{f}(\xi)$, for $i = 1, 2$.

Shepp and Logan phantom

first component $R_1 f$



second component $R_2 f$

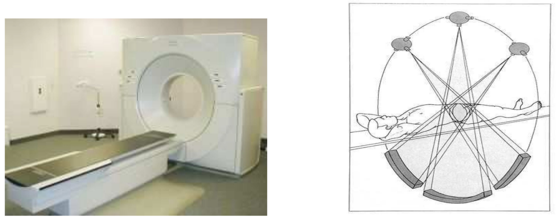
Fourier based Riesz computation :

- Involves a *non local* filtering (pb at frequency 0),
- Requires the knowledge of the *whole image*,
- Computed using *FFT*, complexity : $O(N\ log_2(N))$

Computation of the Riesz Transform (2)

- **Via the Radon domain** [Felsberg 2002]

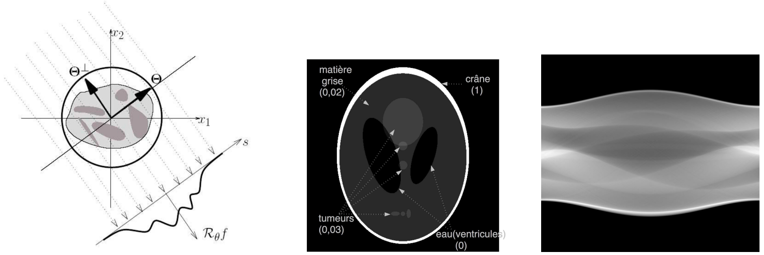
Medical Scan : X-ray tomography
Godfrey N. Hounsfield (ingenior in electronic) and Allan M. Cormack (mathematician) : Nobel Prize in Medicine 1979.



Scan and illustration of its principle : X-ray taken around the patient.

Computation of the Riesz Transform (2)

- **Radon Transform** :



The Radon transform of function $f(x)$ is measured on each detector of direction $\vec{\theta} = (\cos\theta, \sin\theta)$, corresponding to the mean of f along lines $L_{\theta,s}$ of direction $\vec{\theta}^\perp = (-\sin\theta, \cos\theta)$:

$$\mathcal{R}f(\theta, s) = \mathcal{R}_\theta f(s) = \int_{L_{\theta,s}} f(x)\ d\ell = \int_{-\infty}^{+\infty} f(s\vec{\theta} + t\vec{\theta}^\perp)\ dt$$

Computation of the Riesz Transform (2)

- **Inverse Radon transform** : filtered back projection (FBP)

$$f(x) = \mathcal{R}^{-1}\left(\mathcal{R}f(\theta, s)\right) = \int_0^\pi \left(\int_{-\infty}^{+\infty} \widehat{\mathcal{R}_\theta f}(\omega)|\omega|\ e^{2i\pi\omega(x.\vec{\theta})} d\omega \right) d\theta$$

(where $\widehat{\mathcal{R}_\theta f}$ denotes the 1D Fourier transform of $\mathcal{R}_\theta f$.
Remark : involves the *non local* ramp filter $|\omega|$.

- **Original Radon-based Riesz formula** [Felsberg 2002], [Soulard-Carré 2012]

$$\vec{R}f\,(x) = \mathcal{R}^{-1}\left(\left(\mathcal{H}\mathcal{R}_{\vec{\theta}} f\right) \vec{\theta} \right)(x)$$

due to :

$$\vec{R}f\,(x) = \int_0^\pi \left(\int_{-\infty}^{+\infty} \left(\widehat{\mathcal{R}_\theta f}\,(\omega)(-i)\mathrm{sign}(\omega) \right) |\omega| e^{2i\pi\omega(x\cdot\vec{\theta})} \mathrm{d}\omega \right) \vec{\theta}\ \mathrm{d}\theta$$

Remark : involves two *non local* operators : the Hilbert transform $\mathcal{H}$ and the inverse Radon transform $\mathcal{R}^{-1}$.

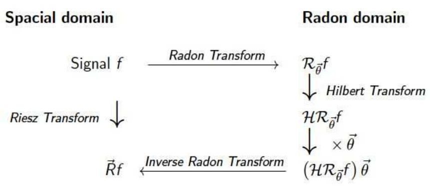
Computation of the Riesz Transform (2)

- **Local Radon-based Riesz formula** [Desbat-P 2015] Since :

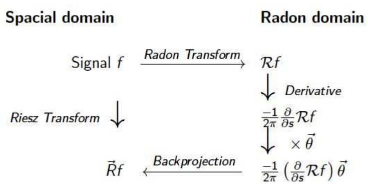
$$\vec{R}f(x) = \int_0^{\pi} \left(\int_{-\infty}^{+\infty} \widehat{\mathcal{R}_\theta f}(\omega)(-i\omega) e^{2i\pi\omega(x\cdot\vec{\theta})} \mathrm{d}\omega \right) \vec{\theta}\, \mathrm{d}\theta$$

Then :

$$\vec{R}f(x) = -\frac{1}{2\pi}\int_0^{\pi} \frac{\partial \mathcal{R}f}{\partial s}\left(\theta, x\cdot\vec{\theta}\right) \vec{\theta}\, \mathrm{d}\theta$$



Non local Radon based Riesz

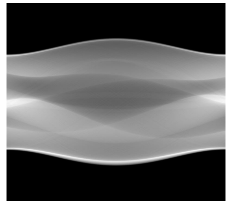


Local Radon based Riesz

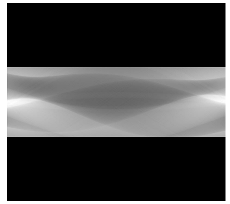
Computation of the Riesz Transform (2)

- **Interest** : local Riesz transform from local Radon data

Phantom and ROI



Radon data

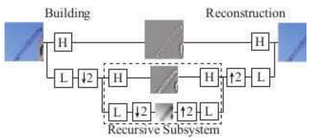


Truncated Radon data

Local $(R_1 f,\ R_2 f)$ with NO ERROR

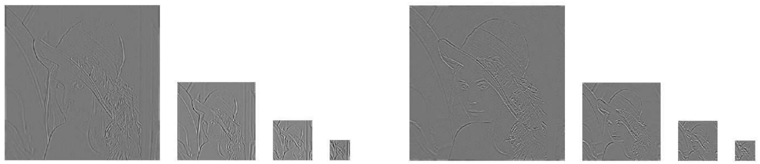
Computation of the Riesz Transform - In direct space (3)

- Pyramidale decomposition [Burt-Adelson 1983]



- **Multiscale Riesz transform** : in each band a, $\widehat{R_i f_a}(\xi) = -\mathrm{i}\,\frac{\xi_i}{|k_a|}\widehat{f_a}(\xi)$

$\rightarrow$ $R_i f \sim \frac{-1}{2\pi k_a}\frac{\partial f_a}{\partial x_i}$



$\rightarrow$ Complexity : linear ! [Wahdwa-Rubinstein-Durand-Freeman 2014] for video magnification

Computation of the Riesz Transform - In direct space (4)

Monogenic Wavelet Transform - [Olhede-Metikas 2009], [Unser-VanDeVille 2009]

- 2D directional CWT

$$c_f(a,b,\alpha) = \int_{\mathbb{R}^2} f(x)\,\overline{\psi_{a,b,\alpha}(x)}\,\mathrm{d}x\ ,\quad \psi_{a,b,\alpha}(x) = \frac{1}{a}\psi\left(r_{-\alpha}\frac{x-b}{a}\right)$$

If ψ is isotropic, $\psi_{a,b,\alpha} = \psi_{a,b,0} = \psi_{a,b}$. Denote $c_f(a,b) = c_f(a,b,0)$.

- Isotropic CWT of the monogenic signal $F = \mathcal{M}f = f + R_1 f\ \mathrm{i} + R_2 f\ \mathrm{j}$

$$c_F(a,b) = (c_f + c_{R_1 f}\ \mathrm{i} + c_{R_2 f}\ \mathrm{j})\,(a,b)$$

- Monogenic Wavelet Transform (ψ real isotropic)

$$c_f^{(M)}(a,b,\alpha) = \int_{\mathbb{R}^2} f(x)\ (\mathcal{M}\psi)_{a,b,\alpha}(x)\,\mathrm{d}x$$

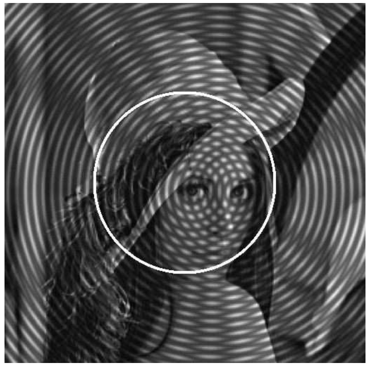
$$c_F(a,b) = \begin{pmatrix} 1 & 0 \\ 0 & -r_\alpha \end{pmatrix} c_f^{(M)}(a,b,\alpha)$$

- Example : $F(x) = A\ e^{(k\cdot x)n_\theta}$, $c_F(a,b) = a\widehat{\psi}(ak)\left(Ae^{(k\cdot b)n_\theta}\right)$

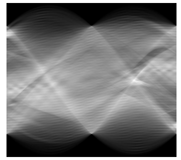
Outline

Local orientations from local Radon data

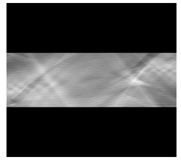
Psychedelic Lenna [Unser-Van De Ville 2009]



(a)



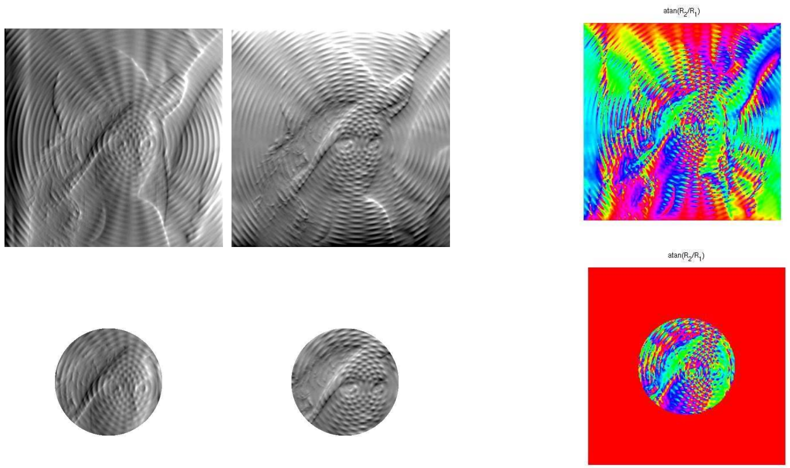
(b)



(c)

(a) Psychedelic Lenna image (of size 512×512*) with the considered ROI (materialized by a white circle) and (b) corresponding sinogram (discretized Radon data : equiangular and equispaced* 806×512 *samples on* $[0, \pi)\times$ *the diagonal of the image). (c) Truncated Radon projections : only the lines passing through the ROI are measured.*

Riesz transform via local Radon formula



$(R_1 f,\ R_2 f)$ *from full (top) and truncated (bottom) Radon data - Orientations*

Decomposition/demodulation of Multicomponent Images

- **Decomposition** : Expand an image $s(x)$ into oscillating modes :

$$s(x) = \sum_{\ell=1}^{L} d_\ell(x) + r_\ell(x)$$

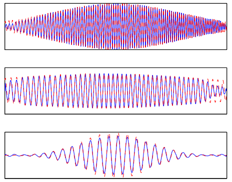
d_ℓ is an Intrinsinc Mode Function (IMF)
(ex : Bidimensional Empirical Mode Decomposition : [Nunes et al 03, Linderhed 09, Damerval-Meignen-Perrier 05, ...])

- **Demodulation** : $d_\ell(x) = A_\ell(x)\ \mathrm{e}^{\varphi_\ell(x) n_{\theta_\ell}(x)}$ using the monogenic signal $\mathcal{M}d_\ell$ of the mode d_ℓ.

Different approaches :
- *2D extension of the Hilbert-Huang Transform (HHT)* : [Huang-Kunoth 2012] [Schmitt-Pustelnik-Borgnat-Flandrin-Condat 2014]
- *2D synchrosqueezing* (2 steps simultaneously) : [Clausel-Oberlin-P. 2014]

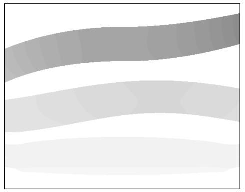
Principle of the 1D synchrosqueezed Wavelet Transform

[Daubechies-Lu-Wu, ACHA 2011]

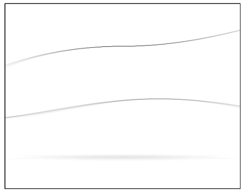


Multicomponent AM-FM signal f

Wavelet Transform $W_f(a, b)$



$\omega_f(a, b)$ (to approximate $\varphi_\ell'(b)$)



SST= $S_f(k, b)$

Principles of the SST 1D - [Daubechies-Lu-Wu, ACHA 2011]

- Example : $f(x) = A\cos(\omega x)$

The CWT of its analytic signal : $F(x) = A\, e^{i\omega x}$:

$$W_F(a, b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{+\infty} F(x)\overline{\psi\left(\frac{x-b}{a}\right)} dx = A\sqrt{a}\ \overline{\hat{\psi}(a\omega)}\ e^{i\omega b}$$

Then

$$\partial_b W_F(a, b) = i\omega W_F(a, b) \rightarrow \omega = -i\frac{\partial_b W_F(a; b)}{W_F(a; b)}$$

and

$$F(b) = \frac{\lambda_\psi}{\sqrt{a_0}} W_F(a_0, b) \quad \text{where } a_0 = \frac{k_0}{\omega} \quad (k_0 \text{ peak wavenumber of } \psi)$$

Principles of the SST 1D - [Daubechies-Lu-Wu, ACHA 2011]

- Example : $f(x) = A\cos(\omega x)$

The CWT of its analytic signal : $F(x) = A\, e^{i\omega x}$:

$$W_F(a, b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{+\infty} F(x)\overline{\psi\left(\frac{x-b}{a}\right)} dx = A\sqrt{a}\ \overline{\hat{\psi}(a\omega)}\ e^{i\omega b}$$

Then

$$\partial_b W_F(a, b) = i\omega W_F(a, b) \rightarrow \omega = -i\frac{\partial_b W_F(a; b)}{W_F(a; b)}$$

and

$$F(b) = \frac{\lambda_\psi}{\sqrt{a_0}} W_F(a_0, b) \quad \text{where } a_0 = \frac{k_0}{\omega} \quad (k_0 \text{ peak wavenumber of } \psi)$$

- **Monocomponent complex signal** : $f(x) = A(x)\exp(i\varphi(x))$,

with slowly varying A, φ.

Candidate instantaneous frequency :

$$\omega_F(a, b) = -i\frac{\partial_b W_F(a, b)}{W_F(a, b)}, \quad \text{when } |W_F(a; b)| > \varepsilon$$

Estimate : $|\omega_F(a, b) - \varphi'(b)| < \varepsilon$ with suitable conditions (C_ε) on A, φ.

Principles of the SST 1D

- **Multicomponent complex signal** $f(x)$: superposition of several IMFs assumed to be slowly varying and well separated in time-frequency domain :

$$f(x) = \sum_{\ell=1}^{L} A_\ell(x)\ \mathrm{e}^{i\varphi_\ell(x)}$$

Synchrosqueezed Wavelet Transform (SST) :

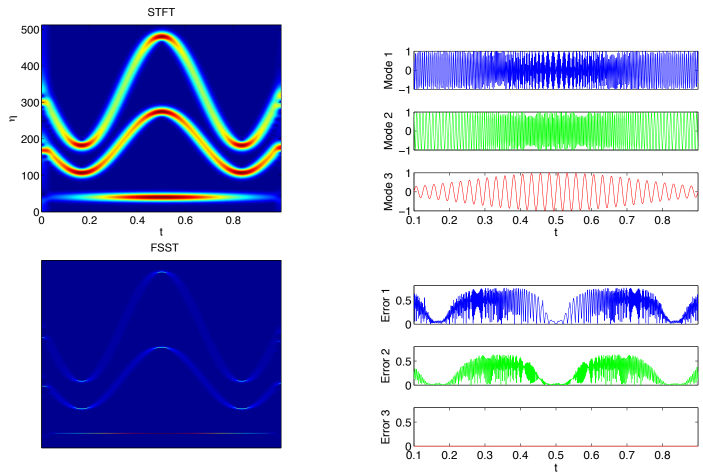
$$S^\delta_{f,\varepsilon}(b, k) = \int_{|W_f(a,b)|>\varepsilon} W_f(a, b)\frac{1}{\delta} h\left(\frac{k - \omega_f(a, b)}{\delta}\right) \frac{\mathrm{d}a}{a^{3/2}}$$

Estimate :

$$\lim_{\delta \to 0} \frac{1}{C_\psi} \int_{\{k;\, |k - \varphi_\ell'(b)| \leq \varepsilon\}} S^\delta_{f,\varepsilon}(b, k)\mathrm{d}k = A_\ell(b)\mathrm{e}^{i\varphi_\ell(b)} + O(\varepsilon)$$

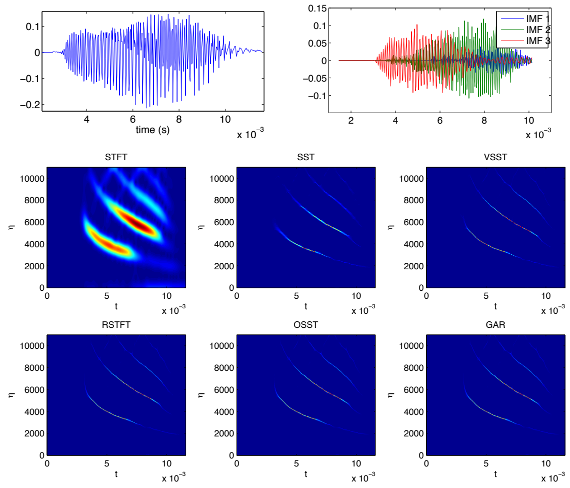
2D extension of the analytic signal $\longrightarrow$ monogenic signal

Example (Fourier) SST 1D [Oberlin-Meignen-Perrier 2014]



Academic signal

Example (Fourier) SST 1D [Oberlin-Meignen-Perrier 2014] [Auger et al, SPM 2013]



Bat echolocation call signal

Principle of 2D SST : WT of the Monogenic Signal

[Clausel-Oberlin-P. 2014]

$$F(x) = A\ e^{(k\cdot x)n_\theta} \qquad k = (k_1, k_2)$$

- Isotropic wavelet transform of F :

$$c_F(a,b) = a\widehat{\psi}(ak)\left(Ae^{(k\cdot b)n_\theta}\right)$$

For $i = 1, 2$:

$$\partial_{b_i} c_F(a,b) = k_i n_\theta \left(a\widehat{\psi}(ak)\right)\left(Ae^{(k\cdot b)n_\theta}\right)$$

- Instantaneous frequency k and orientation n_θ :

$$k_1 n_\theta = \partial_{b_1} c_F(a,b) \times (c_F(a,b))^{-1}$$

$$k_2 n_\theta = \partial_{b_2} c_F(a,b) \times (c_F(a,b))^{-1}$$

- On the "ridge" $a = a_0 = \frac{|k_0|}{|k|}$

$$F(b) = \lambda_\psi a_0\ c_F(a_0, b)$$

Principle of 2D SST : WT of the Monogenic Signal

[Clausel-Oberlin-P. 2014]

$$F(x) = A\ e^{(k\cdot x)n_\theta} \qquad k = (k_1, k_2)$$

- Isotropic wavelet transform of F :

$$c_F(a,b) = a\widehat{\psi}(ak)\left(Ae^{(k\cdot b)n_\theta}\right)$$

For $i = 1, 2$:

$$\partial_{b_i} c_F(a,b) = k_i n_\theta \left(a\widehat{\psi}(ak)\right)\left(Ae^{(k\cdot b)n_\theta}\right)$$

- Instantaneous frequency k and orientation n_θ :

$$k_1 n_\theta = \partial_{b_1} c_F(a,b) \times (c_F(a,b))^{-1}$$

$$k_2 n_\theta = \partial_{b_2} c_F(a,b) \times (c_F(a,b))^{-1}$$

- On the "ridge" $a = a_0 = \frac{|k_0|}{|k|}$

$$F(b) = \lambda_\psi a_0\ c_F(a_0, b)$$

Intrinsic Monogenic Mode Function (IMMF)

- Intrinsic Monogenic Mode Function (IMMF) with accuracy $\varepsilon > 0$:

$$F(x) = A(x)\mathrm{e}^{\varphi(x) n_{\theta(x)}} \quad \text{with} \quad n_{\theta(x)} = \cos(\theta(x))\ \mathrm{i} + \sin(\theta(x))\ \mathrm{j}$$

A, φ, n_θ slowly varying functions ($|\nabla A(x)|, |\nabla \theta(x)|, |\nabla^2 \varphi(x)| < \varepsilon |\nabla \varphi(x)|$).
- A : local amplitude of F
- φ, n_θ : local scalar phase and orientation of F
- $\nabla \varphi$: instantaneous frequency.

- Candidate to approximate the <u>instantaneous frequency</u> :

$$\begin{aligned} \Lambda_1(a,b) &= \partial_{b_1} c_F(a,b) \times (c_F(a,b))^{-1} \\ \Lambda_2(a,b) &= \partial_{b_2} c_F(a,b) \times (c_F(a,b))^{-1} \end{aligned}$$

- Estimate : for $i = 1, 2$,

$$|\Lambda_i(a,b) - \partial_{b_i}\varphi(b) n_{\theta(b)}| \leq \varepsilon \quad \textit{where} \quad |c_F(a,b)| > \varepsilon$$

Monogenic Synchrosqueered Wavelet Transform (MSST)

- Multicomponent signal $F(x)$: superposition of IMMFs of accuracy ε, well separated in the space-frequency domain :

$$F(x) = \sum_{\ell=1}^{L} A_\ell(x)\ \mathrm{e}^{\varphi_\ell(x) n_{\theta_\ell(x)}}$$

- MSST= local CWT-reconstruction at fixed point b, in the ε-vicinity of the estimated instantaneous frequencies (Λ_1, Λ_2) :

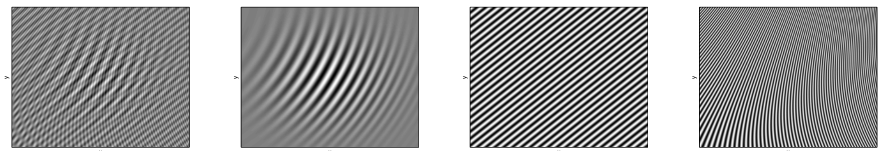
$$S^\delta_{F,\varepsilon}(b,k,n) = \int_{|c_F(a,b)|>\varepsilon} c_F(a,b) \frac{1}{\delta^2} h\left(\frac{k_1 - \mathrm{Re}(\overline{n}\ \Lambda_1(a,b))}{\delta}\right) h\left(\frac{k_2 - \mathrm{Re}(\overline{n}\ \Lambda_2(a,b))}{\delta}\right) \frac{\mathrm{d}a}{a^2}$$

($h \in C^\infty_c$ s.t. $\int h = 1$)

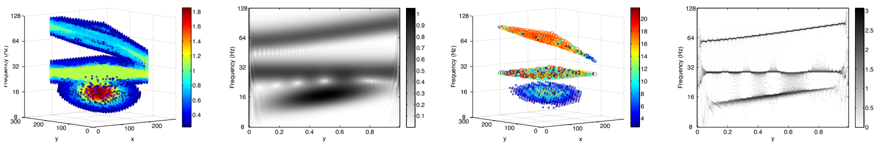
- ℓ^{th}-IMMF estimate ($\hat{\psi}$ compactly supported) :

$$\lim_{\delta \to 0} \frac{2\pi}{\tilde{C}_\psi} \int_{\mathbb{S}^1} \int_{\{k;\ \max_i |k_i n - \partial_{b_i}\varphi_\ell(b) n_{\theta_\ell(b)}| \leq \varepsilon\}} S^\delta_{f,\varepsilon}(b,k,n)\mathrm{d}k\mathrm{d}n = A_\ell(b)\mathrm{e}^{\varphi_\ell(b) n_{\theta_\ell(b)}} + O(\varepsilon)\ .$$

Decomposition/demodulation of Multicomponent Images



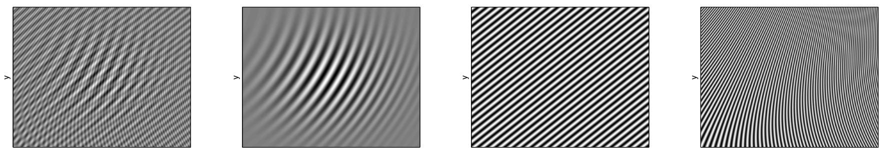
$f(x) \quad = \quad f_1(x) \quad + \quad f_2(x) \quad + \quad f_3(x)$



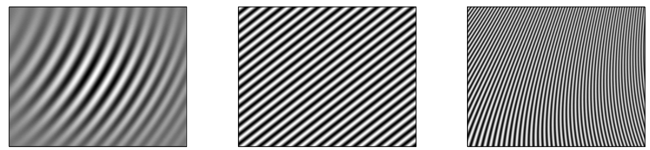
Monogenic WT (modulus 3D, 2D slice) Monogenic SST (modulus 3D, 2D slice)

Monogenic Synchrosqueezed Wavelet Transform of Images

Multicomponent signal : reconstruction of modes



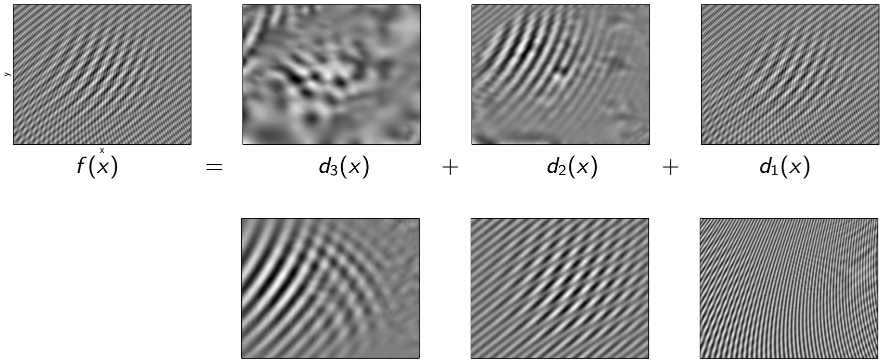
$f(x) \quad = \quad f_1(x) \quad + \quad f_2(x) \quad + \quad f_3(x)$



Reconstructed modes = $f_1(x)$ (MSE=0.05) + $f_2(x)$ (MSE= 0.06) + $f_3(x)$ (MSE=0.034)

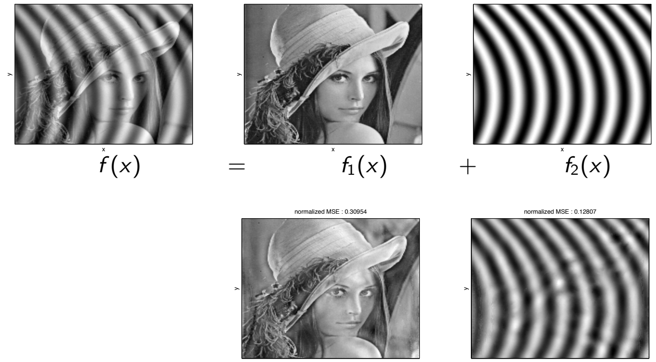
$$\begin{cases} f_1(x_1,x_2) &= e^{-10((x_1-0.5)^2+(x_2-0.5)^2))} \sin(10\pi(x_1^2 + x_2^2 + 2(x_1 + 0.2x_2)) \\ f_2(x_1,x_2) &= 1.2\sin(40\pi(x_1 + x_2)) \\ f_3(x_1,x_2) &= \cos(2\pi(70x_1 + 20x_1^2 + 50x_2 - 20x_2^2 - 41x_1x_2)) \end{cases}$$

Comparison with EMD and EEMB



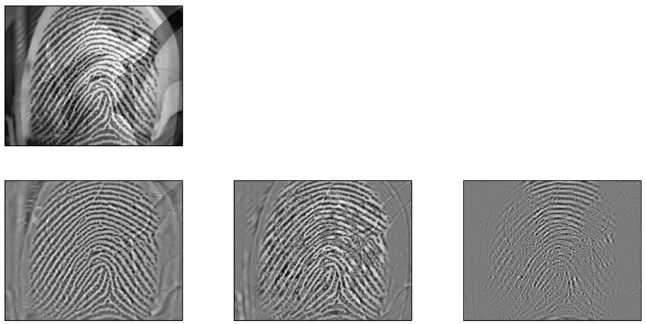
Reconstructed modes = $d_3(x)$ (MSE=0.86) + $d_2(x)$ (MSE= 0.74) + $d_1(x)$ (MSE=0.59)

Extraction of AM-FM modes from a real image



Reconstructed modes = $f_1(x)$ (MSE=0.31) + $f_2(x)$ (MSE= 0.13)

Extraction of AM-FM modes from a real image



Superposition of Lenna and a fingerprint.
Extracted fingerprint by : 2D MSST (left), first mode of EMD (middle), first mode of EEMD (right).

Outline

Conclusion

- **Riesz transform** : easy way to compute local phases and orientations of images, alternative to oriented Gabor filters. ⟶ Medical imaging applications, 3D, ...
- 2D generalization of the Synchrosqueezed Wavelet Method in the Monogenic Signal framework

Conclusion

- **Riesz transform** : easy way to compute local phases and orientations of images, alternative to oriented Gabor filters. ⟶ Medical imaging applications, 3D, ...
- 2D generalization of the Synchrosqueezed Wavelet Method in the Monogenic Signal framework

- MSST allows to link local orientations to instantaneous frequency : new way for detection and characterization of anisotropy in images (application to textures) ⟶ problems still remain when the modes are not well separated in frequency (only in orientation)

Conclusion

- **Riesz transform** : easy way to compute local phases and orientations of images, alternative to oriented Gabor filters. ⟶ Medical imaging applications, 3D, ...
- 2D generalization of the Synchrosqueezed Wavelet Method in the Monogenic Signal framework

- MSST allows to link local orientations to instantaneous frequency : new way for detection and characterization of anisotropy in images (application to textures) ⟶ problems still remain when the modes are not well separated in frequency (only in orientation)

- M. Clausel, T. Oberlin, V. Perrier, *The Monogenic Synchrosqueezed Wavelet Transform : A tool for the Decomposition/Demodulation of AM-FM images*, ACHA (2014).

- L. Desbat, V. Perrier, *On locality of Radon to Riesz transform*, preprint, submitted.