

Itakura-Saito NMF: un modèle probabiliste à facteurs latents pour la transformée de Fourier court-terme

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Outline

Generalities about NMF

- Concept of NMF

- Majorization-minimization algorithms

Itakura-Saito NMF

- A statistical model of the STFT

- Piano decomposition example

Nonnegative dynamical system for speech & audio

- Statistical model

- Speech enhancement

Nonnegative matrix factorization (NMF)

Given a *nonnegative* matrix \mathbf{V} of dimensions $F \times N$, NMF is the problem of finding a factorization

$$\mathbf{V} \approx \mathbf{WH}$$

where \mathbf{W} and \mathbf{H} are *nonnegative* matrices of dimensions $F \times K$ and $K \times N$, respectively.

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Dimensions:

- ▶ If \mathbf{W} tall ($K < F$), NMF produces a low-rank approximation.
- ▶ If \mathbf{W} fat ($K > F$), NMF produces an overcomplete representation (e.g., sparse coding).

An unsupervised part-based representation

Along VQ, PCA or ICA, NMF provides an **unsupervised linear representation** of data

$$\mathbf{v}_n \approx \mathbf{W} \mathbf{h}_n$$

data vector “explanatory variables” “regressors”
 “basis”, “dictionary” “expansion coefficients”
 “patterns” “activation coefficients”

and \mathbf{W} is learnt from the set of data vectors $\mathbf{V} = [\mathbf{v}_1 \dots \mathbf{v}_N]$.

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- ▶ **nonneg. of \mathbf{W}** ensures *interpretability* of the dictionary (features \mathbf{w}_k and data \mathbf{v}_n belong to same space).
- ▶ **nonneg. of \mathbf{H}** tends to produce *part-based* representations because subtractive combinations are forbidden.

Early work by Paatero and Tapper (1994), landmark paper in *Nature* by Lee and Seung (1999).

NMF as a constrained minimization problem

Minimize a measure of fit between data \mathbf{V} and model \mathbf{WH} , subject to nonnegativity of \mathbf{W} and \mathbf{H} :

$$\min_{\mathbf{W}, \mathbf{H} \geq 0} D(\mathbf{V} | \mathbf{WH}) = \sum_{fn} d([\mathbf{V}]_{fn} | [\mathbf{WH}]_{fn})$$

where $d(x|y)$ is a scalar cost function.

Regularization terms are often added to $D(\mathbf{V} | \mathbf{WH})$ to favor certain properties of \mathbf{W} or \mathbf{H} (sparsity, smoothness).

Divergences used in NMF

(selected references)

- ▶ Euclidean distance (Paatero and Tapper, 1994; Lee and Seung, 2001)
- ▶ Kullback-Leibler divergence (Lee and Seung, 1999; Finesso and Spreij, 2006)
- ▶ α -divergence (Cichocki et al., 2008)
- ▶ β -divergence (Cichocki et al., 2006; Févotte and Idier, 2011)
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- ▶ Itakura-Saito divergence (Févotte et al., 2009)

Common NMF algorithm design

- ▶ Block-coordinate update of \mathbf{H} given $\mathbf{W}^{(i-1)}$ and \mathbf{W} given $\mathbf{H}^{(i)}$.

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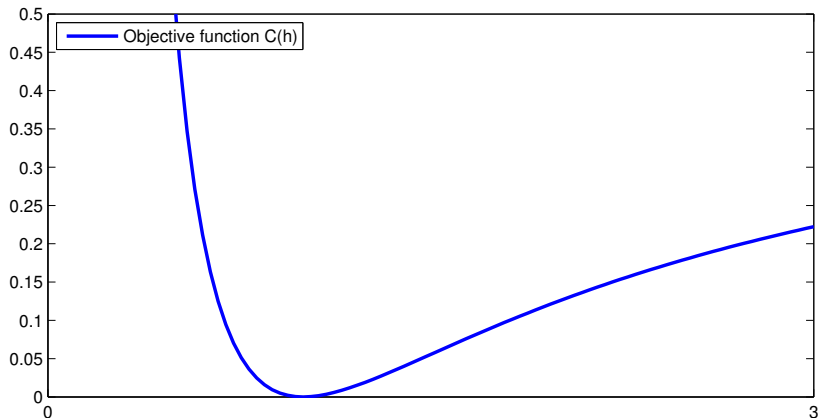
- ▶ In the end we are left with *nonnegative linear regression*

$$\min_{\mathbf{h} \geq 0} C(\mathbf{h}) \stackrel{\text{def}}{=} D(\mathbf{v}|\mathbf{W}\mathbf{h})$$

Numerous references in the image restoration literature (Richardson, 1972; Lucy, 1974; Daube-Witherspoon and Muehllehner, 1986)

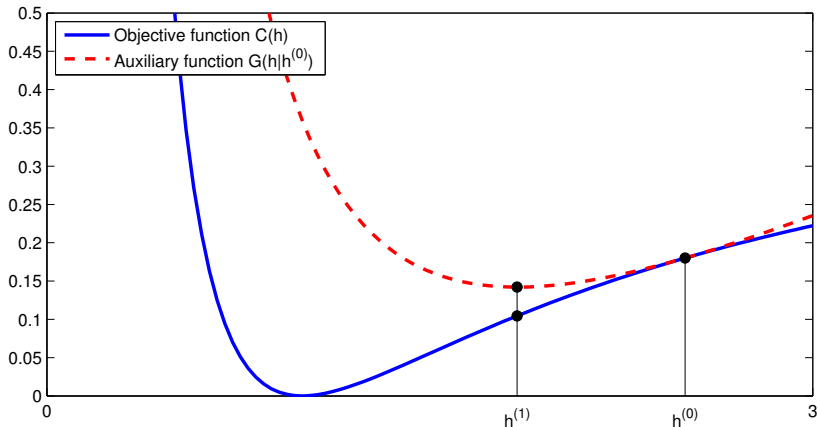
Majorization-minimization (MM)

Build $G(\mathbf{h}|\tilde{\mathbf{h}})$ such that $G(\mathbf{h}|\tilde{\mathbf{h}}) \geq C(\mathbf{h})$ and $G(\tilde{\mathbf{h}}|\tilde{\mathbf{h}}) = C(\tilde{\mathbf{h}})$.
Optimize (iteratively) $G(\mathbf{h}|\tilde{\mathbf{h}})$ instead of $C(\mathbf{h})$.



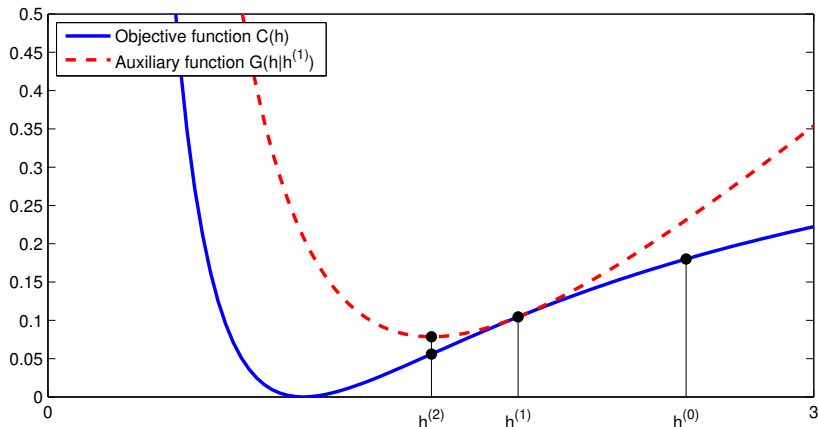
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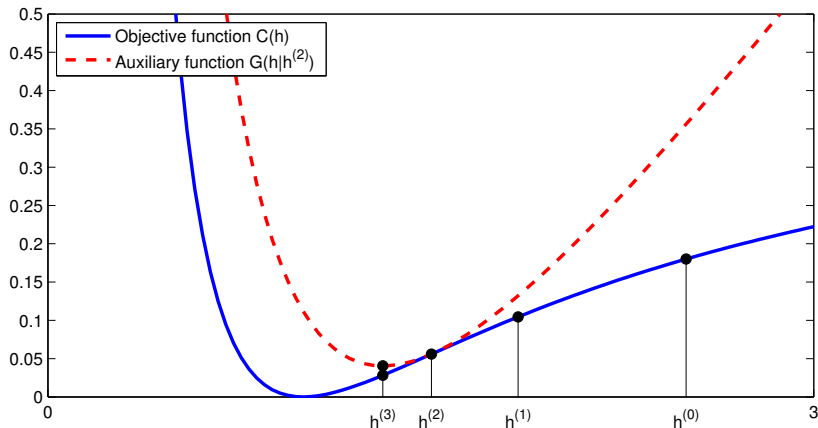
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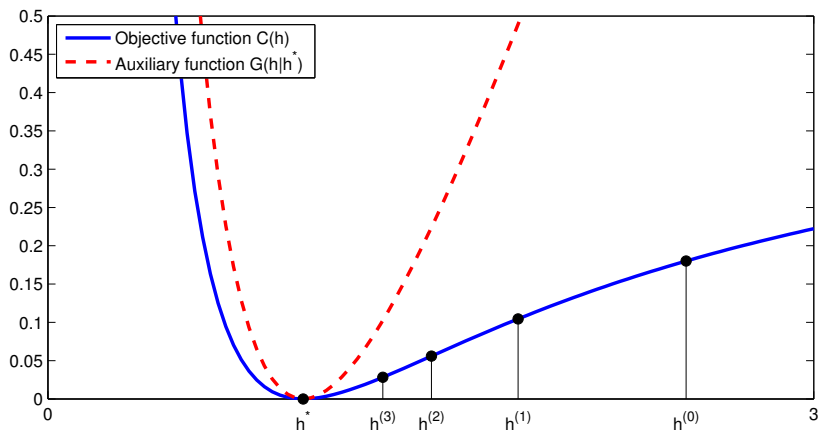
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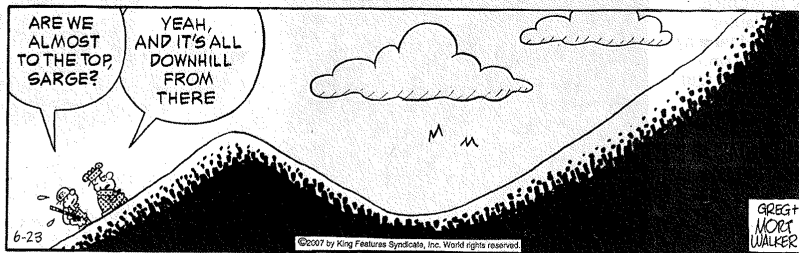


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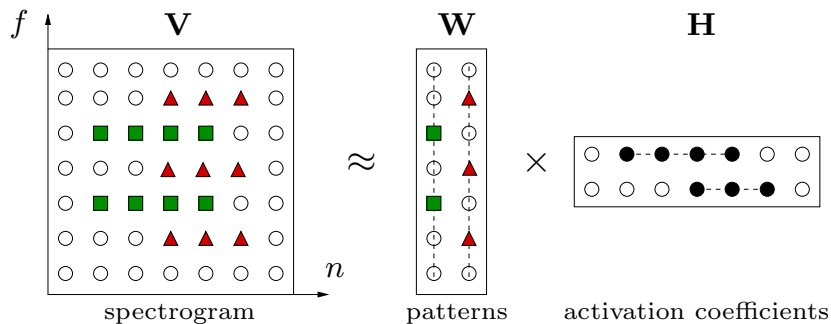
Local convergence



- ▶ If $d(x|y)$ is convex w.r.t to y , $D(\mathbf{V}|\mathbf{WH})$ convex w.r.t either \mathbf{W} or \mathbf{H} but not both.
- ▶ Not even true if $d(x|y)$ not convex w.r.t y .

Application to music signal processing

(Smaragdis and Brown, 2003)



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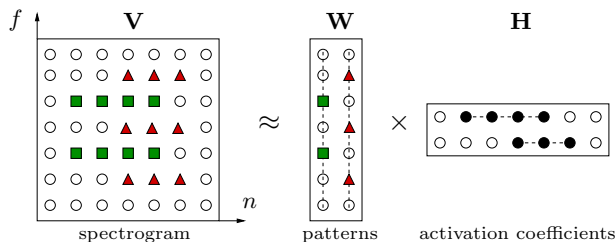
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Model choices



- Magnitude or power spectrogram ?
- Which measure of fit should be used for the factorization ?
- NMF approximates the spectrogram by a sum of rank-one spectrograms. How can we invert these ? What about phase ?

Itakura-Saito NMF: a generative approach

(Févotte, Bertin, and Durrieu, 2009)

Let $\mathbf{X} = \{x_{fn}\}$ be the (complex-valued) STFT of the signal.

Assume

$$x_{fn} = \sum_{k=1}^K c_{k,fn}$$
$$c_{k,fn} \sim \mathcal{N}_c(0, w_{fk} h_{kn})$$

and the components $c_{1,fn}, \dots, c_{K,fn}$ are independent given \mathbf{W} and \mathbf{H} .

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and the components $c_{1,fn}, \dots, c_{K,fn}$ are independent given \mathbf{W} and \mathbf{H} . Then

$$-\log p(\mathbf{X}|\mathbf{W}, \mathbf{H}) = D_{IS}(|\mathbf{X}|^2|\mathbf{W}\mathbf{H}) \quad + \text{cst.}$$

Additivity assumed in the STFT domain. Phase is preserved in the model, though in a noninformative way (uniform distribution).

Related work by Benaroya et al. (2003); Parry and Essa (2007)

Itakura-Saito NMF: a generative approach

(Févotte, Bertin, and Durrieu, 2009)

Main message: Itakura-Saito NMF of the power spectrogram corresponds to maximum likelihood estimation in a well-defined generative composite model of the STFT.

This in particular gives a statistically grounded way of reconstructing the components:

$$\hat{c}_{k,fn} = E\{c_{k,fn} | \mathbf{X}, \mathbf{W}, \mathbf{H}\} = \underbrace{\frac{w_{fk} h_{kn}}{\sum_j w_{fj} h_{jn}}}_{\text{time-freq. mask}} x_{fn}$$

Lossless decomposition: $x_{fn} = \sum_k \hat{c}_{k,fn}$

Itakura-Saito NMF: a generative approach

(Févotte, Bertin, and Durrieu, 2009)

Alternatively, IS-NMF can be interpreted as maximum likelihood in multiplicative noise:

$$v_{fn} = |x_{fn}|^2 = [\mathbf{WH}]_{fn} \cdot \epsilon_{fn}$$

where ϵ_{fn} is Gamma multiplicative noise with mean value 1.

Related work by Abdallah and Plumbley (2004).

Noteworthy properties of the IS divergence

- ▶ The IS divergence is scale-invariant:

$$d_{IS}(\lambda x | \lambda y) = d_{IS}(x | y)$$

Implies higher accuracy in the representation of data with large dynamic range, such as audio spectra. In contrast,

$$d_{EUC}(\lambda x | \lambda y) = \lambda^2 d_{EUC}(x | y)$$

$$d_{KL}(\lambda x | \lambda y) = \lambda d_{KL}(x | y)$$

- ▶ The IS divergence is nonconvex (inflexion at $y = 2x$); was found to lead to more local minima in practice.

Other statistical factor models of the spectrogram

Latent factor models for count data inspired from text analysis:

- ▶ Poisson models (Le Roux et al., 2007; Cemgil, 2009), similar to *GaP* (Canny, 2004)
- ▶ Multinomial models (Shashanka et al., 2008; Smaragdis et al., 2009), similar to *PLSI* (Hofmann, 1999) or *LDA* (Blei et al., 2003; Buntine and Jakulin, 2006)

Not generative models:

- ▶ Data $|x_{fn}|$ is modeled as integer.
- ▶ Additivity is assumed at the magnitude level

$$|x_{fn}| = \sum_k |c_{k,fn}|.$$

Small-scale example



(MIDI numbers : 61, 65, 68, 72)

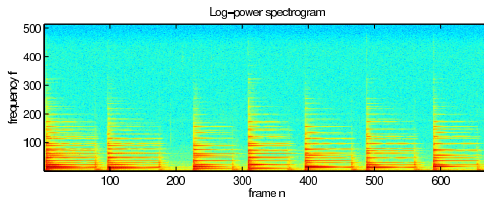
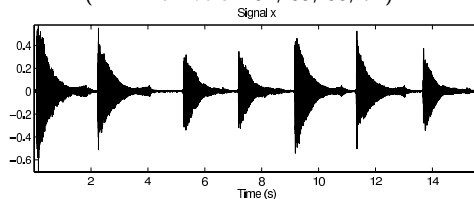
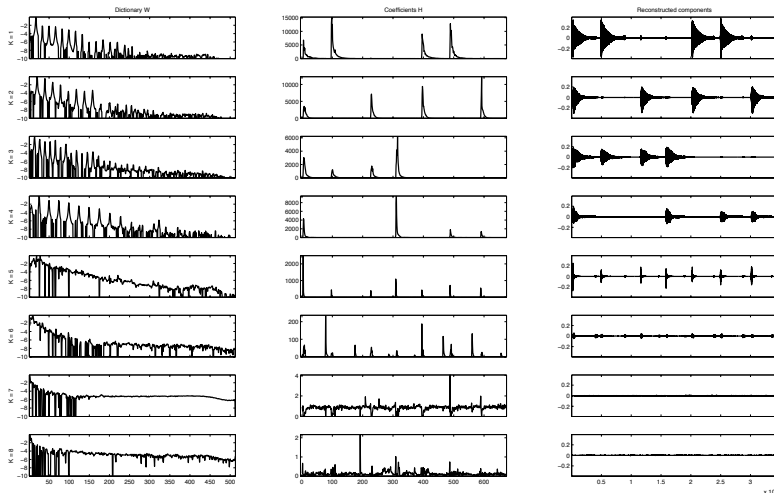
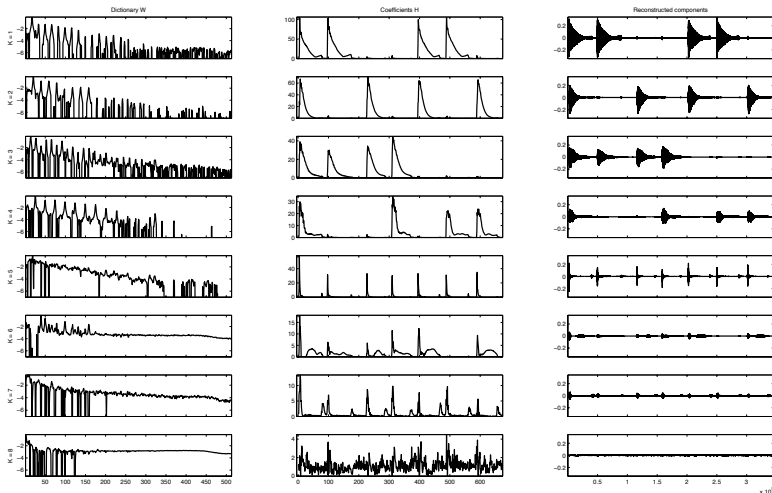


Figure: Three representations of **data**.

IS-NMF on power spectrogram with $K = 8$ 

Pitch estimates: 65.0 68.0 61.0 72.0 0 0 0 0
 (True values: 61, 65, 68, 72)

KL-NMF on magnitude spectrogram with $K = 8$



Pitch estimates: 65.2 68.2 61.0 72.2 0 56.2 0 0
 (True values: 61, 65, 68, 72)

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Motivation

- ▶ Model the temporal structure in audio signals.
 - ▶ more accurate estimation of \mathbf{H} , and \mathbf{W} ,
 - ▶ reduced identifiability ambiguities,
 - ▶ perceptually more pleasant component reconstruction.
- ▶ Many existing models for nonnegative data or for sequences, but hard to gather desirable properties.

**Dynamical models for
real-valued data:**

Linear dynamical system (LDS)
Hidden Markov model (HMM)

**Static models for
nonnegative data:**

Nonnegative matrix factorization
(NMF)

- ▶ **Goal:** bring advantages of LDS, HMM and NMF together in a simple and elegant framework.

Linear dynamical system (LDS)

The classic Gaussian model for real-valued data

$$\mathbf{h}_n = \mathbf{A}\mathbf{h}_{n-1} + \boldsymbol{\xi}_n \quad (\text{state dynamics})$$

$$\mathbf{v}_n = \mathbf{W}\mathbf{h}_n + \boldsymbol{\epsilon}_n \quad (\text{observation model})$$

- ▶ Continuous Markov chain with **real-valued** variables and parameters.
- ▶ **Additive Gaussian** innovations with zero mean value.

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$$\mathbb{E}[\mathbf{h}_n | \mathbf{A}\mathbf{h}_{n-1}] = \mathbf{A}\mathbf{h}_{n-1}$$

$$\mathbb{E}[\mathbf{v}_n | \mathbf{W}\mathbf{h}_n] = \mathbf{W}\mathbf{h}_n$$

Nonnegative dynamical system (NDS)

(Févotte, Le Roux, and Hershey, 2013)

$$\mathbf{h}_n = \mathbf{A}\mathbf{h}_{n-1} \circ \boldsymbol{\xi}_n \quad (\text{state dynamics})$$

$$\mathbf{v}_n = \mathbf{W}\mathbf{h}_n \circ \boldsymbol{\epsilon}_n \quad (\text{observation model})$$

- ▶ Continuous Markov chain with **nonnegative** variables and parameters.
- ▶ **Multiplicative Gamma** innovations with mean value 1.

$$\mathbb{E}[\mathbf{h}_n | \mathbf{A}\mathbf{h}_{n-1}] = \mathbf{A}\mathbf{h}_{n-1}$$

$$\mathbb{E}[\mathbf{v}_n | \mathbf{W}\mathbf{h}_n] = \mathbf{W}\mathbf{h}_n$$

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$$\mathbf{v}_n = \mathbf{W}\mathbf{h}_n \circ \boldsymbol{\epsilon}_n \quad (\text{observation model})$$

- ▶ The observation model underlies an Itakura-Saito (IS) pseudo-likelihood:

$$-\log p(\mathbf{V}|\mathbf{WH}) = D_{IS}(\mathbf{V}|\mathbf{WH}) + cst$$

- ▶ When $\mathbf{A} = \mathbf{I}_K$, we obtain smooth IS-NMF, i.e.,
 $E[h_{kn}|h_{k(n-1)}] = h_{k(n-1)}$. (Févotte, 2011)

Parameter estimation

MAP approach:

$$\min_{\mathbf{W}, \mathbf{A}, \mathbf{H} \geq 0} C(\mathbf{W}, \mathbf{H}, \mathbf{A}) = \underbrace{-\log p(\mathbf{V}|\mathbf{W}\mathbf{H})}_{\text{fit}} - \underbrace{\log p(\mathbf{H}|\mathbf{A})}_{\text{dynamics}}$$

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Optimization:

- ▶ Block-coordinate descent algorithm that updates \mathbf{W} , \mathbf{A} and \mathbf{H} in turn.
- ▶ Adjacent columns of \mathbf{H} are coupled in the optimization; we used a left-to-right block-coordinate descent:

$$\underbrace{\mathbf{h}_1^{(i)} \rightarrow \dots \rightarrow \mathbf{h}_{n-1}^{(i)}}_{\text{already updated}} \rightarrow \mathbf{h}_n \rightarrow \underbrace{\mathbf{h}_{n+1}^{(i-1)} \rightarrow \dots \rightarrow \mathbf{h}_{n+1}^{(i-1)}}_{\text{not yet updated}}$$

- ▶ Updates obtained by majorization-minimization (MM).

Training a speech model

- ▶ Data
 - ▶ Speech from TIMIT, 16 kHz. [exemple 1](#) [exemple 2](#) [exemple 3](#)
 - ▶ 1000 files per gender (\approx 50 minutes per gender).
 - ▶ Speaker-independent, gender-dependent training.

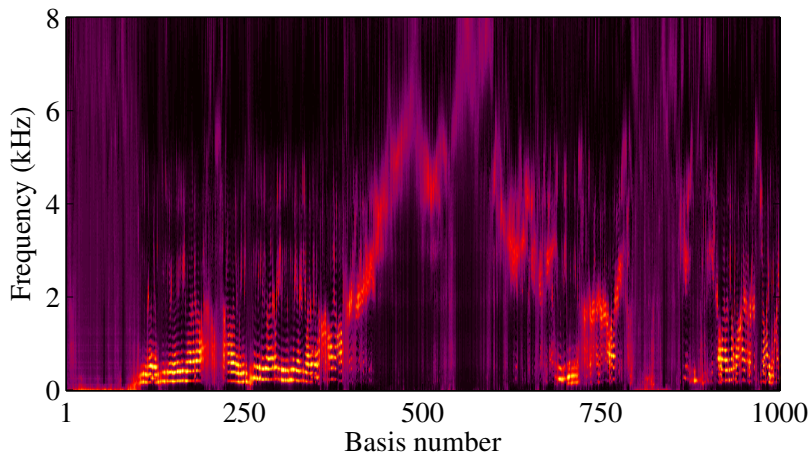
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 - ▶ Sine window with 50% overlap.
 - ▶ Frame length: 32 ms to 60 ms.

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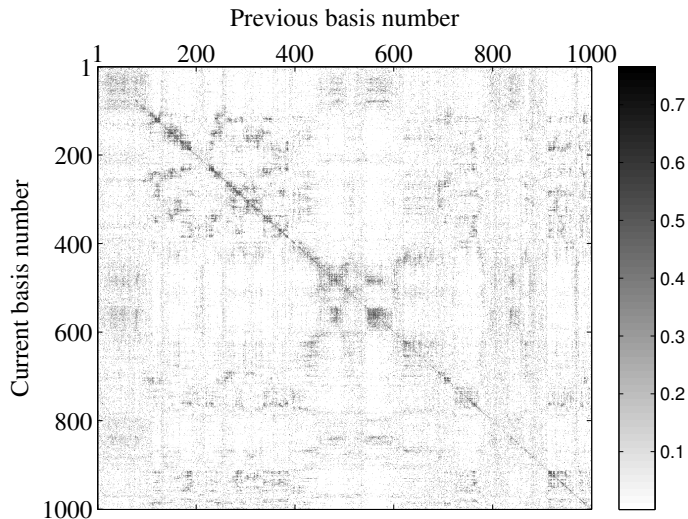
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 - ▶ Frame length: 32 ms to 60 ms.
- ▶ NDS specifications
 - ▶ $K = 1000$
 - ▶ Observation: $\mathbf{v}_t = \mathbf{W}\mathbf{h}_t \circ \epsilon_t$ with exponential innovation (\Leftrightarrow Gaussian modeling of the complex-valued STFT)
 - ▶ Dynamics: $\mathbf{h}_t = \mathbf{A}\mathbf{h}_t \circ \xi_t$ with very sparse innovation
 - \Rightarrow favors very few active coefficients in every frame
 - \Rightarrow dictionary elements look like phonemes
 - \Rightarrow “relaxed” HMM model

Trained dictionary \mathbf{W} (female)



(spectral patterns sorted greedily by similarity)

Trained transition matrix \mathbf{A} (female)



Speech enhancement

$$\underbrace{x}_{\text{noisy signal}} = \underbrace{s}_{\text{clean speech}} + \underbrace{n}_{\text{noise}}$$

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1. Learn $\mathbf{W}^{\text{train}}$, $\mathbf{A}^{\text{train}}$ from clean speech (training data).

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3. Produce the decomposition

$$\mathbf{V} = \mathbf{W}^{\text{train}} \mathbf{H} + \mathbf{W}^{\text{noise}} \mathbf{H}^{\text{noise}}$$

where

- ▶ $E[\mathbf{h}_n | \mathbf{A}^{\text{train}} \mathbf{h}_{n-1}] = \mathbf{A}^{\text{train}} \mathbf{h}_{n-1}$ a priori.
- ▶ $\mathbf{W}^{\text{noise}} \mathbf{H}^{\text{noise}}$ forms a “garbage” NMF model of the noise.

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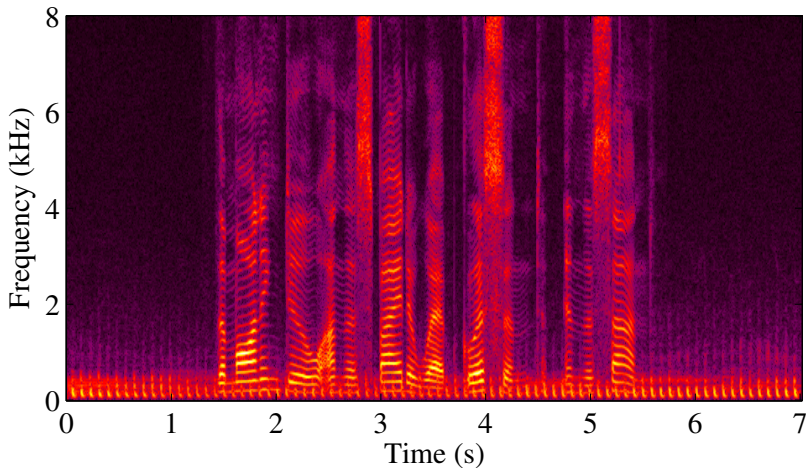
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 - ▶ $\mathbf{W}^{\text{noise}} \mathbf{H}^{\text{noise}}$ forms a “garbage” NMF model of the noise.
4. Produce the source estimate STFT by Wiener filtering

$$\hat{s}_{fn} = \frac{[\mathbf{W}^{\text{train}} \mathbf{H}]_{fn}}{[\mathbf{W}^{\text{train}} \mathbf{H} + \mathbf{W}^{\text{noise}} \mathbf{H}^{\text{noise}}]_{fn}} x_{fn}$$

Speech enhancement results

Helicopter

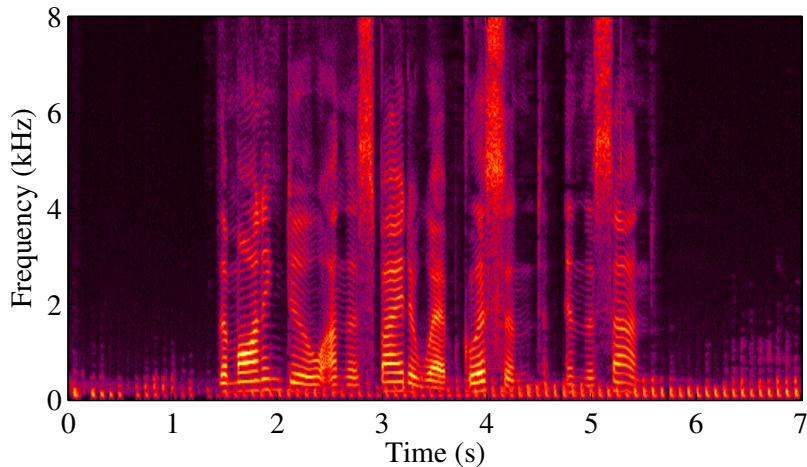
noisy signal



Speech enhancement results

Helicopter

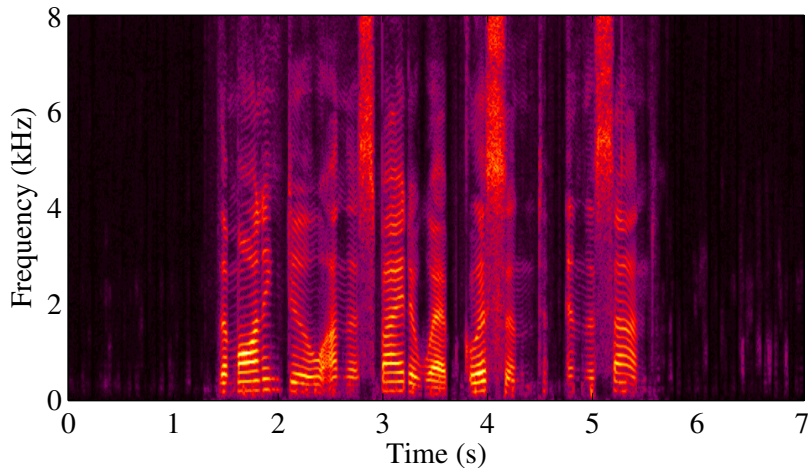
enhanced with OMLSA



Speech enhancement results

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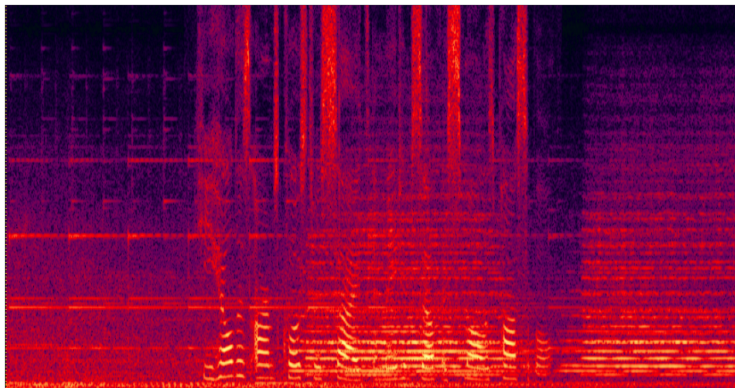
enhanced with NDS



Speech enhancement results

Train

noisy signal

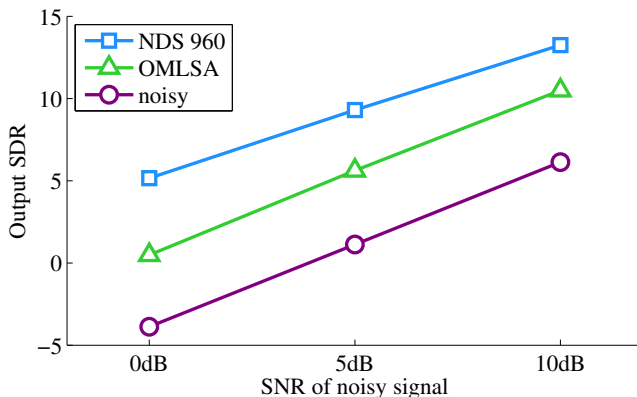


enhanced with OMLSA

enhanced with NDS

Speech enhancement results

- ▶ 150 mixtures: 10 speech files \times 15 texture sounds; 3 SNRs.
- ▶ Compared with state-of-the-art OMLSA (Cohen, 2002, 2003).
- ▶ 2 “garbage” spectral patterns to model the noise.



Conclusions

- ▶ Itakura-Saito NMF of the power spectrogram is underlain by a statistical model of superimposed Gaussian components.
- ▶ This model is relevant to the representation of audio signals.
- ▶ Algorithms can be designed in a principled way in the majorization-minimization setting.
- ▶ Regularized variants, e.g., NDS.

Conclusions

- ▶ Itakura-Saito NMF of the power spectrogram is underlain by a statistical model of superimposed Gaussian components.
- ▶ This model is relevant to the representation of audio signals.
- ▶ Algorithms can be designed in a principled way in the majorization-minimization setting.
- ▶ Regularized variants, e.g., NDS.
- ▶ Possible extension to multichannel data for audio source separation.
- ▶ The latent statistical model opens doors to fully Bayesian approaches that integrates over \mathbf{W} and/or \mathbf{H} (Févotte and Cemgil, 2009; Hoffman et al., 2010; Févotte et al., 2011; Dikmen and Févotte, 2011)

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