Transmission of light through complex Scattering material : calibration and imaging

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Light perturbation :

two regimes of perturbations :

- *turbulence :*
  continuous (weak) phase aberrations → effectively mitigated by adaptive optic

- *turbidity :* strong multiple scattering →   ?

Imaging in depth in scattering media

→ Conventionally : information from only unscattered (*'ballistic'*) light

but

Beer-Lambert Law: Exponential decay of the ballistic light

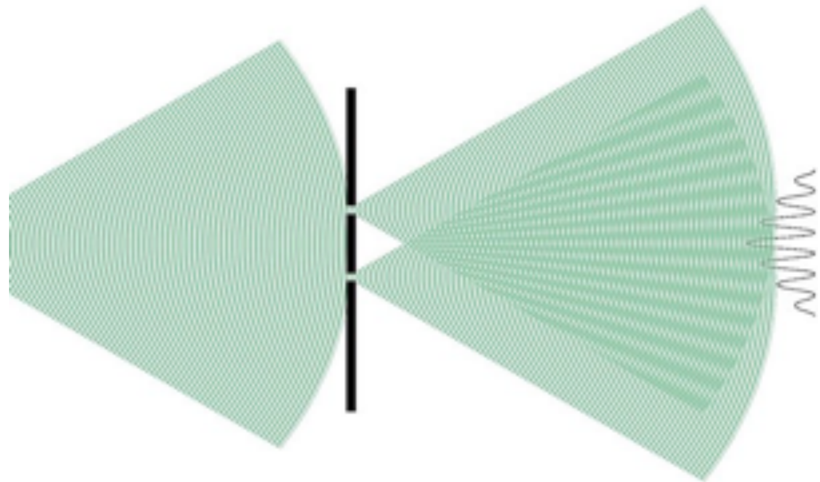
→ No imaging beyond a fews hundreds microns in living tissues

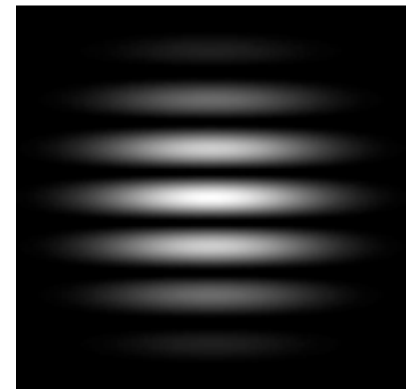
CAN WE GO DEEPER?

Scattering, a coherent process

Young's slit experiment:

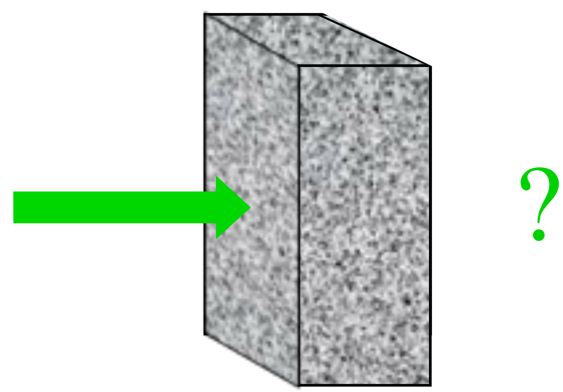
- two wave interference
- Fringes





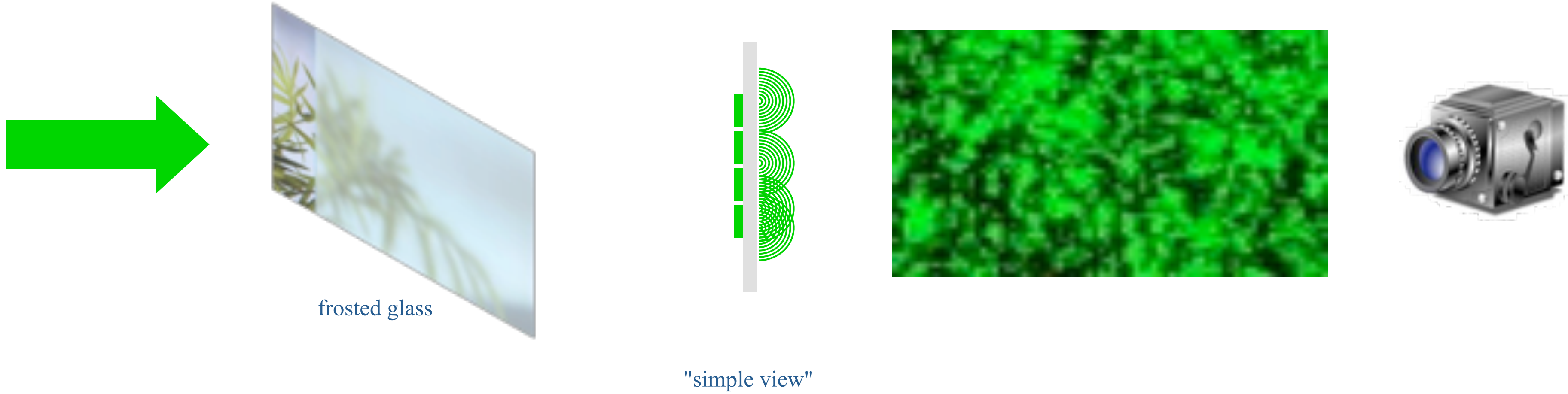
Multiple scattering

thin layer of white paint
(particle size ≤ 1 μm)

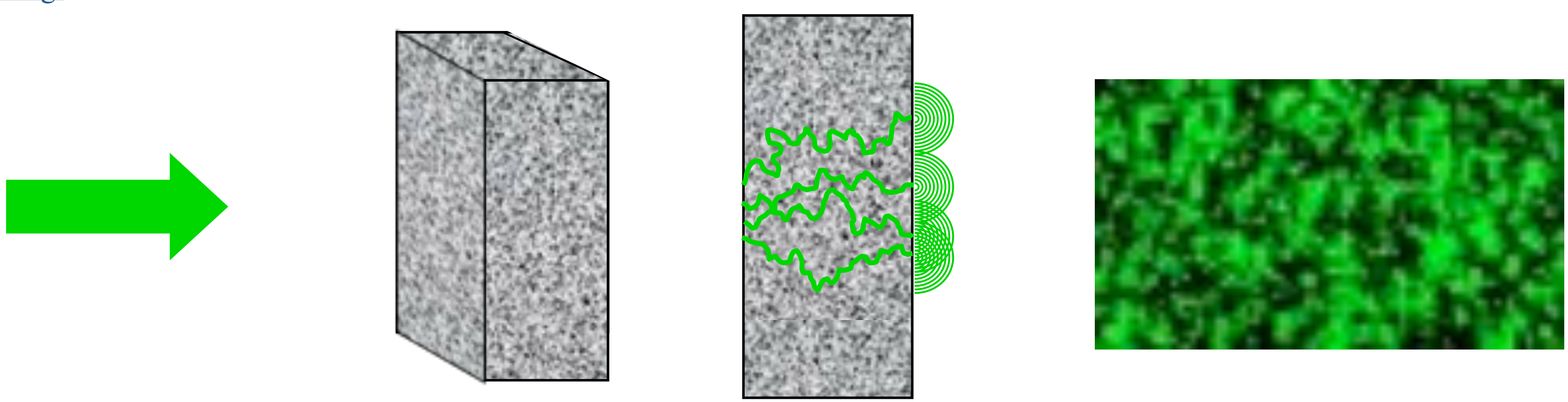


Scattering, a coherent process

Surface scattering :

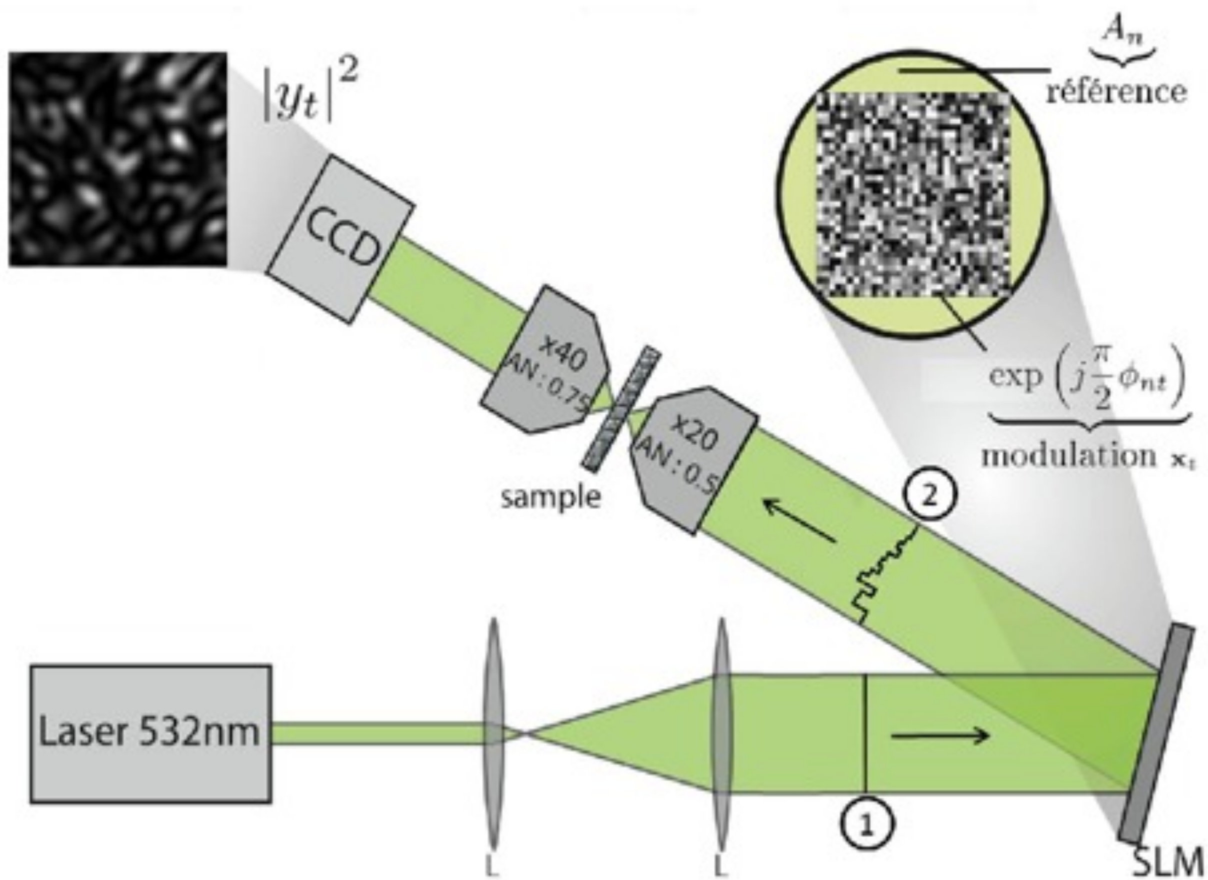


Volume scattering:

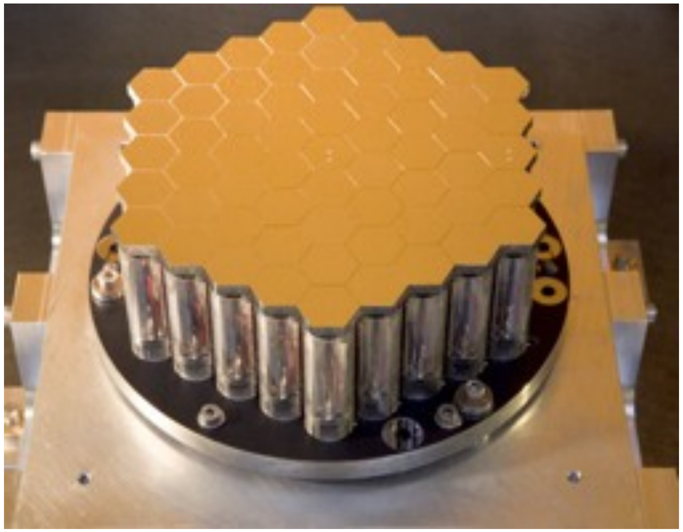


Speckle results from multiple interferences between a multiplicity of random paths

Schematic view of the experimental setup



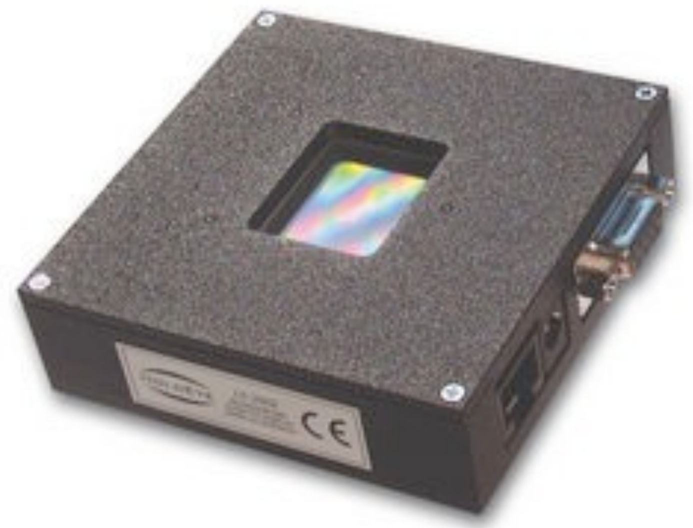
Devices for wavefront control



Deformable mirrors
(piezo, magnetics…)

10-100 actuators (typ.)
course : 10-20 microns
Speed > kHz

Adaptive optics

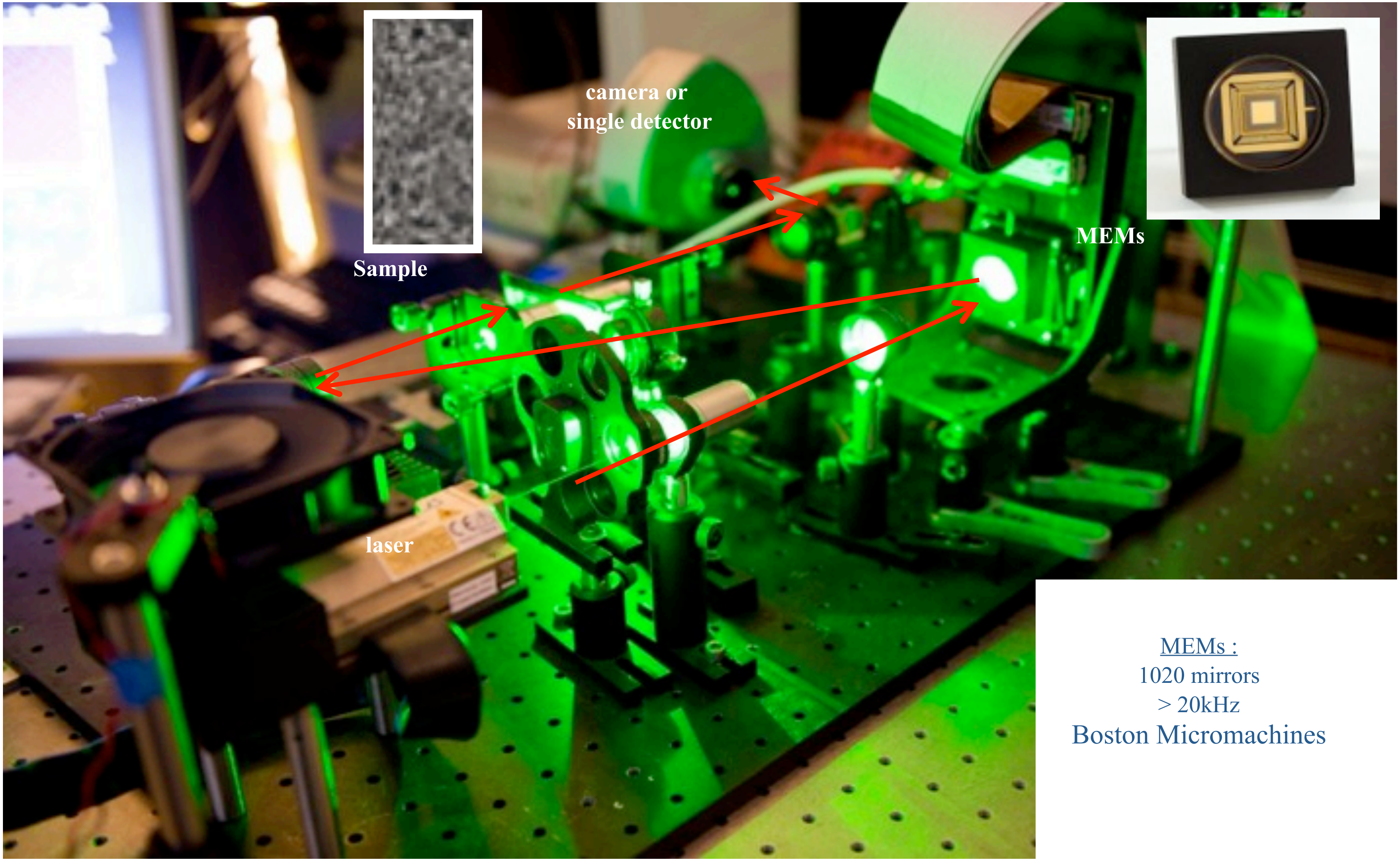


Spatial light modulator (SLM)
(mostly liquid crystals)

Segmented, >1 million pixel
course : 1 microns
speed: 50Hz

Diffractive optics, displays ….

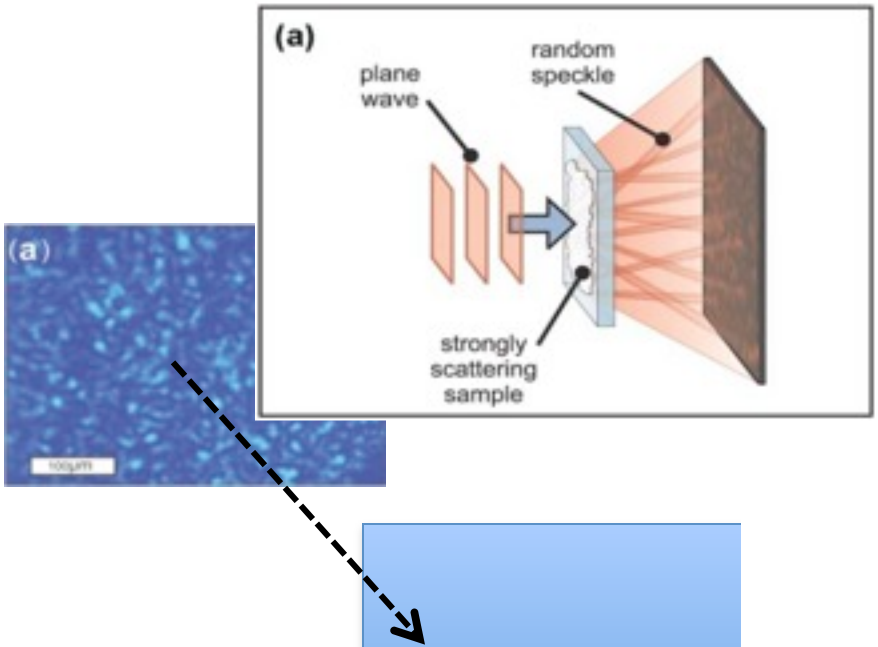
Experimental setup

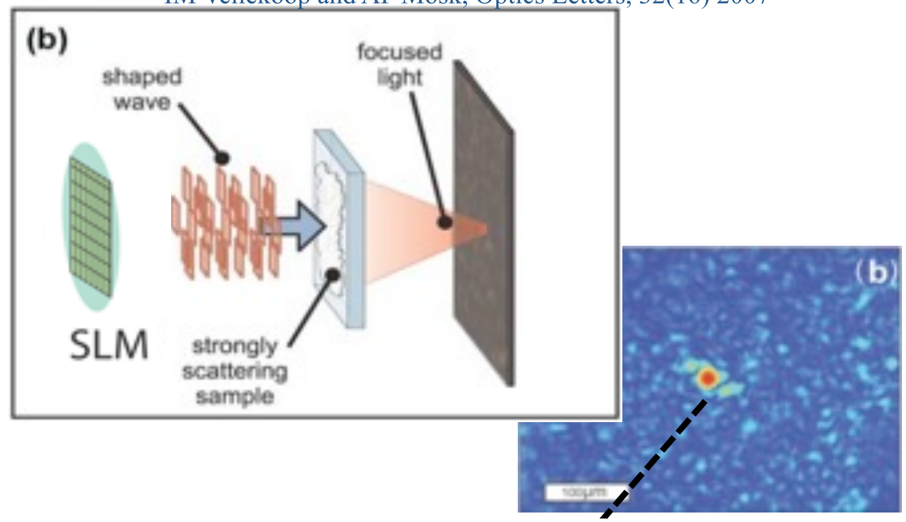


<u>MEMs :</u>
1020 mirrors
> 20kHz
Boston Micromachines

Optimization for focusing through complex media

IM Vellekoop and AP Mosk, Optics Letters, 32(16) 2007

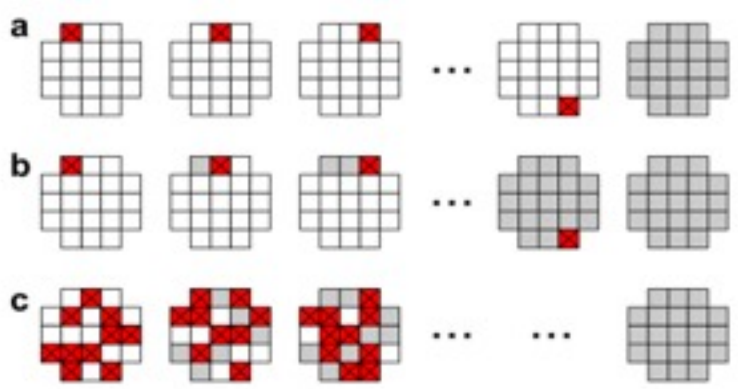


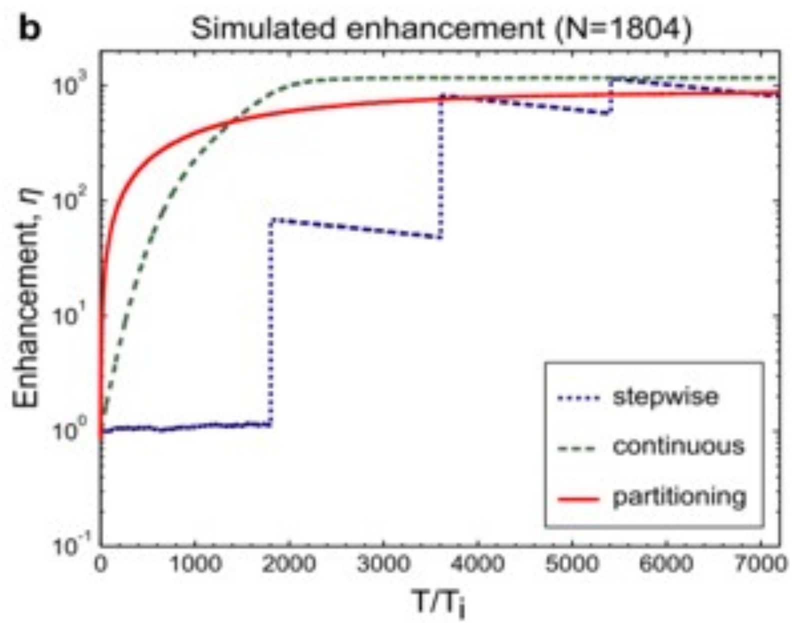


→ It is possible to shape these modes in phase to obtain a constructive interference on a single speckle grain (Equivalent to phase-conjugation)

Optimization for focusing through complex media

<u>different algorithms</u>





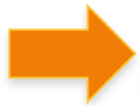
- independant action of pixels → converging optimization
- SNR α number of pixel controlled

I.M. Vellekoop, A.P. Mosk / Optics Communications 281 (2008) 3071–3080

A more general approach : the transmission matrix

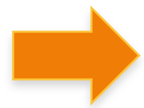
SLM: array of pixels

=



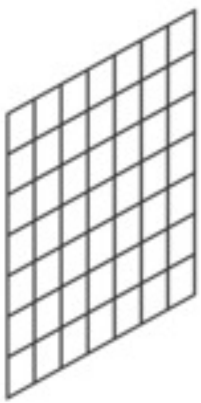
Linear system

=



CCD camera: arrays of pixels

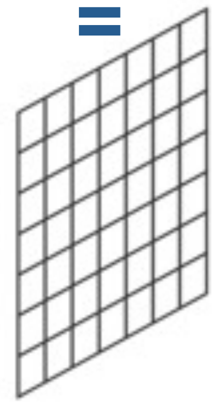
=



N complex-valued amplitudes

$$H = \begin{pmatrix} h_{1,1} & h_{1,2} & \dots & h_{1,N} \\ h_{2,1} & h_{2,2} & \dots & h_{2,N} \\ \vdots & & \ddots & \vdots \\ h_{M,1} & h_{M,2} & \dots & h_{M,N} \end{pmatrix}$$

MxN complex-valued matrix



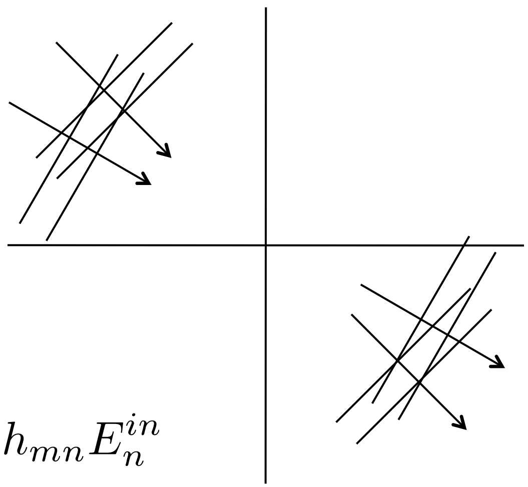
M modulus of complex-valued coefficients

$$\left|E_m^{out}\right| = \left|\sum_n h_{mn} E_n^{in}\right|$$

Sébastien Popoff

The transmission matrix

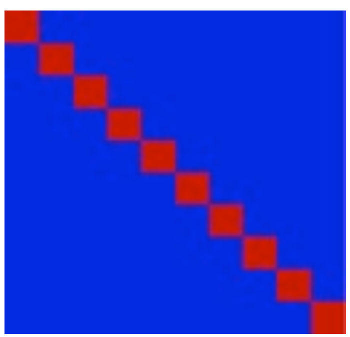
Free space



$$E_m^{out} = \sum_n^{1..N} h_{mn} E_n^{in}$$

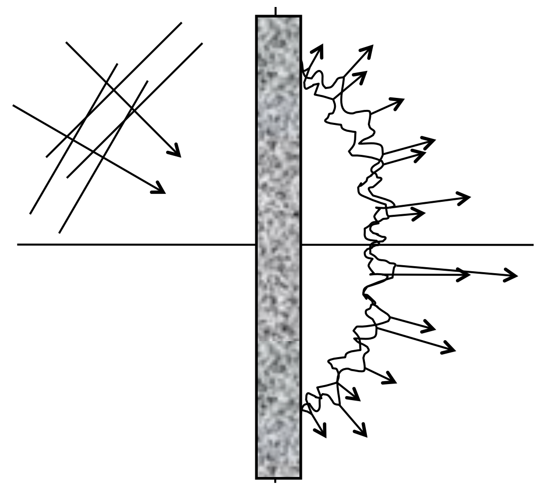
Input **k**

Output **k**



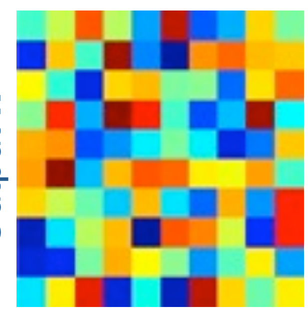
Identity Matrix

Multiple-scattering sample



Input **k**

Output **k**



Seemingly Random Matrix

Speckle interferometry for recovering complex observations

$$I_{out} = |E_{out}|^2$$

Intensity measurement
non-linear

$$e^{i\phi}$$

$$I^{\phi} = |E_{out} + e^{i\phi} E_{ref}|^2$$

4 Steps to extract the amplitude of the speckle on the CCD

$$E_{out} \propto \left(I^0 - I^{\pi}\right) + i \left(I^{3\pi}\right.$$

…but requires interferometric stability during several minutes

$$e^{i\phi}$$

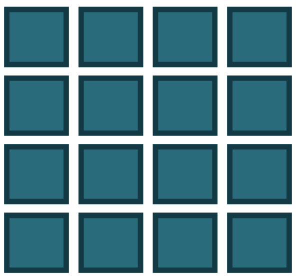
Solution : use part of the speckle as a reference
(the scattering medium is the interferometer)

- Excellent stability
- The reference is a speckle field

- **Measuring the Transmission Matrix (calibration)**
  - Focusing with the TM
    - Fundamental insight on the medium
      - Imaging with the TM

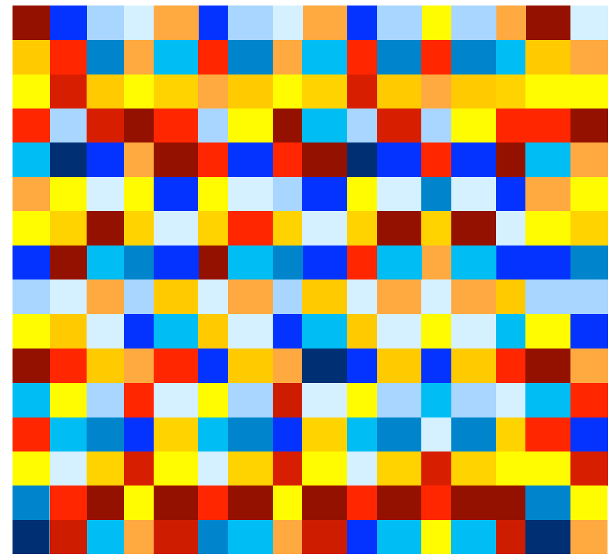
Measuring the Matrix H

Step by step reconstruction



$$E_m^{obs} = E_m^{ref} \sum_n^{1..N} h_{mn} E_n^{in}$$

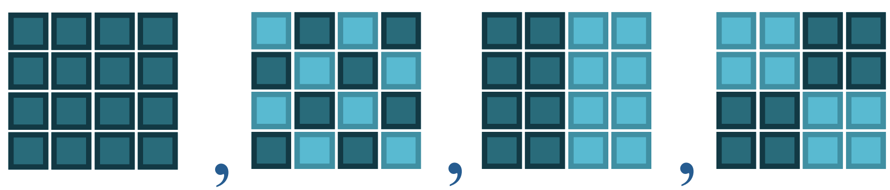
$$H_{obs} = H.\Sigma_{ref}$$



Pixel off   Pixel on

Popoff et. al. use Hadamard basis

(Phase-only SLM,SNR)



etc…

φ=+π/2

φ=-π/2

Least-squares estimation of the TM

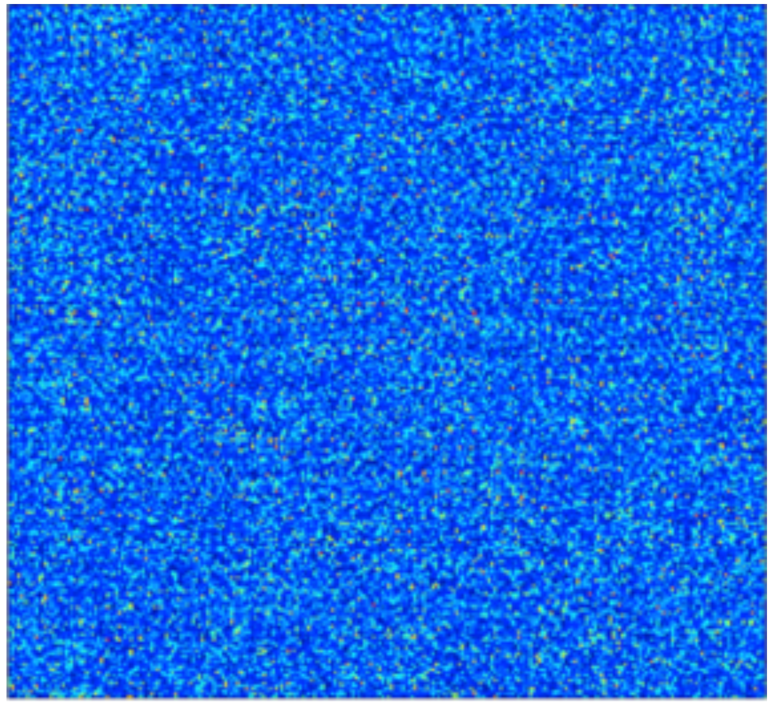
$$Y^* = X^* H^* + E^*$$

$$\hat{h}_m = y_m X^\star \left(X X^\star + \left(\frac{\sigma_m}{\sigma_H} \right)^2 I_M \right)^{-1}$$

- Handles pixel-dependant noise in the observation
- P>N leads to better calibration

And a few minutes later….

paint on a glass slide:
strongly scattering



Transmission matrix

(in amplitude)

"signature" of the random medium
=
Useful as long as the medium is stable

Paint :
>1 hour
biological sample :
<1ms !!

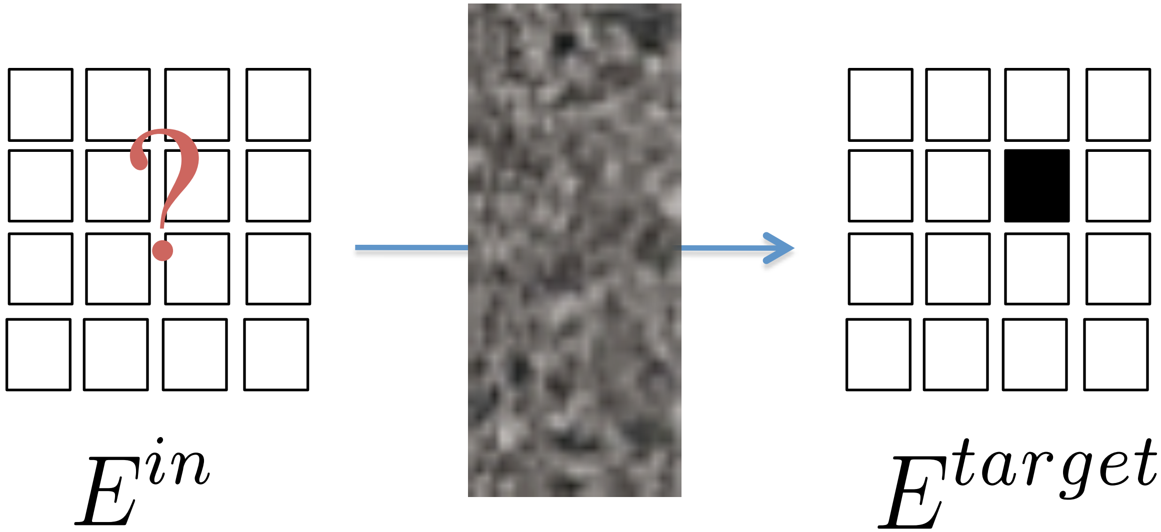
- Measuring the Transmission Matrix

- **Focusing with the TM**

- Fundamental insight on the medium

- Imaging with the TM

Exploiting H : focusing



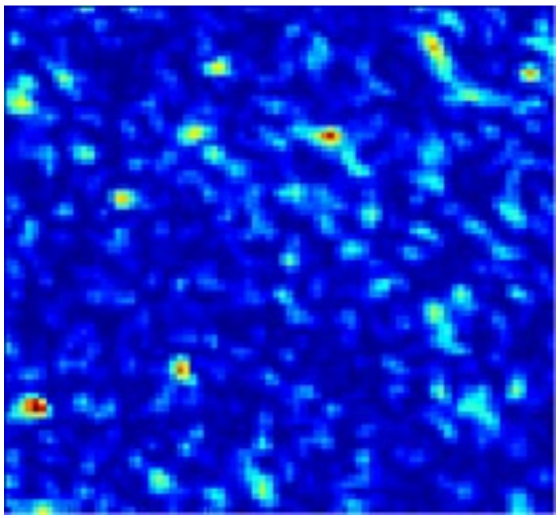
Can H tell us what input will give a given output?

YES: $E^{in} = H^{\dagger} E^{target}$

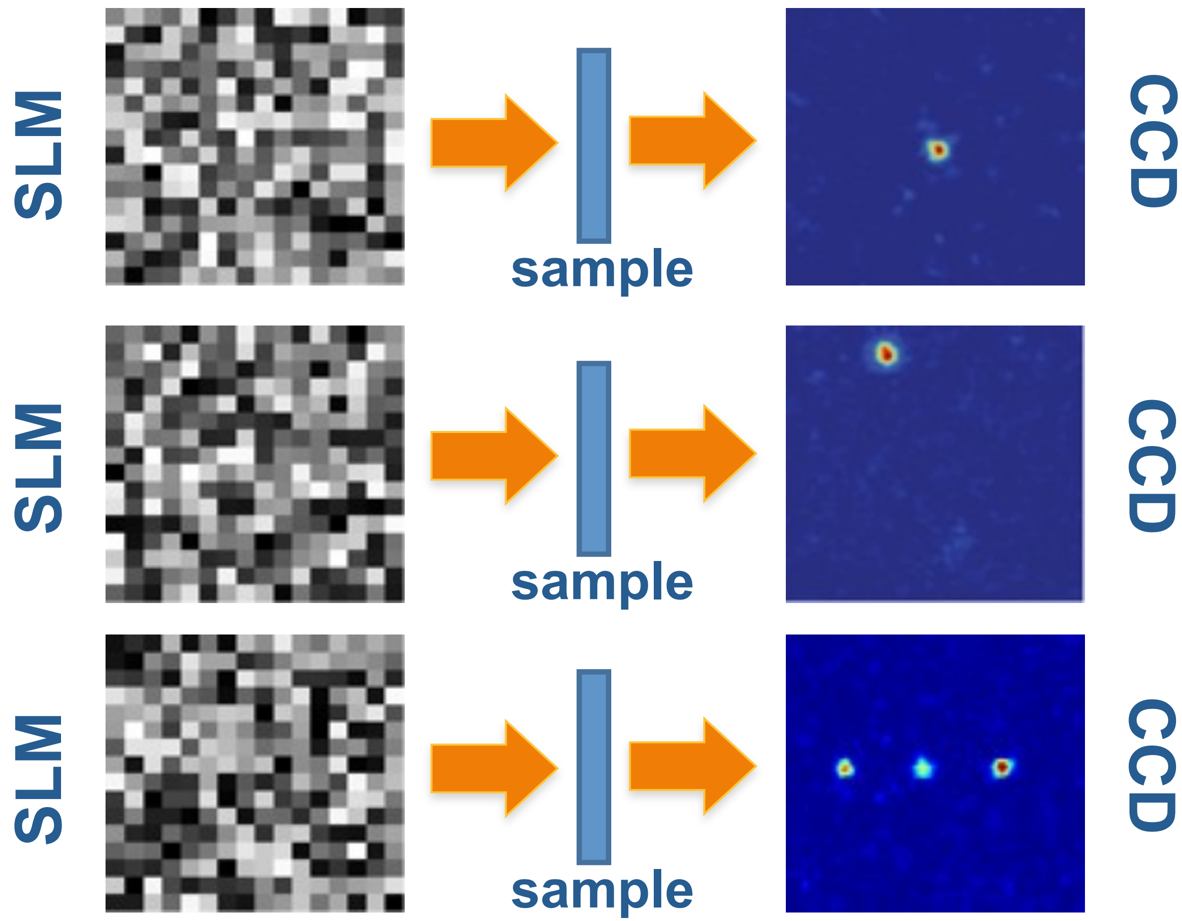
Two interpretations : phase-conjugation
time-reversal

Popoff et al. Phys. Rev. Lett. 104,100601 (2010)

Experimental results: focusing



Plane wave input



Popoff et al. PRL 107, 263901 (2011)

- Measuring the Transmission Matrix
- Focusing with the TM
- **Fundamental insight on the medium**
- Imaging with the TM

Statistical properties of the transmission matrix

Tool: Singular Value Decomposition
(generalization of diagonalization for any Matrix)

$$H = U \Lambda V^*$$

Output basis

Input basis

$$\Lambda = \begin{bmatrix} \lambda_1 & 0 & 0 & 0 \\ 0 & \lambda_2 & 0 & 0 \\ 0 & 0 & \dots & \dots \\ 0 & 0 & \dots & \lambda_N \end{bmatrix}$$

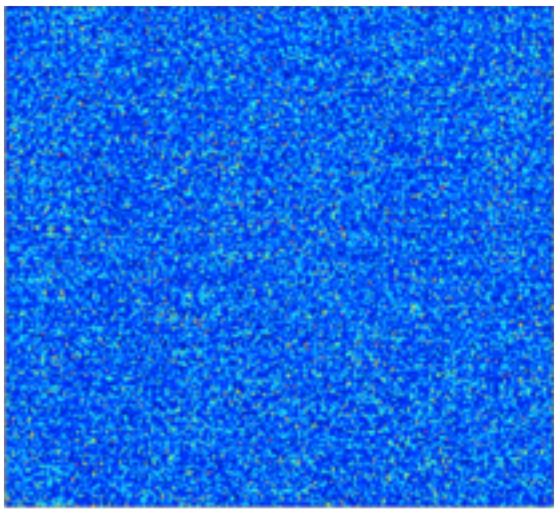
-λ_i >0 represents the **amplitude transmission** through the i^th channel.

-$\Sigma\lambda_i^2$ corresponds to the **total transmittance** for a plane wave

➡ We study the **distribution of (normalized) singular values ρ(λ)**

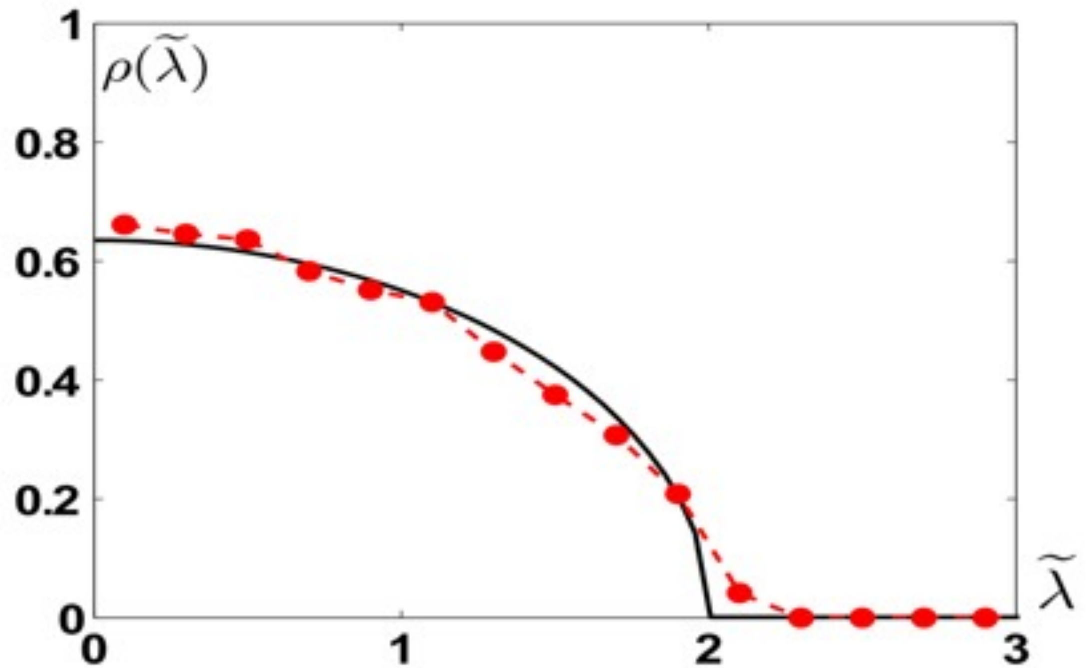
Statistical properties of the transmission matrix

A general Random Matrix Theory prediction : quarter circle law distribution



Transmission matrix

(filtered to remove effect of the reference)



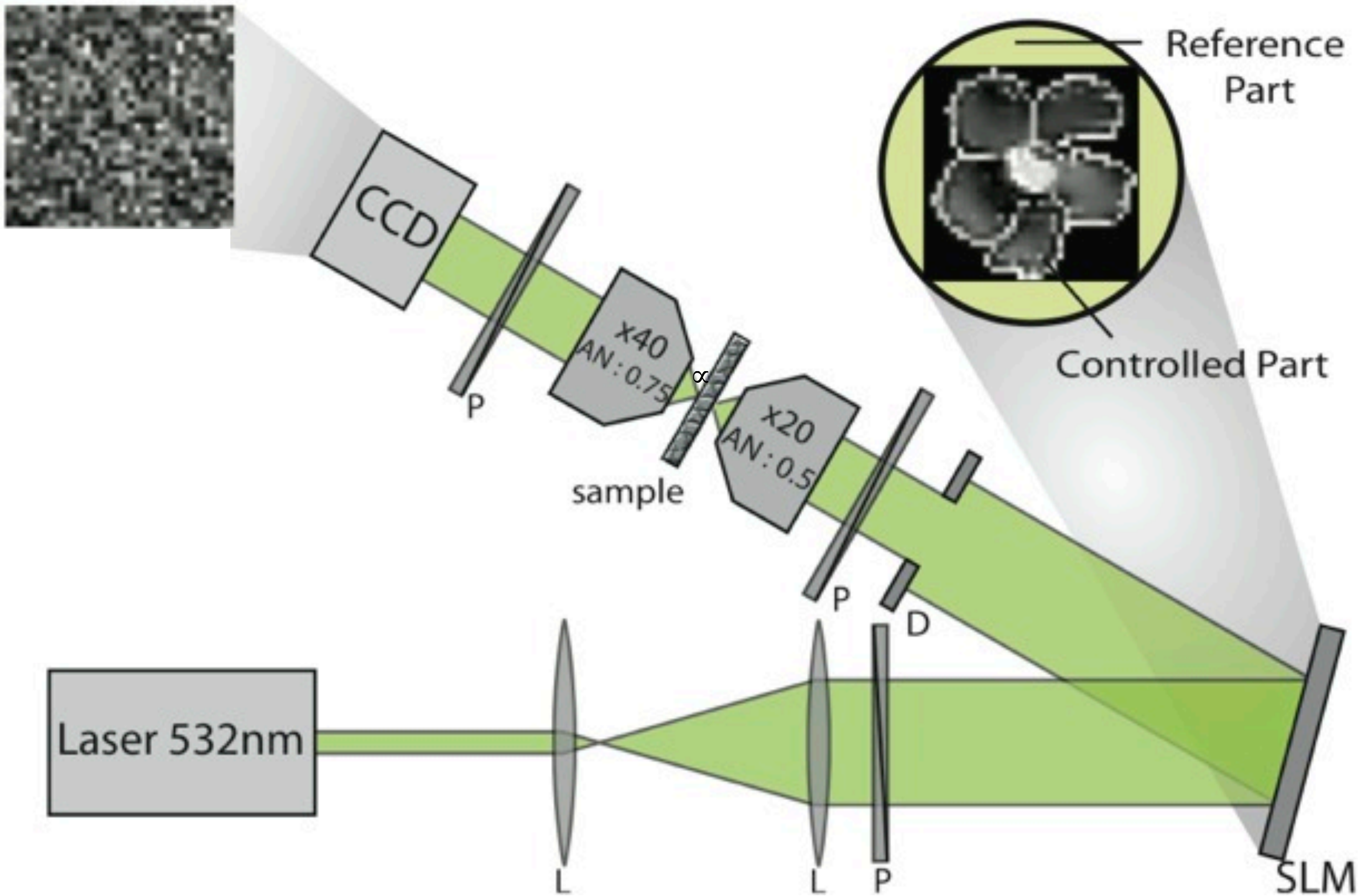
Signature of randomness !

In acoustics:
A. Aubry *et al.*, Phys. Rev. Lett., 102, 84301, (2009)

Popoff et al. Phys. Rev. Lett. 104,100601 (2010)

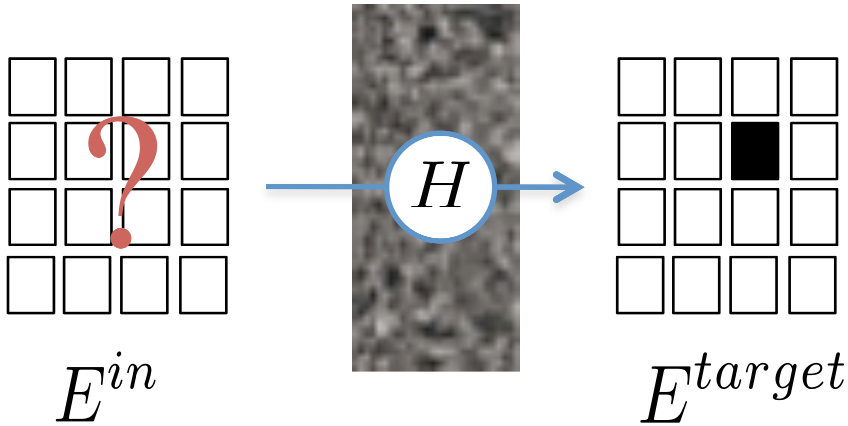
Overview

- Measuring the Transmission Matrix
- Focusing with the TM
- Fundamental insight on the medium
- **Imaging with the TM**


Reference
Part
Controlled Part
CCD
P
x40
AN : 0.75
sample
x20
AN : 0.5
P
D
Laser 532nm
L
L
P
SLM

Using the matrix for imaging

Direct problem

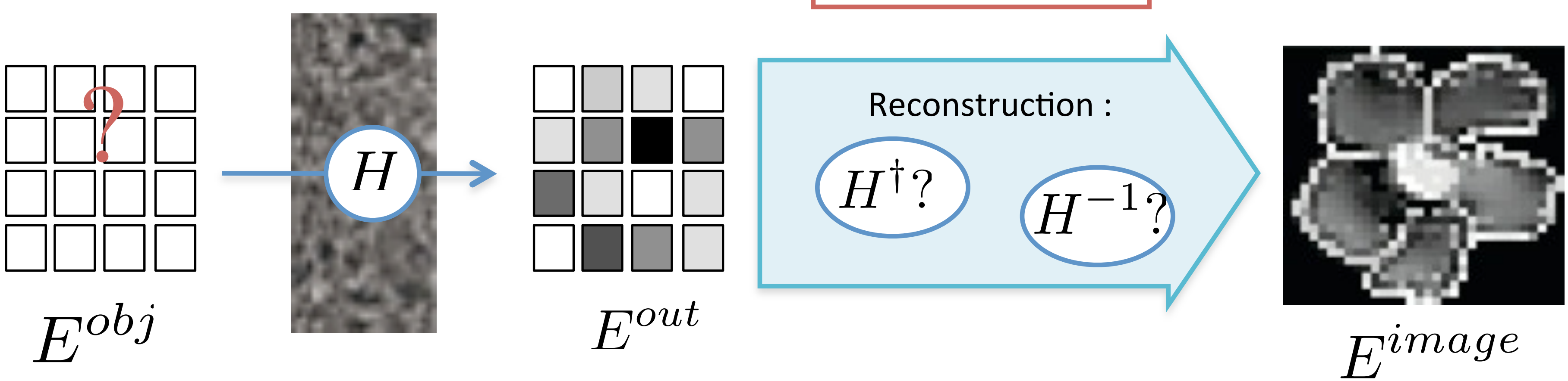


What wavefront to focus to a given target?

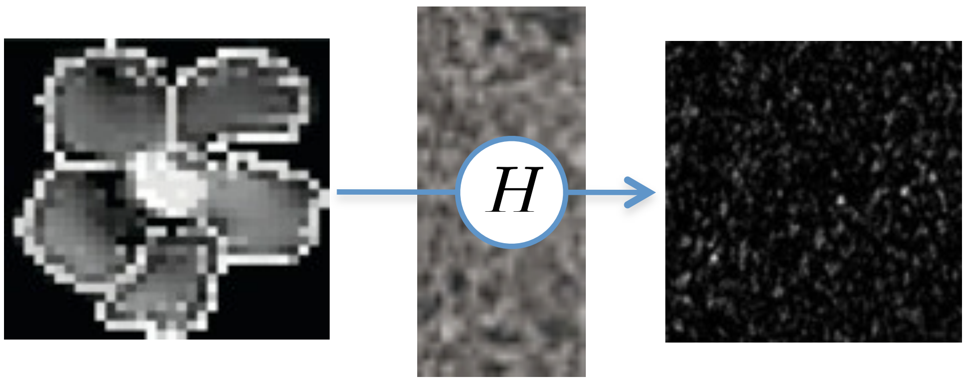
$$E^{in} = H^{\dagger} E^{target}$$

= phase conjugation

Inverse problem



Experimental results : imaging with many pixels



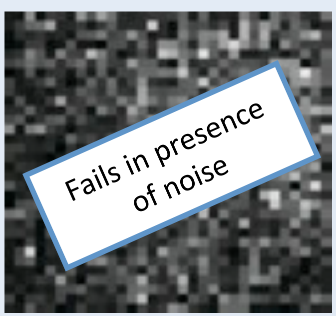
$$H = U \begin{bmatrix} \lambda_1 & 0 & 0 & 0 \\ 0 & \lambda_2 & 0 & 0 \\ 0 & 0 & \dots & \dots \\ 0 & 0 & \dots & \lambda_N \end{bmatrix} V$$

Reconstruction

Inversion

H^{-1}

$\lambda_i \to {}^{1}$



C = 11%

Phase Conjugation

$H^{\dagger}$

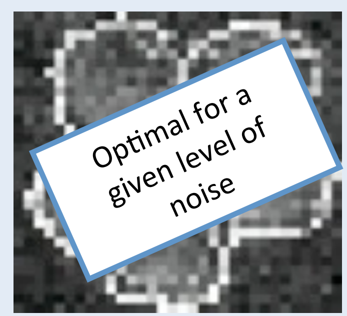
$\lambda_i \to \lambda_i^*$

C = 76%

Tikhonov

$$(H^{\dagger}H + \sigma I)^{-1} H^{\dagger}$$

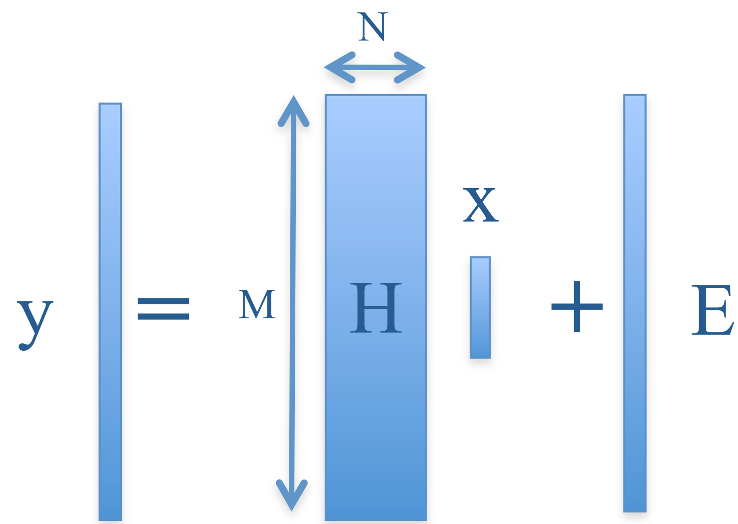
H^{-1} noise $H^{\dagger}$



C = 95%

Popoff et al. *Nat. Commun.* 1:81 doi: 10.1038/ncomms1078 (2010)

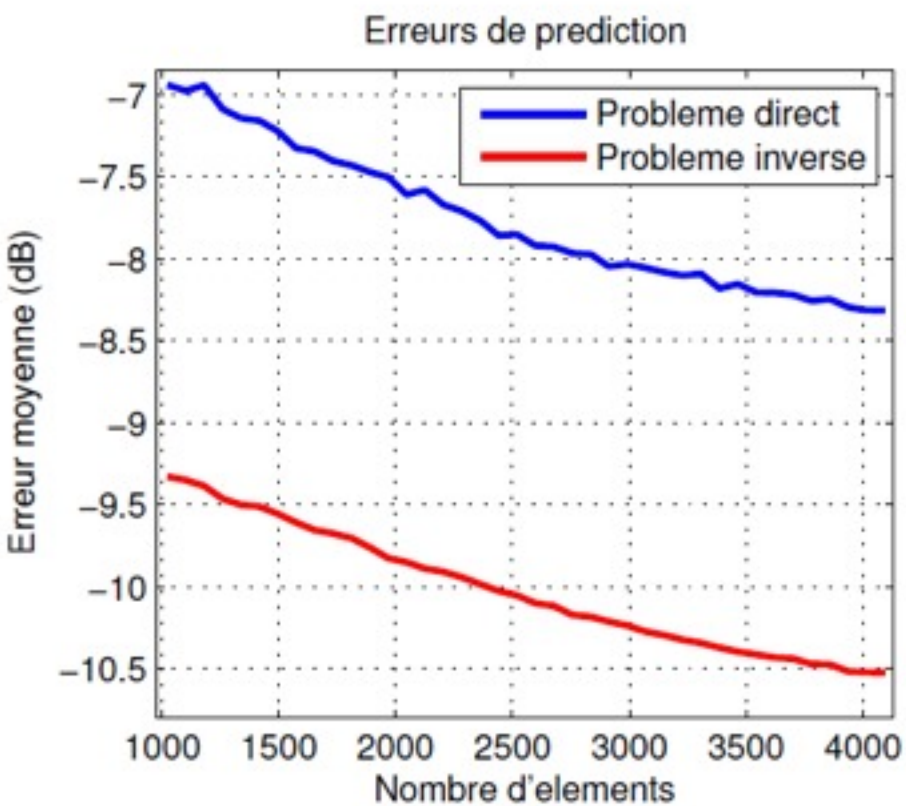
Least-squares inverse problem



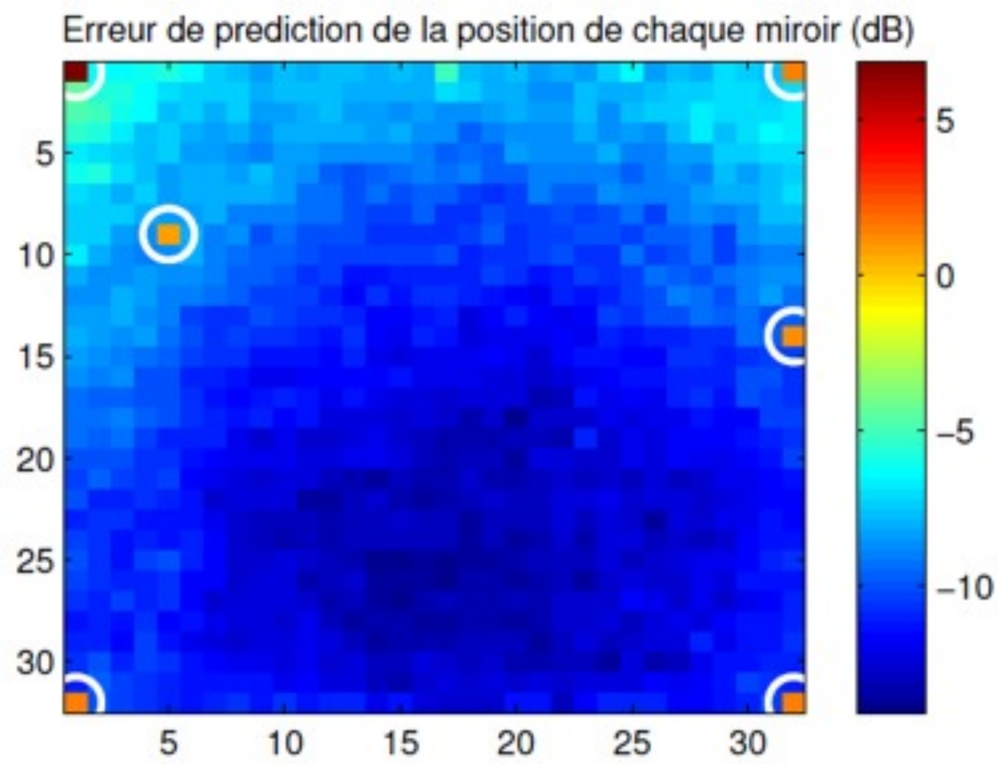
$$\hat{x} = \left(R_x^{-1} + H^\star R_e^{-1} H\right)^{-1} H^\star R_e^{-1} y$$

- Accounts for correlations in the noise and input through correlation matrices R_e and R_x
- Generalizes the Tikhonov regularization

- Calibration quality vs #measurements



- Detecting SLM defaults



To appear in GRETSI 2013

Conclusion and outlook

wavefront shaping : a change of paradigm for complex media

- **Imaging**

  spatial focusing and imaging now demonstrated in many systems

- **Universal sensing matrices**

  scattering media exhibit random matrices properties

  well fitted to some imaging applications

- **Much room for research**
  - Better calibration
  - Better algorithms for reconstruction
  - Handling **intensity** measurements

Team and collaborations

Laurent DAUDET

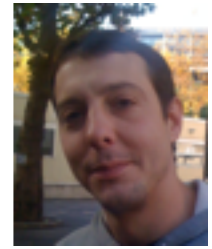
Sylvain GIGAN

David MARTINA

Manuel MOUSSALLAM

Ori KATZ

Sébastien POPOFF



Geoffroy LEROSEY

Igor CARRON

Thank you for your attention !

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