Mesuring the microvibrations of a satellite on the disparity map

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Journée traitement de l'image
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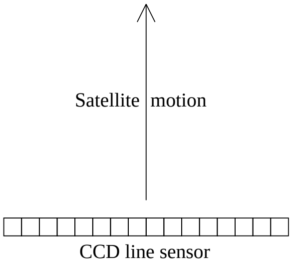
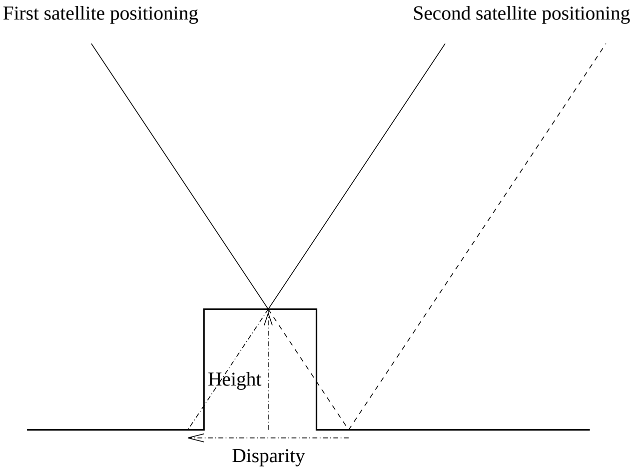
Push-broom imaging system



The image can be distorted
by the satellite microvibrations.

But it can be restored
if the microvibrations are known.



The measured disparity is proportional to the relief.

Image #1

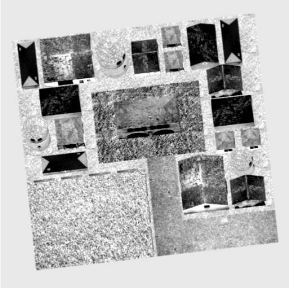
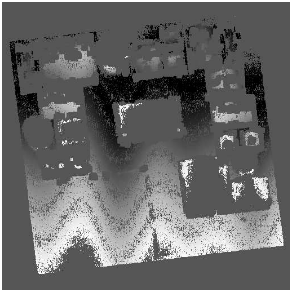


Image #2



The disparity map that is computed by MARC2 software [Sabater et al] is damaged by microvibrations

Objectifs

Keep the relief and microvibrations separate in order to

- obtain a good estimate of the relief,
- measure the microvibrations so as to restore the images.

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Two cases are considered

- Case 1 : System with a single CCD line sensor ⇒ The satellite has to point backward for the second image acquisition ⇒ **Change of vibration mod**.
- Case 2 : System with two CCD line sensors ⇒ The satellite does not have to point backward for the second image acquisitions ⇒ **Some vibration mod** for the two images (with time difference).

The vibrated stereoscopic image pair satisfy the following equations

$$\begin{aligned} I_1[k] &= P_1(k + V_1(k)), \\ I_2[k] &= P_2(k + V_2(k)), \quad \forall k, \end{aligned}$$

where V_1, V_2 are the satellite microvibrations, and P_1, P_2 are the landscapes that are observed from the two positionings of the satellite.

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where V_1, V_2 are the satellite microvibrations, and P_1, P_2 are the landscapes that are observed from the two positionings of the satellite.

$$P_1(x) = P_2(x + H(x)), \quad \forall x,$$

where H is the height of the landscape (directed as the motion of the satellite). Then we have

$$I_1[k] = P_2(k + V_2(k) + H(k + V_1(k))), \quad \forall k.$$

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The disparity D is measured after image interpolation

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One obtains

$$D[k] = H(k + V_1(k)) + V_1(k) - V_2(k + D[k]).$$

Proposed method

$V_i(x) \approx \sum_l A_i[l] e^{i\frac{\pi}{T}l\cdot x}$, where A_i is sparse, $\forall i \in \{1, 2\}$.

case 1 (Two vibration mods). Minimise

$$E_1(A_1, A_2) = \frac{1}{2}\|D - L_1 A_1 + L_2 A_2\|_2^2 + \lambda_1\|A_1\|_1 + \lambda_2\|A_2\|_1,$$

where $\begin{cases} L_1 A[k] = \sum_l A[l] e^{i\frac{\pi}{T}l\cdot k}, \\ L_2 A[k] = \sum_l A[l] e^{i\frac{\pi}{T}l\cdot(k+D[k])}. \end{cases}$

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$$E_1(A_1, A_2) = \frac{1}{2}\|D - L_1 A_1 + L_2 A_2\|_2^2 + \lambda_1 \|A_1\|_1 + \lambda_2 \|A_2\|_1,$$

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case 2 (One vibration mod). Minimise

$$E_2(A_1, A_2) = E_1(A_1, A_2) + \lambda_3 \|A_1 - S_\phi A_2\|_2^2,$$

or

$$E_3(A) = E_1(A, S_\phi A),$$

where $S_\phi A[k] = e^{i\frac{\pi}{T} k \cdot \phi} A[k]$ car $V_2(x) = V_1(x + \phi)$.

- **Direct minimization scheme.**

$$E_1 = F_1 + F_2 \quad \text{where} \begin{cases} F_1(A_1, A_2) = \frac{1}{2}\|D - L_1 A_1 + L_2 A_2\|_2^2, \\ F_2(A_1, A_2) = \lambda_1 \|A_1\|_1 + \lambda_2 \|A_2\|_1. \end{cases}$$

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Iterative Shrinkage Thresholding Algorithm (ISTA)

$$\begin{aligned} (A_1, A_2)^{(n+\frac{1}{2})} &= (A_1, A_2)^{(n)} - \tfrac{1}{L}\nabla F_1\left((A_1, A_2)^{(n)}\right), \\ (A_1, A_2)^{(n+1)} &= \operatorname{prox}_{\frac{1}{L}F_2}\left((A_1, A_2)^{(n+\frac{1}{2})}\right), \end{aligned}$$

$$\text{where} \begin{cases} \nabla F_1(A_1, A_2) = (L_1, -L_2)^*(L_1 A_1 - L_2 A_2 - D), \\ \operatorname{prox}_{\frac{1}{L}F_2}(A_1, A_2) = \tau_{\frac{(\lambda_1, \lambda_2)}{L}}(A_1, A_2) \ \text{(soft thresholding)}. \end{cases}$$

- **Direct minimization scheme.**

$$E_1 = F_1 + F_2 \quad \text{where} \begin{cases} F_1(A_1, A_2) = \frac{1}{2}\|D - L_1 A_1 + L_2 A_2\|_2^2, \\ F_2(A_1, A_2) = \lambda_1 \|A_1\|_1 + \lambda_2 \|A_2\|_1 . \end{cases}$$

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$$\text{where} \begin{cases} \nabla F_1(A_1, A_2) = (L_1, -L_2)^*(L_1 A_1 - L_2 A_2 - D), \\ \mathrm{prox}_{\frac{1}{L}F_2}(A_1, A_2) = \tau_{\frac{(\lambda_1, \lambda_2)}{L}}(A_1, A_2) \text{ (soft thresholding).} \end{cases}$$

$L_1^* L_1$ are $L_2^* L_2$ are Tœplitz matrices, but $L_1^* L_2$ and $L_2^* L_1$ are not.

Minimization

- **Alternating minimization scheme.**

$$
\begin{aligned}
A_1^{(n+1)} &= \arg\min_A \frac{1}{2}\left\| D - L_1 A + L_2 A_2^{(n)} \right\|_2^2 + \lambda_1 \|A\|_1 \\
A_2^{(n+1)} &= \arg\min_A \frac{1}{2}\left\| D - L_1 A_1^{(n+1)} + L_2 A \right\|_2^2 + \lambda_2 \|A\|_1
\end{aligned}
$$

- **Alternating minimization scheme.**

$$A_1^{(n+1)} = \arg\min_A \frac{1}{2}\left\|D - L_1 A + L_2 A_2^{(n)}\right\|_2^2 + \lambda_1 \|A\|_1$$
$$A_2^{(n+1)} = \arg\min_A \frac{1}{2}\left\|D - L_1 A_1^{(n+1)} + L_2 A\right\|_2^2 + \lambda_2 \|A\|_1$$

Each minimization is performed by **ISTA**.

$$A_1^{(n,k+\frac{1}{2})} = A_1^{(n,k)} - \frac{1}{L}\nabla G_1\left(A_1^{(n,k)}\right),$$
$$A_1^{(n,k+1)} = \operatorname{prox}_{\frac{1}{L}G_2}\left(A_1^{(n,k+\frac{1}{2})}\right),$$

où $\begin{cases} \nabla G_1(A) = L_1^* L_1 A - L_1^*(L_2 A_2 + D), \\ \operatorname{prox}_{\frac{1}{L}G_2}(A) = \tau_{\frac{\lambda_1}{L}}(A). \end{cases}$

Minimization

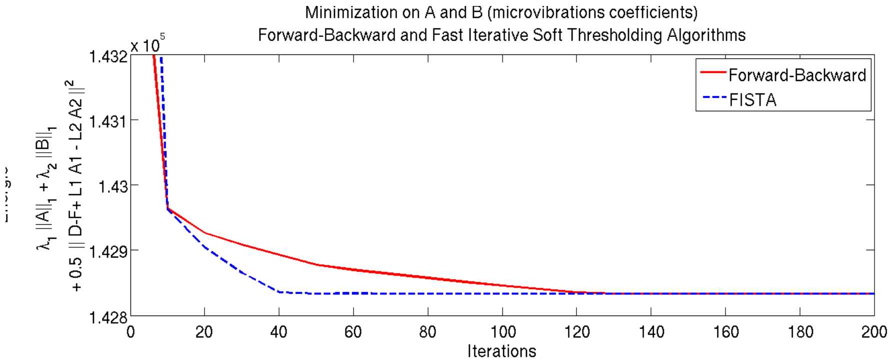
- Fast direct minimization scheme.

Fast Iterative Shrinkage Thresholding Algorithm (FISTA)

$$(B_1,B_2)^{(n+\frac{1}{2})} = (A_1,A_2)^{(n)} - \tfrac{1}{L}\nabla F_1\left((A_1,A_2)^{(n)}\right),$$
$$(B_1,B_2)^{(n+1)} = \mathrm{prox}_{\frac{1}{L}F_2}\left((B_1,B_2)^{(n+\frac{1}{2})}\right),$$
$$(A_1,A_2)^{(n+1)} = (B_1,B_2)^{(n+1)} + \tfrac{t_n-1}{t_{n+1}}\left((B_1,B_2)^{(n+1)} - (B_1,B_2)^{(n)}\right)$$

où
$$F_1(A_1,A_2) = \frac{1}{2}\|D - L_1A_1 + L_2A_2\|_2^2,$$
$$F_2(A_1,A_2) = \lambda_1\|A_1\|_1 + \lambda_2\|A_2\|_1,$$
$$\nabla F_1(A_1,A_2) = (L_1,-L_2)^*(L_1A_1 - L_2A_2 - D),$$
$$\mathrm{prox}_{\frac{1}{L}F_2}(A_1,A_2) = \mathcal{T}_{\frac{(\lambda_1,\lambda_2)}{L}}(A_1,A_2) \text{ (soft thresholding)},$$
$$t_{n+1} = (1+\sqrt{1+4t_n^2})/2.$$

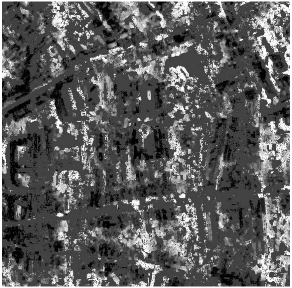
Comparison between **ISTA** and **FISTA**



$E_2(A_1^n, A_2^n)$ / n (number of itérations)

Experimental results

PLEIADES pair



Disparity map computed by MARC2

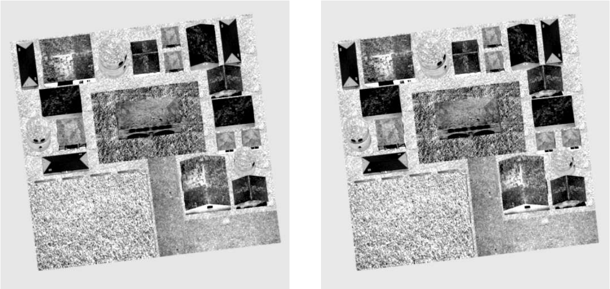
expérimental results

	A_1 , A_2	Frequencies Magnitudes	**0,0048672** **0,008112** **0,25** **0,25**
Case 1 (E_1)	A_1 A_2	Frequencies Magnitudes Frequencies Magnitudes	0,00097 0,00879 0,0205 0,0209 0,018 0,042 0,039 0,016 0,0015 0,0044 **0,0049** 0,0068 **0,0073** 0,100 0,093 **0,40** 0,043 **0,19**

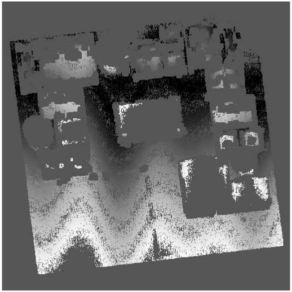
Measured mivrovibrations

The method does not permit to keep the two microvibrations separate.

Experimental results



Village pair



Disparity map
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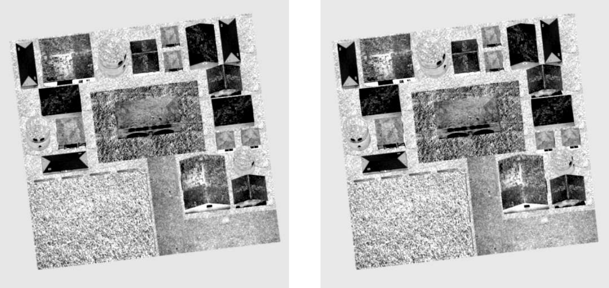
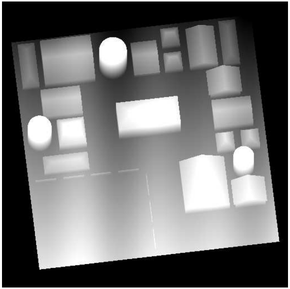
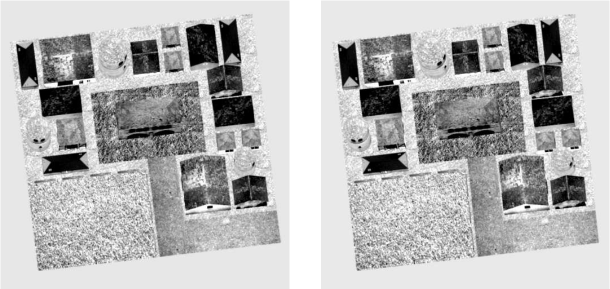


Village pair

Simulated disparity map

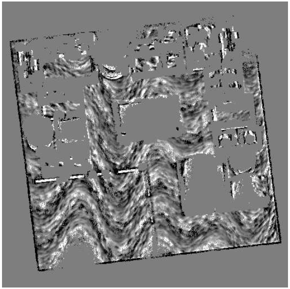


Experimental results



Village pair

Difference between the computed map and the simulated map



Experimental results

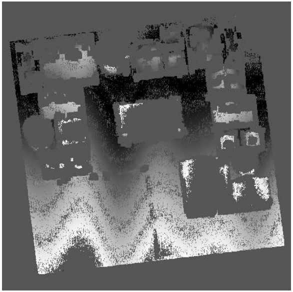
	A_1, A_2	Frequencies Magnitudes	**0,0048672** **0,25**	**0,008112** **0,25**
Case 1 (E_1)	A_1	Frequencies Magnitudes	**0,0049** **0,1523**	0,0098 0,0024
	A_2	Frequencies Magnitudes	0,0039 0,0001	**0,0068** **0,3306**

Experimental results

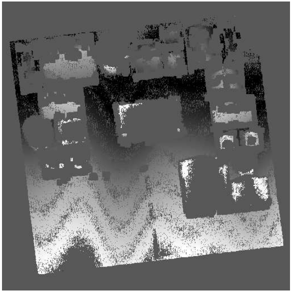
	A_1, A_2	Frequencies	**0,0048672**	**0,008112**	
		Magnitudes	**0,25**	**0,25**	
Case 1 (E_1)	A_1	Frequencies	**0,0049**	0,0098	
		Magnitudes	**0,1523**	0,0024	
	A_2	Frequencies	0,0039	**0,0068**	
		Magnitudes	0,0001	**0,3306**	
Case 2 (E_2)	A_1	Frequencies	**0,0049**	**0,0069**	
		Magnitudes	**0,066**	**0,16**	
	A_2	Frequencies	0,0039	**0,0049**	**0,0068**
		Magnitudes	0,0025	**0,064**	**0,16**

Experimental results

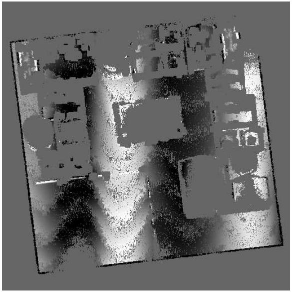
	A_1 , A_2	Frequencies	**0,0048672**	**0,008112**	
		Magnitudes	**0,25**	**0,25**	
Case 1 (E_1)	A_1	Frequencies	**0,0049**	0,0098	
		Magnitudes	**0,1523**	0,0024	
	A_2	Frequencies	0,0039	**0,0068**	
		Magnitudes	0,0001	**0,3306**	
Case 2 (E_2)	A_1	Frequencies	**0,0049**	**0,0069**	
		Magnitudes	**0,066**	**0,16**	
	A_2	Frequencies	0,0039	**0,0049**	**0,0068**
		Magnitudes	0,0025	**0,064**	**0,16**
Case 3 (E_3)	A_1 , A_2	Frequencies	**0,0049**	**0,0068**	
		Magnitudes	**0,066**	**0,16**	



Disparity computed by MARC2



Minimizer of E_3



Difference between the minimizer and the ground truth

Correction of the magnitudes

The ℓ^1 criterion lessen the magnitude of microvibrations
$\Rightarrow$ Approximate the ℓ^0 criterion.

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$\Rightarrow$ Approximate the ℓ^0 criterion.

Two methods have been tested

- **A contrario** method

PLEIADES pair	A_1, A_2	Frequencies Magnitudes	**0,0048672** **0,008112** **0,25** **0,25**
A contrario method	A_1	Frequencies Magnitudes	0,00097 0,0014 0,0044 0,0049 361,36 141,85 8,68 8,65
	A_2	Frequencies Magnitudes	0,00097 0,0014 0,0044 0,0049 361,36 141,83 8,65 8,61

The a contrario method detects low frequency outliers that arise from the relief (the white noise hypothesis is false).

Correction of the magnitudes

The ℓ^1 criterion lessen the magnitude of microvibrations

$\Rightarrow$ Approximate the ℓ^0 criterion.

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- **A contrario** method
- Change the soft thresholding into a **hard thresholding**

Village pair	A_1, A_2	Frequencies Magnitudes	**0,0048672** **0,25**	**0,008112** **0,25**
Case 3 (E_3)	A_1, A_2	Frequencies Magnitudes	**0,0049** **0,30**	**0,0068** **0,34**

The magnitude of microvibrations is no longer lessen. But the measure is still disrupted by the relief.

Correction of the magnitudes

The ℓ^1 criterion lessen the magnitude of microvibrations

$\Rightarrow$ Approximate the ℓ^0 criterion.

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- Change the soft thresholding into a **hard thresholding**

Village pair	A_1, A_2	Frequencies Magnitudes	**0,0048672** **0,25**	**0,008112** **0,25**
Case 3 (E_3)	A_1, A_2	Frequencies Magnitudes	**0,0049** **0,30**	**0,0068** **0,34**

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$\Rightarrow$ **Insert the relief into the model**

New method with regularization on the relief

One assumes that the relief is affine on each connected component of the disparity map.

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case 1 . Minimise

$$E_4(A_1, A_2, H) = \frac{1}{2}\|D - H - L_1 A_1 + L_2 A_2\|_2^2 + \lambda_1 \|A_1\|_1 + \lambda_2 \|A_2\|_1 + \lambda_4 \|\nabla^2 H\|_1,$$

where $\nabla^2 H$ denotes the second derivative of H.

New method with regularization on the relief

One assumes that the relief is affine on each connected component of the disparity map.

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$$E_4(A_1, A_2, H) = \frac{1}{2}\|D - H - L_1 A_1 + L_2 A_2\|_2^2 + \lambda_1\|A_1\|_1 + \lambda_2\|A_2\|_1 + \lambda_4\|\nabla^2 H\|_1,$$

where $\nabla^2 H$ denotes the second derivative of H.

case 2 . Minimise

$$E_5(A_1, A_2, H) = E_4(A_1, A_2, H) + \lambda_3\|A_1 - S_\phi A_2\|_2^2,$$

ou

$$E_6(A, H) = E_4(A, S_\phi A, H).$$

Minimization

Alternating minimization scheme

$$(A_1, A_2)^{(n+1)} = \arg\min_{A_1, A_2} \frac{1}{2}\|D - H^{(n)} - L_1 A_1 + L_2 A_2\|_2^2 + \lambda_1 \|A_1\|_1 + \lambda_2 \|A_2\|_1$$

$$H^{(n+1)} = \arg\min_{H} \frac{1}{2}\|D - H - L_1 A_1^{(n+1)} + L_2 A_2^{(n+1)}\|_2^2 + \lambda_4 \|\nabla^2 H\|_1$$

Minimization

Alternating minimization scheme

$$
\begin{aligned}
&(A_1, A_2)^{(n+1)} \\
= \quad & \arg\min_{A_1, A_2} \frac{1}{2}\|D - H^{(n)} - L_1 A_1 + L_2 A_2\|_2^2 + \lambda_1\|A_1\|_1 + \lambda_2\|A_2\|_1
\end{aligned}
$$

$$
\begin{aligned}
&H^{(n+1)} \\
= \quad & \arg\min_{H} \frac{1}{2}\|D - H - L_1 A_1^{(n+1)} + L_2 A_2^{(n+1)}\|_2^2 + \lambda_4\|\nabla^2 H\|_1
\end{aligned}
$$

The first minimization is performed by **FISTA**,

Minimization

Alternating minimization scheme

$$
\begin{aligned}
&(A_1, A_2)^{(n+1)} \\
= \quad & \arg\min_{A_1, A_2} \frac{1}{2}\|D - H^{(n)} - L_1 A_1 + L_2 A_2\|_2^2 + \lambda_1 \|A_1\|_1 + \lambda_2 \|A_2\|_1
\end{aligned}
$$

$$
\begin{aligned}
&H^{(n+1)} \\
= \quad & \arg\min_{H} \frac{1}{2}\|D - H - L_1 A_1^{(n+1)} + L_2 A_2^{(n+1)}\|_2^2 + \lambda_4 \|\nabla^2 H\|_1
\end{aligned}
$$

The first minimization is performed by **FISTA**,
The seconde minimization is performed by a **dual method**.

$$H^{(n+1)} = U - P_K(U)$$

where $U = D - L_1 A_1^{(n+1)} + L_2 A_2^{(n+1)}$ and P_K is the projection onto

$$K = \left\{ (\nabla^2)^* V \ : \ V \in (\mathbb{R}^N)^4 \text{ et } V^l[k] \in [-\lambda_4, \lambda_4], \ \forall k, \forall l \in \{1, \cdots, 4\} \right\}.$$

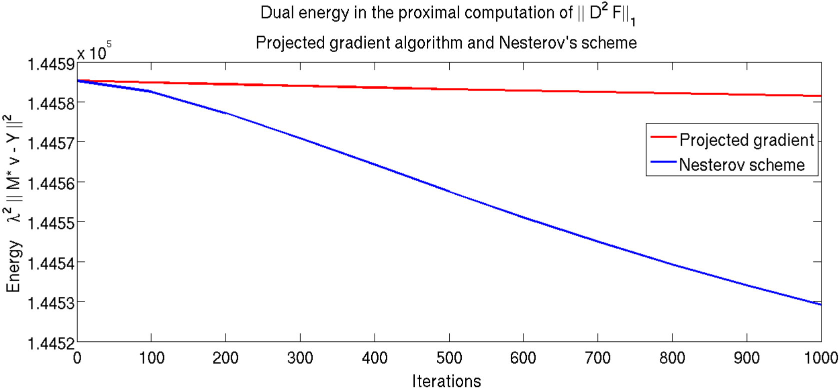
Minimisation

Computation of the projection P_K by

- **Projected gradient**

Computation of the projection P_K by

- **Projected gradient**
- **Nesterov's scheme**

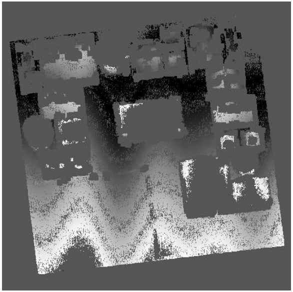


Case 1. Two vibration mods

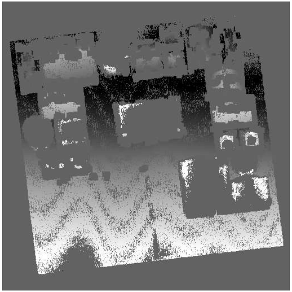
	A_1, A_2	Frequencies	**0,0048672**	**0,008112**	
		Magnitudes	**0,25**	**0,25**	
E_1	A_1	Frequencies	**0,0049**	0,0098	
		Magnitudes	**0,1523**	0,0024	
	A_2	Frequencies	0,0039	**0,0068**	
		Magnitudes	0,0001	**0,3306**	
E_4	A_1	Frequencies	**0.0049**	**0.0078**	**0.0088**
		Magnitudes	**0.2274**	**0.3153**	**0.1108**
	A_2	Frequencies	**0.0049**		
		Magnitudes	**0.0668**		

Case 2. One vibration mod

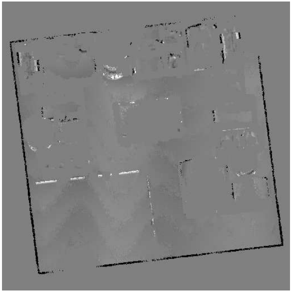
	A_1, A_2	Frequencies	**0,0048672**	**0,008112**	
		Magnitudes	**0,25**	**0,25**	
E_2	A_1	Frequencies	**0,0049**	**0,0069**	
		Magnitudes	**0,066**	**0,16**	
	A_2	Frequencies	0,0039	**0,0049**	**0,0068**
		Magnitudes	0,0025	**0,064**	**0,16**
E_5	A_1	Frequencies	**0.0049**	**0.0078**	**0.0088**
		Magnitudes	**0.21**	**0.17**	**0.08**
	A_2	Frequencies	**0.0048**	**0.0078**	**0.0088**
		Magnitudes	**0.21**	**0.17**	**0.08**



Disparity computed by MARC2



Minimizer of E_5



Difference between the minimizer and the ground truth