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Myocardial Blood Flow Quantification via dynamic PET

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joint work with
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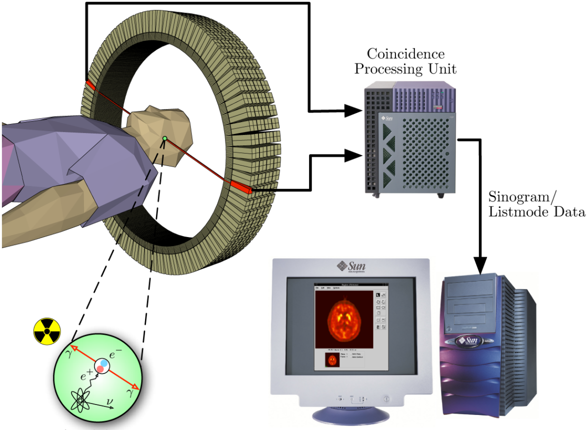
› Positron Emission Tomography



› Inverse Problem in Positron Emission Tomography

- Basic problem:

$$\wp(Ku) = f$$

- f measurements outside the body
- u unkown tracer distribution inside the body
- K PET operator
- Ill-posed inverse problem

› Dynamic Positron Emission Tomography

- In a standard reconstruction process the events during a certain time period are collected and stored into temporal bins
- The amount of events collected, and hence the half-life of the particular tracer, clearly affects the size of the temporal bins
- For myocardial blood flow $H_2{}^{15}O$ is used
  - highly diffusible
  - short half-life
- Each temporal bin has bad statistics (cf. [3] for dealing with similar problems in SPECT)

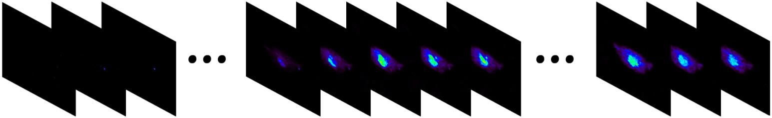
› Dynamic Positron Emission Tomography

- For each temporal bin $f(s,\theta,t)$ an image $u(x,t)$ is computed as the minimizer of $\mathrm{KL}(u)$ with

$$\mathrm{KL}(u) := \int_0^T \int_\Sigma f \log\left(\frac{f}{Ku}\right) + Ku - f \, ds \, d\theta \, dt$$

- $\mathrm{KL}(f)$ is (usually) minimized via the standard EM (Expectation Maximization) algorithm, i.e.

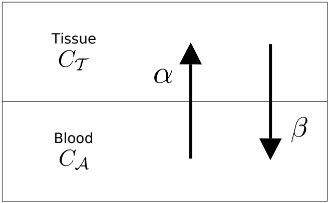
$$u_{k+1} = u_k \frac{K^*}{K^* \mathbf{1}} \left(\frac{f}{Ku_k}\right)$$



› Kinetic Modeling

- Drawback: Every frame $f(x,t)$ for a particular time-step t is computed independently $\Rightarrow$ Temporal correlation among the datasets (the bins) is neglected
- The power of PET: The particular tracer interacts with the body's molecules
- In our case: Exchange of radioactive water between the blood pool and tissue
- This interaction can be modelled via simple mathematical equations
- Spatial regions (Compartments) can be specified, e.g. a single voxel, in which these physiological processes are assumed to take place and for which we can apply the modelling (cf. [5])

› One Tissue Compartment Model



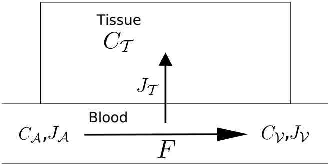
$$\frac{dC_\mathcal{T}}{dt} = \alpha C_\mathcal{A}(t) - \beta C_\mathcal{T}(t)$$

$$C_\mathcal{T}(0) = 0$$

leads to

$$C_\mathcal{T}(t) = \alpha \int_0^t C_\mathcal{A}(\tau) e^{-\beta(t-\tau)}\, d\tau$$

› One Tissue Compartment Model with Flow



Tracer flux: $J_{\{\mathcal{A},\mathcal{T},\mathcal{V}\}} = F \cdot C_{\{\mathcal{A},\mathcal{T},\mathcal{V}\}}$

Use Fick's principle: $J_{\mathcal{A}}(t) = J_{\mathcal{T}}(t) + J_{\mathcal{V}}(t)$

$$J_{\mathcal{T}}(t) = \frac{\mathrm{d}C_{\mathcal{T}}}{\mathrm{d}t} = J_{\mathcal{A}}(t) - J_{\mathcal{V}}(t) = F\big(C_{\mathcal{A}}(t) - C_{\mathcal{V}}(t)\big)$$

Highly diffusible tracer: partition coefficient $\lambda = \frac{C_{\mathcal{T}}}{C_{\mathcal{V}}}$

Leading to

$$\frac{\mathrm{d}C_{\mathcal{T}}(t)}{\mathrm{d}t} = F\left(C_{\mathcal{A}}(t) - \frac{C_{\mathcal{T}}(t)}{\lambda}\right)$$

with

$$C_{\mathcal{T}}(t) = F\int_0^t C_{\mathcal{A}}(\tau)\mathrm{e}^{-\frac{F}{\lambda}(t-\tau)}\,\mathrm{d}\tau$$

› Tissue Fraction and Spillover

Problem: Exact determination of myocardial tissue not possible

- low resolution & heart motion

Solution: Estimate larger region surely containing whole myocardial region

- Incorporate tissue fraction and spillover effects (cf [2])

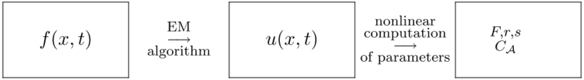
$$G(F, r, s, C_{\mathcal{A}}) = r(x)F(x) \int_0^t C_{\mathcal{A}}(\tau) \mathrm{e}^{-\frac{F(x)}{\lambda}(t-\tau)} \, \mathrm{d}\tau + s(x) C_{\mathcal{A}}(t)$$

Inverse Problem:

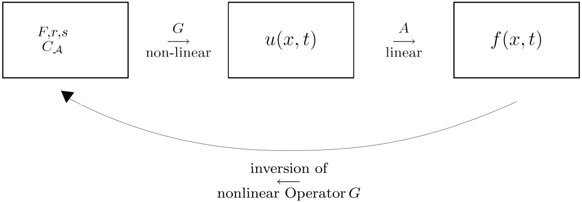
$$\wp(KG(F, r, s, C_{\mathcal{A}})) = f$$

› Inverse Problem of Perfusion Quantification

Previous myocardial perfusion quantification



Myocardial perfusion quantification as an inverse problem

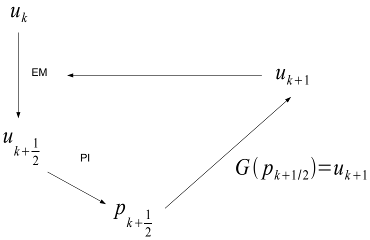


› Variational Model

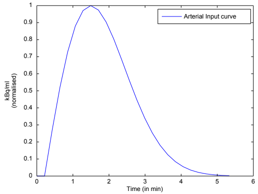
- Regularization is needed due to the ill-posedness

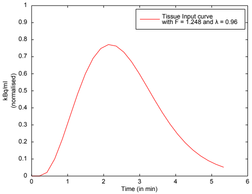
$$KL(u) + \mathcal{R}(p) \to \min_p \quad \text{subject to } u(x,t) = G(p)$$

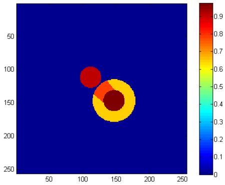
- Advantage: Each parameter can be regularized independently



› Exact Data

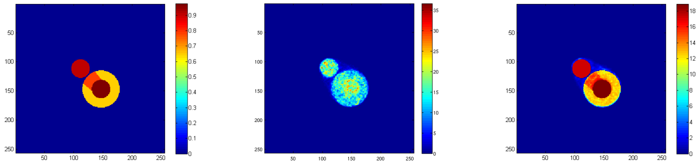






- $F = 1.248 \frac{\text{ml}}{\text{min mg}}$
- $\lambda = 0.96$
- $r = 0.65$
- $s = 0.21$

› Results Exact Data Transformed to Synthetic Data



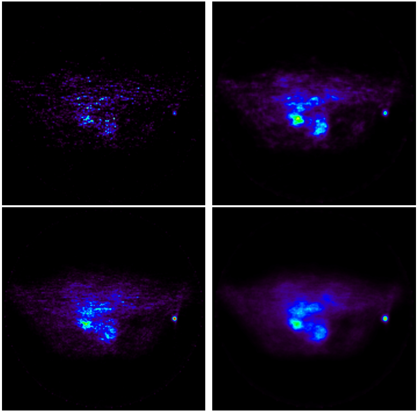
(a) Exact data

(b) EM reconstruction

(c) G(p) with computed optimal parameters

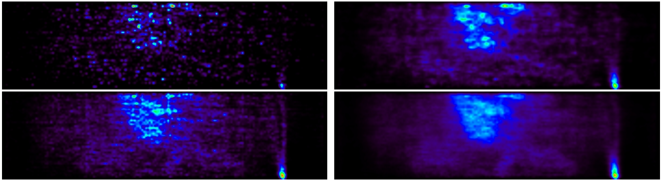
› 4D Results - Real Data

Transversal

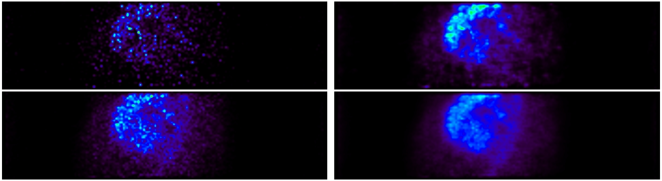


› 4D Results - Real Data

Coronal



Sagittal

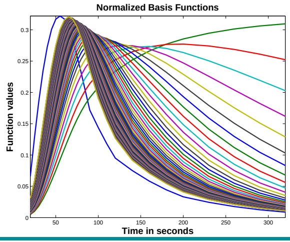


› Linearized Approximation

Define the operator following ideas from A.J. Reader [4]:

$$\big(B(a, C_{\mathcal{A}})\big)(x,t) := \sum_{i=1}^{n} a_i(x)\tilde{b}_i(t)$$

with $n \in \mathbb{N}$, $b_i \in \mathbb{R}_{\geq 0}$, a coefficient vector $\big(a_i(x)\big)_{i \in \{1,\dots,n\}}$ and basis functions $\tilde{b}_i(t) = \int_0^t C_{\mathcal{A}}(\tau) e^{-b_i(t-\tau)}\, d\tau$



› Linearized Inverse Problem

- B is linear in both a and $C_{\mathcal{A}}$
- Usually both parameters are unknown, there might not be a unique solution
- For simplicity we assume the arterial input function to be known
- We have to solve the following inverse problem

$$\wp(KBa) = f$$

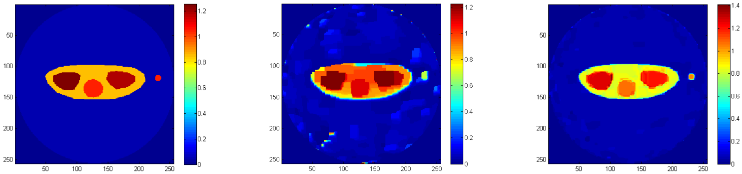
› Linearized Inverse Problem

- We include $\tilde{b}_0(t) := C_{\mathcal{A}}$ with $a_0(x)$ being the corresponding spillover coefficient
- We want a regularization that promotes a sparse solution for each pixel x ($a_n(x)$ should become sparse)
- Ideally we want to obtain a_0 to recover arterial spillover and one a_j to recover a particular perfusion and tissue fraction [1]
- Compare operators G and B:

$$G(F, r, s, C_{\mathcal{A}}) = r(x)F(x)\int_0^t C_{\mathcal{A}}(\tau)e^{-\frac{F(x)}{\lambda}(t-\tau)}\,d\tau + s(x)C_{\mathcal{A}}(t)$$

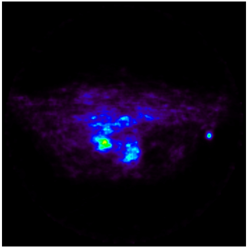
$$Ba = a_j(x)\tilde{b}_j(t) + a_0(x)\tilde{b}_0(t)$$

› Results - synthetic

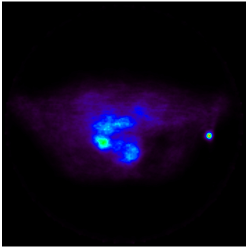


(d) Exact data (e) Static EM (f) Dynamic

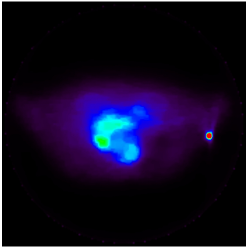
› Results - real data



(g) Standard EM



(h) Full 4D EM



(i) Sparsity based

Thank you for your attention!

Acknowledgements

http://imaging.uni-muenster.de/

http://www.herzforscher.de/

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